

# Computer algebra independent integration tests

1-Algebraic-functions/1.2-Trinomial-products/1.2.3-General/1.2.3.3-d+e-x^n-q-  
a+b-x^n+c-x^-2-n-p

Nasser M. Abbasi

May 14, 2020

Compiled on May 14, 2020 at 12:42am

## Contents

<b>1</b>	<b>Introduction</b>	<b>7</b>
1.1	Listing of CAS systems tested . . . . .	7
1.2	Results . . . . .	8
1.3	Performance . . . . .	11
1.4	list of integrals that has no closed form antiderivative . . . . .	12
1.5	list of integrals solved by CAS but has no known antiderivative . . . . .	12
1.6	list of integrals solved by CAS but failed verification . . . . .	12
1.7	Timing . . . . .	13
1.8	Verification . . . . .	13
1.9	Important notes about some of the results . . . . .	13
1.10	Design of the test system . . . . .	15
<b>2</b>	<b>detailed summary tables of results</b>	<b>17</b>
2.1	List of integrals sorted by grade for each CAS . . . . .	17
2.2	Detailed conclusion table per each integral for all CAS systems . . . . .	19
2.3	Detailed conclusion table specific for Rubi results . . . . .	38
<b>3</b>	<b>Listing of integrals</b>	<b>43</b>
3.1	$\int \frac{d+ex^3}{a+cx^6} dx$ . . . . .	43
3.2	$\int \frac{d+ex^3}{a-cx^6} dx$ . . . . .	50

3.3	$\int \frac{d+ex^4}{a+cx^8} dx$	57
3.4	$\int \frac{d+ex^4}{a-cx^8} dx$	64
3.5	$\int \frac{d+ex^4}{d^2+bx^4+e^2x^8} dx$	71
3.6	$\int \frac{d+ex^4}{d^2+fx^4+e^2x^8} dx$	78
3.7	$\int \frac{d+ex^4}{d^2-bx^4+e^2x^8} dx$	85
3.8	$\int \frac{d+ex^4}{d^2-fx^4+e^2x^8} dx$	91
3.9	$\int \frac{1+x^4}{1+bx^4+x^8} dx$	98
3.10	$\int \frac{1+x^4}{1+3x^4+x^8} dx$	104
3.11	$\int \frac{1+x^4}{1+2x^4+x^8} dx$	110
3.12	$\int \frac{1+x^4}{1+x^4+x^8} dx$	115
3.13	$\int \frac{1+x^4}{1+x^8} dx$	120
3.14	$\int \frac{1+x^4}{1-x^4+x^8} dx$	126
3.15	$\int \frac{1+x^4}{1-2x^4+x^8} dx$	131
3.16	$\int \frac{1+x^4}{1-3x^4+x^8} dx$	135
3.17	$\int \frac{1+x^4}{1-4x^4+x^8} dx$	139
3.18	$\int \frac{1+x^4}{1-5x^4+x^8} dx$	143
3.19	$\int \frac{1+x^4}{1-6x^4+x^8} dx$	147
3.20	$\int \frac{1-x^4}{1+bx^4+x^8} dx$	151
3.21	$\int \frac{1-x^4}{1+3x^4+x^8} dx$	157
3.22	$\int \frac{1-x^4}{1+2x^4+x^8} dx$	163
3.23	$\int \frac{1-x^4}{1+x^4+x^8} dx$	168
3.24	$\int \frac{1-x^4}{1+x^8} dx$	173
3.25	$\int \frac{1-x^4}{1-x^4+x^8} dx$	179
3.26	$\int \frac{1-x^4}{1-2x^4+x^8} dx$	184
3.27	$\int \frac{1-x^4}{1-3x^4+x^8} dx$	188
3.28	$\int \frac{1-x^4}{1-4x^4+x^8} dx$	193
3.29	$\int \frac{1-x^4}{1-5x^4+x^8} dx$	197
3.30	$\int \frac{1-x^4}{1-6x^4+x^8} dx$	201
3.31	$\int \frac{-1+\sqrt{3}+2x^4}{1-x^4+x^8} dx$	205
3.32	$\int \frac{1+(1+\sqrt{3})x^4}{1-x^4+x^8} dx$	210

3.33	$\int \frac{3-2\sqrt{3}+(-3+\sqrt{3})x^4}{1-x^4+x^8} dx$	215
3.34	$\int \frac{d+\frac{e}{x}}{c+\frac{a}{x^2}} dx$	220
3.35	$\int \frac{d+\frac{e}{x}}{c+\frac{a}{x^2}+\frac{b}{x}} dx$	224
3.36	$\int \frac{d+\frac{e}{x^2}}{c+\frac{a}{x^4}} dx$	229
3.37	$\int \frac{d+\frac{e}{x^2}}{c+\frac{a}{x^4}+\frac{b}{x^2}} dx$	235
3.38	$\int \frac{d+\frac{e}{x^3}}{c+\frac{a}{x^6}} dx$	241
3.39	$\int \frac{d+\frac{e}{x^3}}{c+\frac{a}{x^6}+\frac{b}{x^3}} dx$	249
3.40	$\int \frac{d+\frac{e}{x^4}}{c+\frac{a}{x^8}} dx$	255
3.41	$\int \frac{d+\frac{e}{x^4}}{c+\frac{a}{x^8}+\frac{b}{x^4}} dx$	263
3.42	$\int \frac{(d+ex^n)^3}{a+cx^{2n}} dx$	268
3.43	$\int \frac{(d+ex^n)^2}{a+cx^{2n}} dx$	272
3.44	$\int \frac{d+ex^n}{a+cx^{2n}} dx$	276
3.45	$\int \frac{1}{(d+ex^n)(a+cx^{2n})} dx$	280
3.46	$\int \frac{1}{(d+ex^n)^2(a+cx^{2n})} dx$	284
3.47	$\int \frac{d+ex^n}{a-cx^{2n}} dx$	288
3.48	$\int \frac{(d+ex^n)^3}{(a+cx^{2n})^2} dx$	292
3.49	$\int \frac{(d+ex^n)^2}{(a+cx^{2n})^2} dx$	297
3.50	$\int \frac{d+ex^n}{(a+cx^{2n})^2} dx$	302
3.51	$\int \frac{1}{(d+ex^n)(a+cx^{2n})^2} dx$	306
3.52	$\int \frac{1}{(d+ex^n)^2(a+cx^{2n})^2} dx$	311
3.53	$\int \frac{(d+ex^n)^3}{(a+cx^{2n})^3} dx$	316
3.54	$\int \frac{(d+ex^n)^2}{(a+cx^{2n})^3} dx$	321
3.55	$\int \frac{d+ex^n}{(a+cx^{2n})^3} dx$	326

3.56	$\int \frac{1}{(d+ex^n)(a+cx^{2n})^3} dx$	330
3.57	$\int \frac{1}{(d+ex^n)^2(a+cx^{2n})^3} dx$	335
3.58	$\int \frac{1}{(d+ex^n)\sqrt{a+cx^{2n}}} dx$	341
3.59	$\int (d+ex^n)^q (a+cx^{2n})^p dx$	345
3.60	$\int (d+ex^n)^3 (a+cx^{2n})^p dx$	348
3.61	$\int (d+ex^n)^2 (a+cx^{2n})^p dx$	353
3.62	$\int (d+ex^n) (a+cx^{2n})^p dx$	357
3.63	$\int \frac{(a+cx^{2n})^p}{d+ex^n} dx$	361
3.64	$\int \frac{(a+cx^{2n})^p}{(d+ex^n)^2} dx$	365
3.65	$\int \frac{(a+cx^{2n})^p}{(d+ex^n)^3} dx$	370
3.66	$\int (d+ex^n) (a+bx^n+cx^{2n}) dx$	375
3.67	$\int (d+ex^n) (a+bx^n+cx^{2n})^2 dx$	379
3.68	$\int (d+ex^n) (a+bx^n+cx^{2n})^3 dx$	385
3.69	$\int \frac{(d+ex^n)^3}{a+bx^n+cx^{2n}} dx$	396
3.70	$\int \frac{(d+ex^n)^2}{a+bx^n+cx^{2n}} dx$	400
3.71	$\int \frac{d+ex^n}{a+bx^n+cx^{2n}} dx$	404
3.72	$\int \frac{1}{(d+ex^n)(a+bx^n+cx^{2n})} dx$	408
3.73	$\int \frac{1}{(d+ex^n)^2(a+bx^n+cx^{2n})} dx$	412
3.74	$\int \frac{1}{(d+ex^n)^3(a+bx^n+cx^{2n})} dx$	417
3.75	$\int \frac{(d+ex^n)^3}{(a+bx^n+cx^{2n})^2} dx$	422
3.76	$\int \frac{(d+ex^n)^2}{(a+bx^n+cx^{2n})^2} dx$	428
3.77	$\int \frac{d+ex^n}{(a+bx^n+cx^{2n})^2} dx$	434
3.78	$\int \frac{1}{(d+ex^n)(a+bx^n+cx^{2n})^2} dx$	439
3.79	$\int \frac{1}{(d+ex^n)^2(a+bx^n+cx^{2n})^2} dx$	444
3.80	$\int \frac{(d+ex^n)^3}{(a+bx^n+cx^{2n})^3} dx$	450
3.81	$\int \frac{(d+ex^n)^2}{(a+bx^n+cx^{2n})^3} dx$	456
3.82	$\int \frac{d+ex^n}{(a+bx^n+cx^{2n})^3} dx$	462

3.83	$\int \frac{1}{(d+ex^n)(a+bx^n+cx^{2n})^3} dx$	467
3.84	$\int \frac{1}{(d+ex^n)^2(a+bx^n+cx^{2n})^3} dx$	475
3.85	$\int (d+ex^n) \sqrt{a+bx^n+cx^{2n}} dx$	485
3.86	$\int (d+ex^n) (a+bx^n+cx^{2n})^{3/2} dx$	490
3.87	$\int \frac{d+ex^n}{\sqrt{a+bx^n+cx^{2n}}} dx$	495
3.88	$\int \frac{d+ex^n}{(a+bx^n+cx^{2n})^{3/2}} dx$	500
3.89	$\int \frac{d+ex^n}{(a+bx^n+cx^{2n})^{5/2}} dx$	505
3.90	$\int (d+ex^n)^q (a+bx^n+cx^{2n})^p dx$	510
3.91	$\int (d+ex^n)^3 (a+bx^n+cx^{2n})^p dx$	513
3.92	$\int (d+ex^n)^2 (a+bx^n+cx^{2n})^p dx$	518
3.93	$\int (d+ex^n) (a+bx^n+cx^{2n})^p dx$	523
3.94	$\int \frac{(a+bx^n+cx^{2n})^p}{d+ex^n} dx$	528
3.95	$\int \frac{(a+bx^n+cx^{2n})^p}{(d+ex^n)^2} dx$	531
3.96	$\int \frac{(a+bx^n+cx^{2n})^p}{(d+ex^n)^3} dx$	534

#### 4 Listing of Grading functions

537



# Chapter 1

## Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from <https://rulebasedintegration.org>

The number of integrals in this report is [ 96 ]. This is test number [ 47 ].

### 1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.1 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12 on windows 10.
3. Maple 2020 (64 bit) on windows 10.
4. Maxima 5.43 on Linux. (via sagemath 8.9)
5. Fricas 1.3.6 on Linux (via sagemath 9.0)
6. Sympy 1.5 under Python 3.7.3 using Anaconda distribution.
7. Giac/Xcas 1.5 on Linux. (via sagemath 8.9)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

## 1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or Hypergeometric2F1 functions. RootSum and RootOf are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100. ( 96 )	% 0. ( 0 )
Mathematica	% 95.83 ( 92 )	% 4.17 ( 4 )
Maple	% 51.04 ( 49 )	% 48.96 ( 47 )
Maxima	% 9.38 ( 9 )	% 90.62 ( 87 )
Fricas	% 48.96 ( 47 )	% 51.04 ( 49 )
Sympy	% 45.83 ( 44 )	% 54.17 ( 52 )
Giac	% 35.42 ( 34 )	% 64.58 ( 62 )

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

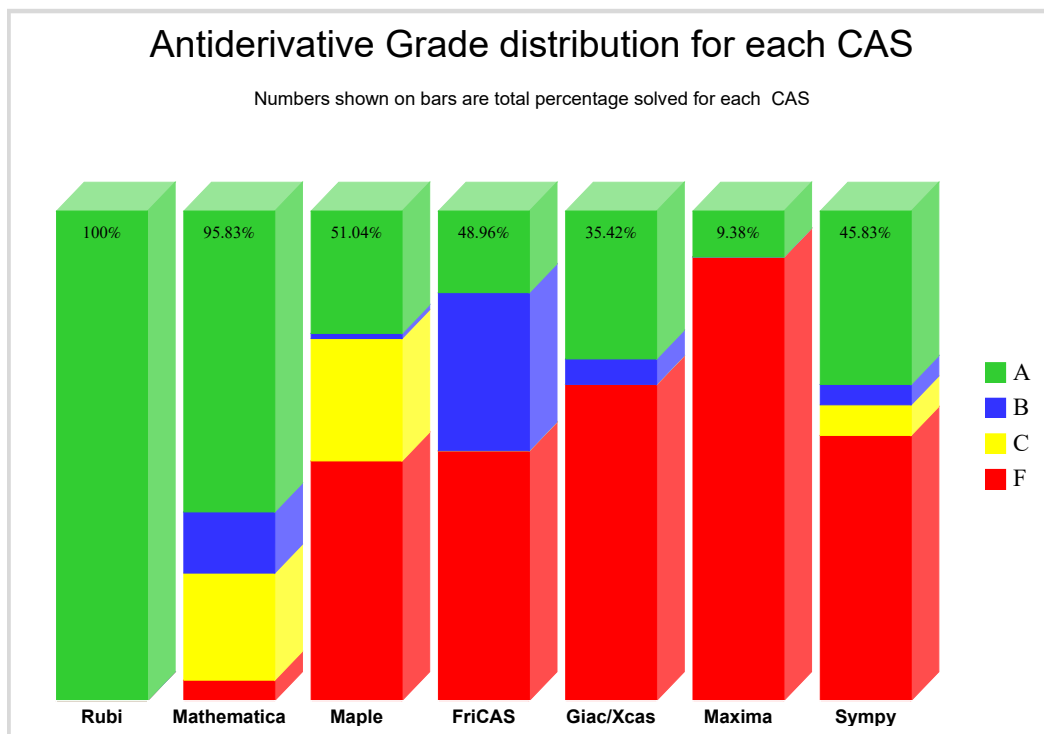


grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons <ol style="list-style-type: none"> <li>1. antiderivative contains a hypergeometric function and the optimal antiderivative does not.</li> <li>2. antiderivative contains a special function and the optimal antiderivative does not.</li> <li>3. antiderivative contains the imaginary unit and the optimal antiderivative does not.</li> </ol>
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

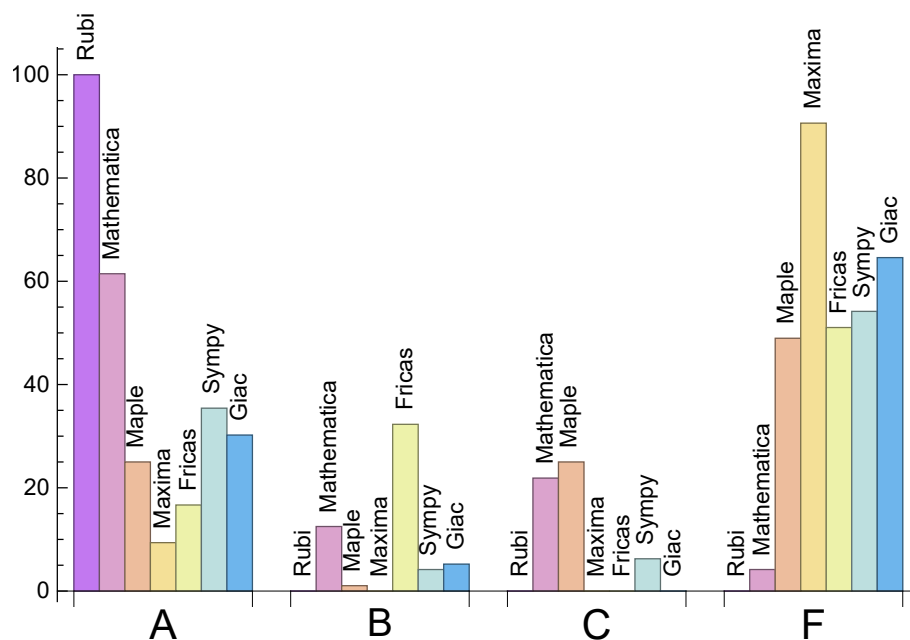
Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

System	% A grade	% B grade	% C grade	% F grade
Rubi	100.	0.	0.	0.
Mathematica	61.46	12.5	21.88	4.17
Maple	25.	1.04	25.	48.96
Maxima	9.38	0.	0.	90.62
Fricas	16.67	32.29	0.	51.04
Sympy	35.42	4.17	6.25	54.17
Giac	30.21	5.21	0.	64.58

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



### 1.3 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.64	361.82	0.95	288.	1.
Mathematica	1.06	1502.09	1.79	134.5	0.82
Maple	0.03	89.8	0.51	53.	0.25
Maxima	0.66	29.67	0.6	0.	0.
Fricas	1.56	2380.49	7.68	1161.	6.76
Sympy	5.86	404.73	2.46	124.	0.42
Giac	1.	354.97	1.73	198.	1.15

## 1.4 list of integrals that has no closed form antiderivative

{59, 90, 94, 95, 96}

## 1.5 list of integrals solved by CAS but has no known antiderivative

**Rubi** {}

**Mathematica** {}

**Maple** {}

**Maxima** {}

**Fricas** {}

**Sympy** {}

**Giac** {}

## 1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

**Rubi** {}

**Mathematica** {12, 23, 83, 84, 85, 86, 87, 88, 89, 91, 92, 93}

**Maple** Verification phase not implemented yet.

**Maxima** Verification phase not implemented yet.

**Fricas** Verification phase not implemented yet.

**Sympy** Verification phase not implemented yet.

**Giac** Verification phase not implemented yet.

## 1.7 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime')
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

## 1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

## 1.9 Important notes about some of the results

### 1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using `sagemath` (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 30 such integrals out of total 705, or about 4 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. If the output was an exception `ValueError` then this is most likely due to this reason.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See <https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/> for reference.

## 1.9.2 Important note about FriCAS and Giac/X-CAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

## 1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size is determined as follows.

For Fricas, Giac and Maxima (all called via `sagemath`) the following code is used

#see <https://stackoverflow.com/questions/25202346/how-to-obtain-leaf-count-expression-size-in>

```
def tree(expr):
    if expr.operator() is None:
        return expr
    else:
        return [expr.operator()+map(tree, expr.operands())

try:
    # 1.35 is a fudge factor since this estimate of leaf count is bit lower than
    #what it should be compared to Mathematica's
    leafCount = round(1.35*len(flatten(tree(anti))))
except Exception as ee:
    leafCount =1
```

For Sympy, called directly from Python, the following code is used

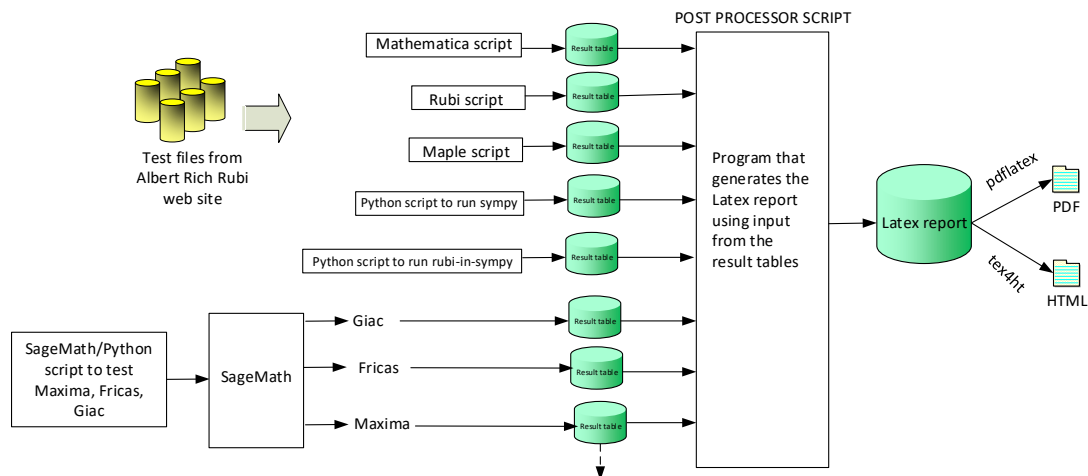
```
try:
    # 1.7 is a fudge factor since it is low side from actual leaf count
    leafCount = round(1.7*count_ops(anti))

except Exception as ee:
    leafCount =1
```

When these cas systems have a builtin function to find the leaf size of expressions, it will be used instead, and these tests run again.

## 1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. It contains 13 fields. This is description of each record (line)

1. integer, the problem number.
2. integer. 0 or 1 for failed or passed. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be "A", "B", "C", or "F"
13. String. The optimal antiderivative in CAS own syntax.

### High level overview of the CAS independent integration test build system

Nasser M. Abbasi  
June 22, 2018



# Chapter 2

## detailed summary tables of results

### 2.1 List of integrals sorted by grade for each CAS

#### 2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96 }

B grade: { }

C grade: { }

F grade: { }

#### 2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 11, 13, 15, 16, 19, 22, 24, 26, 27, 30, 34, 35, 36, 37, 38, 40, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 59, 60, 61, 62, 66, 67, 68, 69, 70, 71, 72, 73, 74, 85, 87, 88, 90, 91, 92, 93, 94, 95, 96 }

B grade: { 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 86, 89 }

C grade: { 5, 6, 7, 8, 9, 10, 12, 14, 17, 18, 20, 21, 23, 25, 28, 29, 31, 32, 33, 39, 41 }

F grade: { 58, 63, 64, 65 }

## 2.1.3 Maple

A grade: { 1, 2, 11, 12, 15, 16, 19, 22, 23, 26, 27, 30, 34, 35, 36, 38, 59, 66, 67, 68, 90, 94, 95, 96 }

B grade: { 37 }

C grade: { 3, 4, 5, 6, 7, 8, 9, 10, 13, 14, 17, 18, 20, 21, 24, 25, 28, 29, 31, 32, 33, 39, 40, 41 }

F grade: { 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 60, 61, 62, 63, 64, 65, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 91, 92, 93 }

## 2.1.4 Maxima

A grade: { 11, 15, 22, 26, 59, 90, 94, 95, 96 }

B grade: { }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 12, 13, 14, 16, 17, 18, 19, 20, 21, 23, 24, 25, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 91, 92, 93 }

## 2.1.5 FriCAS

A grade: { 11, 12, 14, 22, 23, 26, 31, 32, 33, 34, 35, 59, 90, 94, 95, 96 }

B grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 13, 15, 16, 17, 18, 19, 20, 21, 24, 25, 27, 28, 29, 30, 36, 37, 38, 40, 66, 67, 68 }

C grade: { }

F grade: { 39, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 60, 61, 62, 63, 64, 65, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 91, 92, 93 }

## 2.1.6 Sympy

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 24, 25, 27, 28, 29, 30, 31, 36, 38, 40, 66, 67, 68 }

B grade: { 26, 34, 35, 37 }

C grade: { 12, 23, 42, 43, 44, 47 }

F grade: { 32, 33, 39, 41, 45, 46, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96 }

## 2.1.7 Giac

A grade: { 1, 2, 3, 10, 11, 13, 14, 15, 16, 19, 21, 22, 24, 25, 27, 30, 31, 32, 33, 34, 35, 36, 38, 40, 59, 90, 94, 95, 96 }

B grade: { 4, 26, 66, 67, 68 }

C grade: { }

F grade: { 5, 6, 7, 8, 9, 12, 17, 18, 20, 23, 28, 29, 37, 39, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 60, 61, 62, 63, 64, 65, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 91, 92, 93 }

## 2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table, the column **normalized size** is defined as  $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	305	305	334	329	0	6589	165	397
normalized size	1	1.	1.1	1.08	0.	21.6	0.54	1.3
time (sec)	N/A	0.249	0.098	0.076	0.	2.608	2.139	1.124

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	323	323	337	386	0	6541	168	424
normalized size	1	1.	1.04	1.2	0.	20.25	0.52	1.31
time (sec)	N/A	0.189	0.11	0.12	0.	2.413	2.183	1.142

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	754	754	534	34	0	6978	199	811
normalized size	1	1.	0.71	0.05	0.	9.25	0.26	1.08
time (sec)	N/A	1.247	0.623	0.023	0.	3.366	12.622	1.189

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	329	329	425	39	0	6946	202	855
normalized size	1	1.	1.29	0.12	0.	21.11	0.61	2.6
time (sec)	N/A	0.209	0.136	0.015	0.	3.305	12.459	1.221

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	791	791	67	53	0	6279	136	0
normalized size	1	1.	0.08	0.07	0.	7.94	0.17	0.
time (sec)	N/A	0.863	0.045	0.059	0.	1.945	5.785	0.

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	791	791	67	53	0	6279	136	0
normalized size	1	1.	0.08	0.07	0.	7.94	0.17	0.
time (sec)	N/A	0.805	0.036	0.049	0.	1.856	5.415	0.

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	349	349	69	55	0	6279	136	0
normalized size	1	1.	0.2	0.16	0.	17.99	0.39	0.
time (sec)	N/A	0.422	0.045	0.031	0.	1.9	5.869	0.

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	751	751	69	55	0	6278	136	0
normalized size	1	1.	0.09	0.07	0.	8.36	0.18	0.
time (sec)	N/A	0.925	0.04	0.034	0.	1.948	5.335	0.

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	411	411	55	42	0	3723	75	0
normalized size	1	1.	0.13	0.1	0.	9.06	0.18	0.
time (sec)	N/A	0.292	0.027	0.052	0.	1.56	2.249	0.

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	451	451	55	42	0	3098	24	342
normalized size	1	1.	0.12	0.09	0.	6.87	0.05	0.76
time (sec)	N/A	0.407	0.014	0.007	0.	1.624	1.061	1.357

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	64	58	97	302	73	97
normalized size	1	1.	0.75	0.68	1.14	3.55	0.86	1.14
time (sec)	N/A	0.045	0.02	0.001	1.501	1.357	0.147	1.124

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	C	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	140	140	135	109	0	716	190	0
normalized size	1	1.	0.96	0.78	0.	5.11	1.36	0.
time (sec)	N/A	0.095	0.185	0.016	0.	1.439	0.693	0.

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	347	347	258	27	0	3092	19	333
normalized size	1	1.	0.74	0.08	0.	8.91	0.05	0.96
time (sec)	N/A	0.247	0.182	0.006	0.	1.509	1.124	1.179

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	331	331	55	42	0	1196	20	331
normalized size	1	1.	0.17	0.13	0.	3.61	0.06	1.
time (sec)	N/A	0.235	0.016	0.006	0.	1.462	1.23	1.156

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	31	42	36	123	26	39
normalized size	1	1.	1.15	1.56	1.33	4.56	0.96	1.44
time (sec)	N/A	0.008	0.014	0.011	1.5	1.231	0.142	1.096

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	131	96	0	815	49	198
normalized size	1	1.	1.	0.73	0.	6.22	0.37	1.51
time (sec)	N/A	0.086	0.08	0.029	0.	1.373	0.883	1.232

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	157	53	40	0	1062	24	0
normalized size	1	1.	0.34	0.25	0.	6.76	0.15	0.
time (sec)	N/A	0.087	0.013	0.008	0.	1.359	0.179	0.

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	171	171	55	42	0	1812	24	0
normalized size	1	1.	0.32	0.25	0.	10.6	0.14	0.
time (sec)	N/A	0.151	0.013	0.008	0.	1.453	0.19	0.

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	111	78	0	616	49	166
normalized size	1	1.	0.95	0.67	0.	5.26	0.42	1.42
time (sec)	N/A	0.057	0.045	0.051	0.	1.278	0.861	1.214

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	511	511	57	44	0	3725	76	0
normalized size	1	1.	0.11	0.09	0.	7.29	0.15	0.
time (sec)	N/A	0.359	0.025	0.003	0.	1.513	2.263	0.

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	411	411	57	44	0	2790	26	320
normalized size	1	1.	0.14	0.11	0.	6.79	0.06	0.78
time (sec)	N/A	0.321	0.014	0.006	0.	1.585	1.054	1.338

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	90	68	111	371	82	111
normalized size	1	1.	0.93	0.7	1.14	3.82	0.85	1.14
time (sec)	N/A	0.052	0.064	0.008	1.483	1.318	0.171	1.11

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	C	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	140	140	129	109	0	444	148	0
normalized size	1	1.	0.92	0.78	0.	3.17	1.06	0.
time (sec)	N/A	0.099	0.177	0.012	0.	1.365	0.612	0.

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	347	347	257	29	0	3092	20	333
normalized size	1	1.	0.74	0.08	0.	8.91	0.06	0.96
time (sec)	N/A	0.27	0.164	0.006	0.	1.469	1.121	1.197

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	355	355	57	44	0	2313	26	342
normalized size	1	1.	0.16	0.12	0.	6.52	0.07	0.96
time (sec)	N/A	0.277	0.015	0.006	0.	1.52	1.258	1.173

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	25	10	23	66	17	26
normalized size	1	1.	1.92	0.77	1.77	5.08	1.31	2.
time (sec)	N/A	0.005	0.005	0.003	1.479	1.302	0.123	1.128

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	129	110	0	905	51	198
normalized size	1	1.	1.	0.85	0.	7.02	0.4	1.53
time (sec)	N/A	0.118	0.077	0.02	0.	1.411	0.879	1.21



Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	165	55	42	0	1004	26	0
normalized size	1	1.	0.33	0.25	0.	6.08	0.16	0.
time (sec)	N/A	0.104	0.014	0.007	0.	1.417	0.182	0.

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	169	57	44	0	1751	26	0
normalized size	1	1.	0.34	0.26	0.	10.36	0.15	0.
time (sec)	N/A	0.142	0.013	0.006	0.	1.556	0.186	0.

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	114	90	0	686	51	182
normalized size	1	1.	0.91	0.72	0.	5.49	0.41	1.46
time (sec)	N/A	0.067	0.054	0.026	0.	1.347	0.868	1.205

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	71	47	0	315	136	144
normalized size	1	1.	0.53	0.35	0.	2.33	1.01	1.07
time (sec)	N/A	0.124	0.036	0.051	0.	1.408	1.003	1.162

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	164	164	72	62	0	355	0	166
normalized size	1	1.	0.44	0.38	0.	2.16	0.	1.01
time (sec)	N/A	0.095	0.037	0.039	0.	1.389	0.	1.142

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	180	180	89	62	0	436	0	177
normalized size	1	1.	0.49	0.34	0.	2.42	0.	0.98
time (sec)	N/A	0.122	0.046	0.01	0.	1.437	0.	1.16

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	49	43	0	238	112	58
normalized size	1	1.	1.	0.88	0.	4.86	2.29	1.18
time (sec)	N/A	0.03	0.023	0.007	0.	1.272	0.439	1.097

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	86	161	0	643	423	115
normalized size	1	1.	1.	1.87	0.	7.48	4.92	1.34
time (sec)	N/A	0.081	0.09	0.003	0.	1.299	1.268	1.101

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	253	253	293	266	0	1489	109	347
normalized size	1	1.	1.16	1.05	0.	5.89	0.43	1.37
time (sec)	N/A	0.211	0.106	0.01	0.	1.362	0.839	1.102

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	208	208	251	560	0	5042	428	0
normalized size	1	1.	1.21	2.69	0.	24.24	2.06	0.
time (sec)	N/A	0.543	0.191	0.027	0.	1.909	10.624	0.

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	311	311	346	334	0	6369	167	406
normalized size	1	1.	1.11	1.07	0.	20.48	0.54	1.31
time (sec)	N/A	0.29	0.115	0.044	0.	2.898	2.481	1.147

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	716	716	88	67	0	0	0	0
normalized size	1	1.	0.12	0.09	0.	0.	0.	0.
time (sec)	N/A	1.634	0.056	0.013	0.	0.	0.	0.

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	753	753	551	45	0	6888	204	873
normalized size	1	1.	0.73	0.06	0.	9.15	0.27	1.16
time (sec)	N/A	1.436	0.918	0.003	0.	5.356	18.942	1.229

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	433	433	88	67	0	0	0	0
normalized size	1	1.	0.2	0.15	0.	0.	0.	0.
time (sec)	N/A	0.989	0.078	0.005	0.	0.	0.	0.

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	127	0	0	0	337	0
normalized size	1	1.	0.9	0.	0.	0.	2.39	0.
time (sec)	N/A	0.145	0.247	0.078	0.	0.	10.355	0.

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	107	0	0	0	207	0
normalized size	1	1.	1.	0.	0.	0.	1.93	0.
time (sec)	N/A	0.097	0.151	0.062	0.	0.	7.577	0.

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	83	0	0	0	153	0
normalized size	1	1.	1.	0.	0.	0.	1.84	0.
time (sec)	N/A	0.027	0.039	0.045	0.	0.	5.797	0.

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	152	131	0	0	0	0	0
normalized size	1	1.	0.86	0.	0.	0.	0.	0.
time (sec)	N/A	0.111	0.144	0.079	0.	0.	0.	0.

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	205	205	186	0	0	0	0	0
normalized size	1	1.	0.91	0.	0.	0.	0.	0.
time (sec)	N/A	0.174	0.267	0.141	0.	0.	0.	0.

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	81	0	0	0	158	0
normalized size	1	1.	1.	0.	0.	0.	1.95	0.
time (sec)	N/A	0.028	0.061	0.043	0.	0.	6.497	0.

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	288	288	188	0	0	0	0	0
normalized size	1	1.	0.65	0.	0.	0.	0.	0.
time (sec)	N/A	0.252	0.282	0.072	0.	0.	0.	0.

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	203	203	136	0	0	0	0	0
normalized size	1	1.	0.67	0.	0.	0.	0.	0.
time (sec)	N/A	0.167	0.16	0.069	0.	0.	0.	0.

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	83	0	0	0	0	0
normalized size	1	1.	0.62	0.	0.	0.	0.	0.
time (sec)	N/A	0.054	0.042	0.066	0.	0.	0.	0.

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	333	333	227	0	0	0	0	0
normalized size	1	1.	0.68	0.	0.	0.	0.	0.
time (sec)	N/A	0.224	0.291	0.139	0.	0.	0.	0.

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	410	410	298	0	0	0	0	0
normalized size	1	1.	0.73	0.	0.	0.	0.	0.
time (sec)	N/A	0.379	0.503	0.146	0.	0.	0.	0.

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	424	424	188	0	0	0	0	0
normalized size	1	1.	0.44	0.	0.	0.	0.	0.
time (sec)	N/A	0.383	0.307	0.088	0.	0.	0.	0.

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	272	272	136	0	0	0	0	0
normalized size	1	1.	0.5	0.	0.	0.	0.	0.
time (sec)	N/A	0.247	0.183	0.081	0.	0.	0.	0.

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	184	184	83	0	0	0	0	0
normalized size	1	1.	0.45	0.	0.	0.	0.	0.
time (sec)	N/A	0.101	0.048	0.079	0.	0.	0.	0.

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	582	582	346	0	0	0	0	0
normalized size	1	1.	0.59	0.	0.	0.	0.	0.
time (sec)	N/A	0.416	0.464	0.17	0.	0.	0.	0.

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	701	701	426	0	0	0	0	0
normalized size	1	1.	0.61	0.	0.	0.	0.	0.
time (sec)	N/A	0.692	0.742	0.197	0.	0.	0.	0.

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	171	171	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.166	0.122	0.062	0.	0.	0.	0.

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.008	0.162	0.148	0.	0.	0.	0.

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	299	299	213	0	0	0	0	0
normalized size	1	1.	0.71	0.	0.	0.	0.	0.
time (sec)	N/A	0.16	0.2	0.112	0.	0.	0.	0.

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	217	217	171	0	0	0	0	0
normalized size	1	1.	0.79	0.	0.	0.	0.	0.
time (sec)	N/A	0.101	0.109	0.094	0.	0.	0.	0.

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	110	0	0	0	0	0
normalized size	1	1.	0.81	0.	0.	0.	0.	0.
time (sec)	N/A	0.06	0.05	0.093	0.	0.	0.	0.

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	167	167	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.142	0.081	0.092	0.	0.	0.	0.

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	261	261	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.235	0.103	0.076	0.	0.	0.	0.

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	357	357	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.338	0.294	0.086	0.	0.	0.	0.

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	57	66	0	315	656	279
normalized size	1	1.	0.92	1.06	0.	5.08	10.58	4.5
time (sec)	N/A	0.039	0.147	0.01	0.	1.43	1.402	1.137

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	123	138	0	1161	3128	1118
normalized size	1	1.	0.93	1.05	0.	8.8	23.7	8.47
time (sec)	N/A	0.102	0.255	0.013	0.	1.389	13.201	1.139



Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	218	218	205	226	0	2761	9190	2881
normalized size	1	1.	0.94	1.04	0.	12.67	42.16	13.22
time (sec)	N/A	0.201	0.442	0.014	0.	1.455	108.281	1.238

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	308	308	295	0	0	0	0	0
normalized size	1	1.	0.96	0.	0.	0.	0.	0.
time (sec)	N/A	0.699	0.876	0.052	0.	0.	0.	0.

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	224	224	216	0	0	0	0	0
normalized size	1	1.	0.96	0.	0.	0.	0.	0.
time (sec)	N/A	0.476	0.532	0.036	0.	0.	0.	0.

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	154	134	0	0	0	0	0
normalized size	1	1.	0.87	0.	0.	0.	0.	0.
time (sec)	N/A	0.12	0.075	0.021	0.	0.	0.	0.

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	243	243	200	0	0	0	0	0
normalized size	1	1.	0.82	0.	0.	0.	0.	0.
time (sec)	N/A	0.469	0.447	0.06	0.	0.	0.	0.

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	368	368	327	0	0	0	0	0
normalized size	1	1.	0.89	0.	0.	0.	0.	0.
time (sec)	N/A	0.707	0.941	0.136	0.	0.	0.	0.

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	552	552	509	0	0	0	0	0
normalized size	1	1.	0.92	0.	0.	0.	0.	0.
time (sec)	N/A	1.023	1.873	0.163	0.	0.	0.	0.

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	750	750	2462	0	0	0	0	0
normalized size	1	1.	3.28	0.	0.	0.	0.	0.
time (sec)	N/A	2.938	4.671	0.063	0.	0.	0.	0.

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	543	543	1835	0	0	0	0	0
normalized size	1	1.	3.38	0.	0.	0.	0.	0.
time (sec)	N/A	1.818	2.783	0.059	0.	0.	0.	0.

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	362	328	1381	0	0	0	0	0
normalized size	1	0.91	3.81	0.	0.	0.	0.	0.
time (sec)	N/A	0.63	1.768	0.053	0.	0.	0.	0.

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	726	726	8045	0	0	0	0	0
normalized size	1	1.	11.08	0.	0.	0.	0.	0.
time (sec)	N/A	1.927	7.273	0.174	0.	0.	0.	0.

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1129	1129	11657	0	0	0	0	0
normalized size	1	1.	10.33	0.	0.	0.	0.	0.
time (sec)	N/A	3.341	8.105	0.221	0.	0.	0.	0.

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1707	1707	9060	0	0	0	0	0
normalized size	1	1.	5.31	0.	0.	0.	0.	0.
time (sec)	N/A	5.253	7.752	0.104	0.	0.	0.	0.

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1191	1191	7402	0	0	0	0	0
normalized size	1	1.	6.21	0.	0.	0.	0.	0.
time (sec)	N/A	4.01	6.839	0.1	0.	0.	0.	0.

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	713	713	5848	0	0	0	0	0
normalized size	1	1.	8.2	0.	0.	0.	0.	0.
time (sec)	N/A	1.663	6.535	0.094	0.	0.	0.	0.

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	1708	1708	30258	0	0	0	0	0
normalized size	1	1.	17.72	0.	0.	0.	0.	0.
time (sec)	N/A	5.074	8.656	0.329	0.	0.	0.	0.

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	2446	2446	39502	0	0	0	0	0
normalized size	1	1.	16.15	0.	0.	0.	0.	0.
time (sec)	N/A	8.935	10.109	0.448	0.	0.	0.	0.

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	292	292	424	0	0	0	0	0
normalized size	1	1.	1.45	0.	0.	0.	0.	0.
time (sec)	N/A	0.355	1.595	0.062	0.	0.	0.	0.

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	294	294	690	0	0	0	0	0
normalized size	1	1.	2.35	0.	0.	0.	0.	0.
time (sec)	N/A	0.348	4.543	0.059	0.	0.	0.	0.

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	292	292	245	0	0	0	0	0
normalized size	1	1.	0.84	0.	0.	0.	0.	0.
time (sec)	N/A	0.338	0.349	0.018	0.	0.	0.	0.

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	298	298	414	0	0	0	0	0
normalized size	1	1.	1.39	0.	0.	0.	0.	0.
time (sec)	N/A	0.346	1.532	0.013	0.	0.	0.	0.

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	298	298	6648	0	0	0	0	0
normalized size	1	1.	22.31	0.	0.	0.	0.	0.
time (sec)	N/A	0.347	6.558	0.013	0.	0.	0.	0.

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.01	0.306	0.125	0.	0.	0.	0.

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	606	606	438	0	0	0	0	0
normalized size	1	1.	0.72	0.	0.	0.	0.	0.
time (sec)	N/A	0.622	1.503	0.099	0.	0.	0.	0.

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	447	447	338	0	0	0	0	0
normalized size	1	1.	0.76	0.	0.	0.	0.	0.
time (sec)	N/A	0.461	1.037	0.083	0.	0.	0.	0.

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	288	288	243	0	0	0	0	0
normalized size	1	1.	0.84	0.	0.	0.	0.	0.
time (sec)	N/A	0.287	0.688	0.085	0.	0.	0.	0.

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.011	0.159	0.085	0.	0.	0.	0.

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.011	0.197	0.059	0.	0.	0.	0.

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.012	0.71	0.074	0.	0.	0.	0.

## 2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio  $\frac{\text{number of rules}}{\text{integrand size}}$  is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [38] had the largest ratio of [ 0.5882 ]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	12	8	1.	17	0.471
2	A	13	7	1.	18	0.389
3	A	19	6	1.	17	0.353
4	A	13	10	1.	18	0.556
5	A	19	6	1.	26	0.231
6	A	19	6	1.	26	0.231
7	A	7	4	1.	27	0.148
8	A	19	6	1.	27	0.222
9	A	19	6	1.	18	0.333
10	A	19	7	1.	18	0.389
11	A	10	7	1.	18	0.389
12	A	19	6	1.	16	0.375
13	A	19	6	1.	13	0.462
14	A	19	6	1.	18	0.333
15	A	5	5	1.	18	0.278
16	A	7	4	1.	18	0.222
17	A	7	4	1.	18	0.222
18	A	7	4	1.	18	0.222
19	A	7	4	1.	18	0.222
20	A	19	6	1.	20	0.3
21	A	19	7	1.	20	0.35
22	A	11	8	1.	20	0.4
23	A	19	6	1.	18	0.333
24	A	19	6	1.	15	0.4
25	A	19	6	1.	20	0.3
26	A	5	5	1.	20	0.25
27	A	7	4	1.	20	0.2
28	A	7	4	1.	20	0.2
29	A	7	4	1.	20	0.2
30	A	7	4	1.	20	0.2
31	A	9	6	1.	25	0.24

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
32	A	9	6	1.	26	0.231
33	A	9	6	1.	33	0.182
34	A	5	5	1.	17	0.294
35	A	6	6	1.	22	0.273
36	A	11	8	1.	17	0.471
37	A	5	4	1.	22	0.182
38	A	14	10	1.	17	0.588
39	A	15	9	1.	22	0.409
40	A	21	8	1.	17	0.471
41	A	9	6	1.	22	0.273
42	A	5	4	1.	21	0.19
43	A	5	4	1.	21	0.19
44	A	3	3	1.	19	0.158
45	A	6	4	1.	21	0.19
46	A	7	4	1.	21	0.19
47	A	3	3	1.	20	0.15
48	A	9	5	1.	21	0.238
49	A	7	5	1.	21	0.238
50	A	4	4	1.	19	0.21
51	A	10	5	1.	21	0.238
52	A	11	5	1.	21	0.238
53	A	11	5	1.	21	0.238
54	A	8	5	1.	21	0.238
55	A	5	4	1.	19	0.21
56	A	15	5	1.	21	0.238
57	A	16	5	1.	21	0.238
58	A	6	5	1.	23	0.217
59	A	0	0	0.	0	0.
60	A	10	5	1.	21	0.238
61	A	8	5	1.	21	0.238
62	A	6	5	1.	19	0.263
63	A	6	5	1.	21	0.238

Continued on next page



Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
64	A	8	5	1.	21	0.238
65	A	10	5	1.	21	0.238
66	A	2	1	1.	22	0.045
67	A	2	1	1.	24	0.042
68	A	2	1	1.	24	0.042
69	A	5	3	1.	26	0.115
70	A	5	3	1.	26	0.115
71	A	3	2	1.	24	0.083
72	A	6	3	1.	26	0.115
73	A	7	3	1.	26	0.115
74	A	8	3	1.	26	0.115
75	A	9	4	1.	26	0.154
76	A	9	5	1.	26	0.192
77	A	4	3	0.91	24	0.125
78	A	10	4	1.	26	0.154
79	A	11	4	1.	26	0.154
80	A	11	4	1.	26	0.154
81	A	11	5	1.	26	0.192
82	A	5	3	1.	24	0.125
83	A	15	4	1.	26	0.154
84	A	16	4	1.	26	0.154
85	A	6	5	1.	26	0.192
86	A	6	5	1.	26	0.192
87	A	6	5	1.	26	0.192
88	A	6	5	1.	26	0.192
89	A	6	5	1.	26	0.192
90	A	0	0	0.	0	0.
91	A	10	5	1.	26	0.192
92	A	8	5	1.	26	0.192
93	A	6	5	1.	24	0.208
94	A	0	0	0.	0	0.
95	A	0	0	0.	0	0.

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
96	A	0	0	0.	0	0.

# Chapter 3

## Listing of integrals

### 3.1 $\int \frac{d+ex^3}{a+cx^6} dx$

**Optimal.** Leaf size=305

$$-\frac{(\sqrt{3}\sqrt{cd} - \sqrt{ae}) \log(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{cx} + \sqrt[3]{a} + \sqrt[3]{cx^2})}{12a^{5/6}c^{2/3}} + \frac{(\sqrt{ae} + \sqrt{3}\sqrt{cd}) \log(\sqrt{3}\sqrt[6]{a}\sqrt[6]{cx} + \sqrt[3]{a} + \sqrt[3]{cx^2})}{12a^{5/6}c^{2/3}} - \frac{(\sqrt{3}\sqrt{ae} + \sqrt{cd})}{6a^{5/6}}$$

```
[Out] (d*ArcTan[(c^(1/6)*x)/a^(1/6)])/(3*a^(5/6)*c^(1/6)) - ((Sqrt[c]*d + Sqrt[3]
*Sqrt[a]*e)*ArcTan[Sqrt[3] - (2*c^(1/6)*x)/a^(1/6)])/(6*a^(5/6)*c^(2/3)) +
((Sqrt[c]*d - Sqrt[3]*Sqrt[a]*e)*ArcTan[Sqrt[3] + (2*c^(1/6)*x)/a^(1/6)])/(
6*a^(5/6)*c^(2/3)) - (e*Log[a^(1/3) + c^(1/3)*x^2])/(6*a^(1/3)*c^(2/3)) - (
(Sqrt[3]*Sqrt[c]*d - Sqrt[a]*e)*Log[a^(1/3) - Sqrt[3]*a^(1/6)*c^(1/6)*x + c
^(1/3)*x^2])/(12*a^(5/6)*c^(2/3)) + ((Sqrt[3]*Sqrt[c]*d + Sqrt[a]*e)*Log[a^
(1/3) + Sqrt[3]*a^(1/6)*c^(1/6)*x + c^(1/3)*x^2])/(12*a^(5/6)*c^(2/3))
```

---

**Rubi [A]** time = 0.248568, antiderivative size = 305, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.471$ , Rules used = {1416, 635, 203, 260, 634, 617, 204, 628}

$$-\frac{(\sqrt{3}\sqrt{cd} - \sqrt{ae}) \log(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{cx} + \sqrt[3]{a} + \sqrt[3]{cx^2})}{12a^{5/6}c^{2/3}} + \frac{(\sqrt{ae} + \sqrt{3}\sqrt{cd}) \log(\sqrt{3}\sqrt[6]{a}\sqrt[6]{cx} + \sqrt[3]{a} + \sqrt[3]{cx^2})}{12a^{5/6}c^{2/3}} - \frac{(\sqrt{3}\sqrt{ae} + \sqrt{cd})}{6a^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^3)/(a + c*x^6), x]
```

```
[Out] (d*ArcTan[(c^(1/6)*x)/a^(1/6)]/(3*a^(5/6)*c^(1/6)) - ((Sqrt[c]*d + Sqrt[3]
*Sqrt[a]*e)*ArcTan[Sqrt[3] - (2*c^(1/6)*x)/a^(1/6)]/(6*a^(5/6)*c^(2/3)) +
((Sqrt[c]*d - Sqrt[3]*Sqrt[a]*e)*ArcTan[Sqrt[3] + (2*c^(1/6)*x)/a^(1/6)]/(
6*a^(5/6)*c^(2/3)) - (e*Log[a^(1/3) + c^(1/3)*x^2])/(6*a^(1/3)*c^(2/3)) - (
(Sqrt[3]*Sqrt[c]*d - Sqrt[a]*e)*Log[a^(1/3) - Sqrt[3]*a^(1/6)*c^(1/6)*x + c
^(1/3)*x^2])/(12*a^(5/6)*c^(2/3)) + ((Sqrt[3]*Sqrt[c]*d + Sqrt[a]*e)*Log[a^
(1/3) + Sqrt[3]*a^(1/6)*c^(1/6)*x + c^(1/3)*x^2])/(12*a^(5/6)*c^(2/3))
```

### Rule 1416

```
Int[((d_) + (e_.)*(x_)^3)/((a_) + (c_.)*(x_)^6), x_Symbol] := With[{q = Rt[
c/a, 6]}, Dist[1/(3*a*q^2), Int[(q^2*d - e*x)/(1 + q^2*x^2), x], x] + (Dist
[1/(6*a*q^2), Int[(2*q^2*d - (Sqrt[3]*q^3*d - e)*x)/(1 - Sqrt[3]*q*x + q^2*
x^2), x], x] + Dist[1/(6*a*q^2), Int[(2*q^2*d + (Sqrt[3]*q^3*d + e)*x)/(1 +
Sqrt[3]*q*x + q^2*x^2), x], x])) /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 +
a*e^2, 0] && PosQ[c/a]
```

### Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] := Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && !NiceSqrtQ[-(a*c)]
```

### Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

### Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

### Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

### Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
```

], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4\*a\*c]) /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

### Rule 204

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

### Rule 628

Int[((d\_) + (e\_)\*(x\_))/((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2), x\_Symbol] := Simp[(d\*Log[RemoveContent[a + b\*x + c\*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

### Rubi steps

$$\begin{aligned} \int \frac{d + ex^3}{a + cx^6} dx &= \frac{\int \frac{2\sqrt[3]{cd} - \left(\frac{\sqrt{3}\sqrt{cd} - e}{\sqrt{a}}\right)x}{1 - \frac{\sqrt{3}\sqrt[6]{cx}}{\sqrt[6]{a}} + \frac{\sqrt[3]{cx^2}}{\sqrt[3]{a}}} dx}{6a^{2/3}\sqrt[3]{c}} + \frac{\int \frac{2\sqrt[3]{cd} + \left(\frac{\sqrt{3}\sqrt{cd} + e}{\sqrt{a}}\right)x}{1 + \frac{\sqrt{3}\sqrt[6]{cx}}{\sqrt[6]{a}} + \frac{\sqrt[3]{cx^2}}{\sqrt[3]{a}}} dx}{6a^{2/3}\sqrt[3]{c}} + \frac{\int \frac{\sqrt[3]{cd} - ex}{1 + \frac{\sqrt[3]{cx^2}}{\sqrt[3]{a}}} dx}{3a^{2/3}\sqrt[3]{c}} \\ &= \frac{d \int \frac{1}{1 + \frac{\sqrt[3]{cx^2}}{\sqrt[3]{a}}} dx}{3a} - \frac{e \int \frac{x}{1 + \frac{\sqrt[3]{cx^2}}{\sqrt[3]{a}}} dx}{3a^{2/3}\sqrt[3]{c}} - \frac{(\sqrt{3}\sqrt{cd} - \sqrt{ae}) \int \frac{-\frac{\sqrt{3}\sqrt[6]{c}}{\sqrt[6]{a}} + \frac{2\sqrt[3]{cx}}{\sqrt[3]{a}}}{1 - \frac{\sqrt{3}\sqrt[6]{cx}}{\sqrt[6]{a}} + \frac{\sqrt[3]{cx^2}}{\sqrt[3]{a}}} dx}{12a^{5/6}c^{2/3}} + \frac{(\sqrt{3}\sqrt{cd} + \sqrt{ae}) \int \frac{\frac{\sqrt{3}\sqrt[6]{c}}{\sqrt[6]{a}} + \frac{2\sqrt[3]{cx}}{\sqrt[3]{a}}}{1 + \frac{\sqrt{3}\sqrt[6]{cx}}{\sqrt[6]{a}} + \frac{\sqrt[3]{cx^2}}{\sqrt[3]{a}}} dx}{12a^{5/6}c^{2/3}} \\ &= \frac{d \tan^{-1}\left(\frac{\sqrt[6]{cx}}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt[6]{c}} - \frac{e \log(\sqrt[3]{a} + \sqrt[3]{cx^2})}{6\sqrt[3]{ac^{2/3}}} - \frac{(\sqrt{3}\sqrt{cd} - \sqrt{ae}) \log(\sqrt[3]{a} - \sqrt{3}\sqrt[6]{a}\sqrt[6]{cx} + \sqrt[3]{cx^2})}{12a^{5/6}c^{2/3}} + \frac{(\sqrt{3}\sqrt{cd} + \sqrt{ae}) \log(\sqrt[3]{a} + \sqrt{3}\sqrt[6]{a}\sqrt[6]{cx} + \sqrt[3]{cx^2})}{12a^{5/6}c^{2/3}} \\ &= \frac{d \tan^{-1}\left(\frac{\sqrt[6]{cx}}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt[6]{c}} - \frac{(\sqrt{cd} + \sqrt{3}\sqrt{ae}) \tan^{-1}\left(\sqrt{3} - \frac{2\sqrt[6]{cx}}{\sqrt[6]{a}}\right)}{6a^{5/6}c^{2/3}} + \frac{(\sqrt{cd} - \sqrt{3}\sqrt{ae}) \tan^{-1}\left(\sqrt{3} + \frac{2\sqrt[6]{cx}}{\sqrt[6]{a}}\right)}{6a^{5/6}c^{2/3}} - \frac{e \log(\sqrt[3]{a} + \sqrt[3]{cx^2})}{6\sqrt[3]{ac^{2/3}}} \end{aligned}$$

**Mathematica [A]** time = 0.0981448, size = 334, normalized size = 1.1

$$\frac{(\sqrt{3}\sqrt[6]{a}\sqrt{cd} - a^{2/3}e) \log(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{cx} + \sqrt[3]{a} + \sqrt[3]{cx^2})}{12ac^{2/3}} - \frac{(-a^{2/3}e - \sqrt{3}\sqrt[6]{a}\sqrt{cd}) \log(\sqrt{3}\sqrt[6]{a}\sqrt[6]{cx} + \sqrt[3]{a} + \sqrt[3]{cx^2})}{12ac^{2/3}} + \frac{(\sqrt{3}a^{2/3})}{12ac^{2/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e\*x^3)/(a + c\*x^6),x]

[Out]  $(d \operatorname{ArcTan}[(c^{1/6}x)/a^{1/6}]) / (3a^{5/6}c^{1/6}) + ((a^{1/6} \operatorname{Sqrt}[c]d + \operatorname{Sqrt}[3]a^{2/3}e) \operatorname{ArcTan}[(-\operatorname{Sqrt}[3]a^{1/6}) + 2c^{1/6}x/a^{1/6}]) / (6a^{2/3}) + ((a^{1/6} \operatorname{Sqrt}[c]d - \operatorname{Sqrt}[3]a^{2/3}e) \operatorname{ArcTan}[(\operatorname{Sqrt}[3]a^{1/6}) + 2c^{1/6}x/a^{1/6}]) / (6a^{2/3}) - (e \operatorname{Log}[a^{1/3} + c^{1/3}x^2]) / (6a^{1/3}c^{2/3}) - ((\operatorname{Sqrt}[3]a^{1/6} \operatorname{Sqrt}[c]d - a^{2/3}e) \operatorname{Log}[a^{1/3} - \operatorname{Sqrt}[3]a^{1/6}c^{1/6}x + c^{1/3}x^2]) / (12a^{2/3}) - ((-\operatorname{Sqrt}[3]a^{1/6} \operatorname{Sqrt}[c]d - a^{2/3}e) \operatorname{Log}[a^{1/3} + \operatorname{Sqrt}[3]a^{1/6}c^{1/6}x + c^{1/3}x^2]) / (12a^{2/3})$

**Maple [A]** time = 0.076, size = 329, normalized size = 1.1

$$\frac{c\sqrt{3}d}{12a^2} \left(\frac{a}{c}\right)^{\frac{7}{6}} \ln\left(x^2 + \sqrt{3}\sqrt[6]{\frac{a}{c}}x + \sqrt[3]{\frac{a}{c}}\right) + \frac{e}{12a} \left(\frac{a}{c}\right)^{\frac{2}{3}} \ln\left(x^2 + \sqrt{3}\sqrt[6]{\frac{a}{c}}x + \sqrt[3]{\frac{a}{c}}\right) + \frac{d}{6a} \sqrt[6]{\frac{a}{c}} \arctan\left(2x\frac{1}{\sqrt[6]{\frac{a}{c}}} + \sqrt{3}\right) - \frac{\sqrt{3}e}{6a} \left(\frac{a}{c}\right)^{\frac{1}{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x^3+d)/(c\*x^6+a),x)

[Out]  $1/12*c*(a/c)^{7/6}/a^2*\ln(x^2+3^{1/2}*(a/c)^{1/6}*x+(a/c)^{1/3})*3^{1/2}*d+1/12*(a/c)^{2/3}/a*\ln(x^2+3^{1/2}*(a/c)^{1/6}*x+(a/c)^{1/3})*e+1/6*(a/c)^{1/6}/a*\arctan(2*x/(a/c)^{1/6}+3^{1/2})*d-1/6*(a/c)^{2/3}/a*\arctan(2*x/(a/c)^{1/6}+3^{1/2})*3^{1/2}*e+1/12/a*\ln(x^2-3^{1/2}*(a/c)^{1/6}*x+(a/c)^{1/3})*(a/c)^{2/3}*e-1/12/a*\ln(x^2-3^{1/2}*(a/c)^{1/6}*x+(a/c)^{1/3})*3^{1/2}*(a/c)^{1/6}*d+1/6/a*(a/c)^{2/3}*\arctan(2*x/(a/c)^{1/6}-3^{1/2})*3^{1/2}*e+1/6/a*(a/c)^{1/6}*\arctan(2*x/(a/c)^{1/6}-3^{1/2})*d-1/6*(a/c)^{2/3}/a*e*\ln(x^2+(a/c)^{1/3}))+1/3*(a/c)^{1/6}/a*d*\arctan(x/(a/c)^{1/6})$

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^3+d)/(c\*x^6+a),x, algorithm="maxima")

[Out] Exception raised: ValueError

---

**Fricas [B]** time = 2.60776, size = 6589, normalized size = 21.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^3+d)/(c\*x^6+a),x, algorithm="fricas")

[Out]  $\frac{1}{3}\sqrt{3}\left(\frac{a^2c^2\sqrt{-(c^2d^6-6acd^4e^2+9a^2d^2e^4)}}{a^5c^3} + 3cd^2e - ae^3\right)\sqrt[3]{\frac{1}{2}\arctan\left(\frac{2(\sqrt{3}(a^4c^4d^2 - a^5c^3e^2)\sqrt{-(c^2d^6-6acd^4e^2+9a^2d^2e^4)}}}{(a^5c^3)}\right) - 2\sqrt{3}(a^2c^3d^4e - 3a^3c^2d^2e^3)\sqrt{\left((c^3d^7 - ac^2d^5e^2 - 5a^2cd^3e^4 - 3a^3d^6e^6)\right)x^2 + (2a^5c^3d^3e\sqrt{-(c^2d^6-6acd^4e^2+9a^2d^2e^4)}}}{a^5c^3} + a^2c^3d^5 - 4a^3c^2d^3e^2 + 3a^4cd^4e^4)\left(\frac{a^2c^2\sqrt{-(c^2d^6-6acd^4e^2+9a^2d^2e^4)}}{a^5c^3} + 3cd^2e - ae^3\right)\sqrt[3]{\frac{1}{2}\arctan\left(\frac{1}{2}\sqrt{\frac{3}{c^3d^7 - ac^2d^5e^2 - 5a^2cd^3e^4 - 3a^3d^6e^6}}\right) + 3cd^2e - ae^3\sqrt{\frac{1}{2}\sqrt{\frac{3}{c^3d^7 - ac^2d^5e^2 - 5a^2cd^3e^4 - 3a^3d^6e^6}}}\right)\sqrt[3]{\frac{1}{2}\arctan\left(\frac{1}{2}\sqrt{\frac{3}{c^3d^7 - ac^2d^5e^2 - 5a^2cd^3e^4 - 3a^3d^6e^6}}\right) + 3cd^2e - ae^3\sqrt{\frac{1}{2}\sqrt{\frac{3}{c^3d^7 - ac^2d^5e^2 - 5a^2cd^3e^4 - 3a^3d^6e^6}}}\right) - 2(\sqrt{3}(a^4c^4d^2 - a^5c^3e^2)x\sqrt{-(c^2d^6-6acd^4e^2+9a^2d^2e^4)}}}{a^5c^3} - 2\sqrt{3}(a^2c^3d^4e - 3a^3c^2d^2e^3)x)\left(\frac{a^2c^2\sqrt{-(c^2d^6-6acd^4e^2+9a^2d^2e^4)}}{a^5c^3} + 3cd^2e - ae^3\right)\sqrt[3]{\frac{1}{2}\arctan\left(\frac{1}{2}\sqrt{\frac{3}{c^3d^7 - ac^2d^5e^2 - 5a^2cd^3e^4 - 3a^3d^6e^6}}\right) + 3cd^2e - ae^3\sqrt{\frac{1}{2}\sqrt{\frac{3}{c^3d^7 - ac^2d^5e^2 - 5a^2cd^3e^4 - 3a^3d^6e^6}}}\right) - \sqrt{3}\frac{(c^3d^7 - ac^2d^5e^2 - 5a^2cd^3e^4 - 3a^3d^6e^6)}{(c^3d^7 - ac^2d^5e^2 - 5a^2cd^3e^4 - 3a^3d^6e^6)} - \frac{1}{3}\sqrt{3}\left(-\frac{a^2c^2\sqrt{-(c^2d^6-6acd^4e^2+9a^2d^2e^4)}}{a^5c^3} - 3cd^2e + ae^3\right)\sqrt[3]{\frac{1}{2}\arctan\left(\frac{1}{2}\sqrt{\frac{3}{c^3d^7 - ac^2d^5e^2 - 5a^2cd^3e^4 - 3a^3d^6e^6}}\right) + 3cd^2e - ae^3\sqrt{\frac{1}{2}\sqrt{\frac{3}{c^3d^7 - ac^2d^5e^2 - 5a^2cd^3e^4 - 3a^3d^6e^6}}}\right) + 2\sqrt{3}(a^2c^3d^4e - 3a^3c^2d^2e^3)\sqrt{\left((c^3d^7 - ac^2d^5e^2 - 5a^2cd^3e^4 - 3a^3d^6e^6)\right)x^2 - (2a^5c^3d^3e\sqrt{-(c^2d^6-6acd^4e^2+9a^2d^2e^4)}}}{a^5c^3} - a^2c^3d^5 + 4a^3c^2d^3e^2 - 3a^4cd^4e^4)\left(-\frac{a^2c^2\sqrt{-(c^2d^6-6acd^4e^2+9a^2d^2e^4)}}{a^5c^3} - 3cd^2e + ae^3\right)\sqrt[3]{\frac{1}{2}\arctan\left(\frac{1}{2}\sqrt{\frac{3}{c^3d^7 - ac^2d^5e^2 - 5a^2cd^3e^4 - 3a^3d^6e^6}}\right) + 3cd^2e - ae^3\sqrt{\frac{1}{2}\sqrt{\frac{3}{c^3d^7 - ac^2d^5e^2 - 5a^2cd^3e^4 - 3a^3d^6e^6}}}\right) + \left(\frac{a^4c^3d^2e + a^5c^2e^3}{a^5c^3}\right)x\sqrt{\frac{1}{2}\sqrt{\frac{3}{c^3d^7 - ac^2d^5e^2 - 5a^2cd^3e^4 - 3a^3d^6e^6}}}\left(-\frac{a^2c^2\sqrt{-(c^2d^6-6acd^4e^2+9a^2d^2e^4)}}{a^5c^3} - 3cd^2e + ae^3\right)\sqrt[3]{\frac{1}{2}\arctan\left(\frac{1}{2}\sqrt{\frac{3}{c^3d^7 - ac^2d^5e^2 - 5a^2cd^3e^4 - 3a^3d^6e^6}}\right) + 3cd^2e - ae^3\sqrt{\frac{1}{2}\sqrt{\frac{3}{c^3d^7 - ac^2d^5e^2 - 5a^2cd^3e^4 - 3a^3d^6e^6}}}\right) - 2(\sqrt{3}(a^4c^4d^2 - a^5c^3e^2)x\sqrt{-(c^2d^6-6acd^4e^2+9a^2d^2e^4)}}}{a^5c^3} + 2\sqrt{3}(a^2c^3d^4e - 3a^3c^2d^2e^3)x)\left(-\frac{a^2c^2\sqrt{-(c^2d^6-6acd^4e^2+9a^2d^2e^4)}}{a^5c^3} - 3cd^2e + ae^3\right)\sqrt[3]{\frac{1}{2}\arctan\left(\frac{1}{2}\sqrt{\frac{3}{c^3d^7 - ac^2d^5e^2 - 5a^2cd^3e^4 - 3a^3d^6e^6}}\right) + 3cd^2e - ae^3\sqrt{\frac{1}{2}\sqrt{\frac{3}{c^3d^7 - ac^2d^5e^2 - 5a^2cd^3e^4 - 3a^3d^6e^6}}}\right) -$

$$\begin{aligned}
& 3*c*d^2*e + a*e^3)/(a^2*c^2))^{(2/3)} + \text{sqrt}(3)*(c^3*d^7 - a*c^2*d^5*e^2 - 5* \\
& a^2*c*d^3*e^4 - 3*a^3*d*e^6))/(c^3*d^7 - a*c^2*d^5*e^2 - 5*a^2*c*d^3*e^4 - \\
& 3*a^3*d*e^6)) - 1/12*((a^2*c^2*\text{sqrt}(-(c^2*d^6 - 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4) \\
& ^4)/(a^5*c^3)) + 3*c*d^2*e - a*e^3)/(a^2*c^2))^{(1/3)}*\text{log}(-(c^3*d^7 - a*c^2* \\
& d^5*e^2 - 5*a^2*c*d^3*e^4 - 3*a^3*d*e^6)*x^2 - (2*a^5*c^3*d*e*\text{sqrt}(-(c^2*d^6 \\
& - 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^5*c^3)) + a^2*c^3*d^5 - 4*a^3*c^2*d^3 \\
& *e^2 + 3*a^4*c*d*e^4)*((a^2*c^2*\text{sqrt}(-(c^2*d^6 - 6*a*c*d^4*e^2 + 9*a^2*d^2* \\
& e^4)/(a^5*c^3)) + 3*c*d^2*e - a*e^3)/(a^2*c^2))^{(2/3)} + ((a^4*c^3*d^2*e + a \\
& ^5*c^2*e^3)*x*\text{sqrt}(-(c^2*d^6 - 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^5*c^3)) + \\
& (a*c^3*d^6 - 2*a^2*c^2*d^4*e^2 - 3*a^3*c*d^2*e^4)*x)*((a^2*c^2*\text{sqrt}(-(c^2*d^ \\
& ^6 - 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^5*c^3)) + 3*c*d^2*e - a*e^3)/(a^2*c^ \\
& 2))^{(1/3)}) - 1/12*(-(a^2*c^2*\text{sqrt}(-(c^2*d^6 - 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4) \\
& )/(a^5*c^3)) - 3*c*d^2*e + a*e^3)/(a^2*c^2))^{(1/3)}*\text{log}(-(c^3*d^7 - a*c^2*d^ \\
& 5*e^2 - 5*a^2*c*d^3*e^4 - 3*a^3*d*e^6)*x^2 + (2*a^5*c^3*d*e*\text{sqrt}(-(c^2*d^6 \\
& - 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^5*c^3)) - a^2*c^3*d^5 + 4*a^3*c^2*d^3*e \\
& ^2 - 3*a^4*c*d*e^4)*(-(a^2*c^2*\text{sqrt}(-(c^2*d^6 - 6*a*c*d^4*e^2 + 9*a^2*d^2*e \\
& ^4)/(a^5*c^3)) - 3*c*d^2*e + a*e^3)/(a^2*c^2))^{(2/3)} - ((a^4*c^3*d^2*e + a^ \\
& 5*c^2*e^3)*x*\text{sqrt}(-(c^2*d^6 - 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^5*c^3)) - ( \\
& a*c^3*d^6 - 2*a^2*c^2*d^4*e^2 - 3*a^3*c*d^2*e^4)*x)*(-(a^2*c^2*\text{sqrt}(-(c^2*d^ \\
& ^6 - 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^5*c^3)) - 3*c*d^2*e + a*e^3)/(a^2*c^ \\
& 2))^{(1/3)}) + 1/6*((a^2*c^2*\text{sqrt}(-(c^2*d^6 - 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/ \\
& (a^5*c^3)) + 3*c*d^2*e - a*e^3)/(a^2*c^2))^{(1/3)}*\text{log}(-(c^2*d^5 - 2*a*c*d^3* \\
& e^2 - 3*a^2*d*e^4)*x - (a^4*c^2*e*\text{sqrt}(-(c^2*d^6 - 6*a*c*d^4*e^2 + 9*a^2*d^ \\
& 2*e^4)/(a^5*c^3)) + a*c^2*d^4 - 3*a^2*c*d^2*e^2)*((a^2*c^2*\text{sqrt}(-(c^2*d^6 - \\
& 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^5*c^3)) + 3*c*d^2*e - a*e^3)/(a^2*c^2))^{( \\
& 1/3)}) + 1/6*(-(a^2*c^2*\text{sqrt}(-(c^2*d^6 - 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^ \\
& 5*c^3)) - 3*c*d^2*e + a*e^3)/(a^2*c^2))^{(1/3)}*\text{log}(-(c^2*d^5 - 2*a*c*d^3*e^2 \\
& - 3*a^2*d*e^4)*x + (a^4*c^2*e*\text{sqrt}(-(c^2*d^6 - 6*a*c*d^4*e^2 + 9*a^2*d^2*e \\
& ^4)/(a^5*c^3)) - a*c^2*d^4 + 3*a^2*c*d^2*e^2)*(-(a^2*c^2*\text{sqrt}(-(c^2*d^6 - 6 \\
& *a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^5*c^3)) - 3*c*d^2*e + a*e^3)/(a^2*c^2))^{(1 \\
& /3)})
\end{aligned}$$


---

**Sympy [A]** time = 2.13879, size = 165, normalized size = 0.54

$$\text{RootSum}\left(46656t^6a^5c^4 + t^3(432a^4c^2e^3 - 1296a^3c^3d^2e) + a^3e^6 + 3a^2cd^2e^4 + 3ac^2d^4e^2 + c^3d^6, \left(t \mapsto t \log\left(x + \frac{-1296t^4a^4}{\dots}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x\*\*3+d)/(c\*x\*\*6+a), x)

[Out] RootSum(46656\*\_t\*\*6\*a\*\*5\*c\*\*4 + \_t\*\*3\*(432\*a\*\*4\*c\*\*2\*e\*\*3 - 1296\*a\*\*3\*c\*\*3\*d\*\*2\*e) + a\*\*3\*e\*\*6 + 3\*a\*\*2\*c\*d\*\*2\*e\*\*4 + 3\*a\*c\*\*2\*d\*\*4\*e\*\*2 + c\*\*3\*d\*\*6,



```
Lambda(_t, _t*log(x + (-1296*_t**4*a**4*c**2*e - 6*_t*a**3*e**4 + 36*_t*a**2*c*d**2*e**2 - 6*_t*a*c**2*d**4)/(3*a**2*d*e**4 + 2*a*c*d**3*e**2 - c**2*d**5))))
```

**Giac [A]** time = 1.12413, size = 397, normalized size = 1.3

$$\frac{(ac^5)^{\frac{1}{6}} d \arctan\left(\frac{x}{\left(\frac{a}{c}\right)^{\frac{1}{6}}}\right)}{3ac} - \frac{(ac^5)^{\frac{2}{3}} |c| e \log\left(x^2 + \left(\frac{a}{c}\right)^{\frac{1}{3}}\right)}{6ac^5} + \frac{\left((ac^5)^{\frac{1}{6}} c^3 d - \sqrt{3} (ac^5)^{\frac{2}{3}} e\right) \arctan\left(\frac{2x + \sqrt{3} \left(\frac{a}{c}\right)^{\frac{1}{6}}}{\left(\frac{a}{c}\right)^{\frac{1}{6}}}\right)}{6ac^4} + \frac{\left((ac^5)^{\frac{1}{6}} c^3 d + \sqrt{3} (ac^5)^{\frac{2}{3}} e\right) \arctan\left(\frac{2x - \sqrt{3} \left(\frac{a}{c}\right)^{\frac{1}{6}}}{\left(\frac{a}{c}\right)^{\frac{1}{6}}}\right)}{6ac^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^3+d)/(c*x^6+a),x, algorithm="giac")
```

```
[Out] 1/3*(a*c^5)^(1/6)*d*arctan(x/(a/c)^(1/6))/(a*c) - 1/6*(a*c^5)^(2/3)*abs(c)*e*log(x^2 + (a/c)^(1/3))/(a*c^5) + 1/6*((a*c^5)^(1/6)*c^3*d - sqrt(3)*(a*c^5)^(2/3)*e)*arctan((2*x + sqrt(3)*(a/c)^(1/6))/(a/c)^(1/6))/(a*c^4) + 1/6*((a*c^5)^(1/6)*c^3*d + sqrt(3)*(a*c^5)^(2/3)*e)*arctan((2*x - sqrt(3)*(a/c)^(1/6))/(a/c)^(1/6))/(a*c^4) + 1/12*(sqrt(3)*(a*c^5)^(1/6)*c^3*d + (a*c^5)^(2/3)*e)*log(x^2 + sqrt(3)*x*(a/c)^(1/6) + (a/c)^(1/3))/(a*c^4) - 1/12*(sqrt(3)*(a*c^5)^(1/6)*c^3*d - (a*c^5)^(2/3)*e)*log(x^2 - sqrt(3)*x*(a/c)^(1/6) + (a/c)^(1/3))/(a*c^4)
```

## 3.2 $\int \frac{d+ex^3}{a-cx^6} dx$

**Optimal.** Leaf size=323

$$\frac{(\sqrt{ae} + \sqrt{cd}) \log(\sqrt[6]{a}\sqrt[6]{cx} + \sqrt[3]{a} + \sqrt[3]{cx^2})}{12a^{5/6}c^{2/3}} - \frac{(\sqrt{ae} + \sqrt{cd}) \log(\sqrt[6]{a} - \sqrt[6]{cx})}{6a^{5/6}c^{2/3}} + \frac{(\sqrt{ae} + \sqrt{cd}) \tan^{-1}\left(\frac{\sqrt[6]{a+2\sqrt[6]{cx}}}{\sqrt{3}\sqrt[6]{a}}\right)}{2\sqrt{3}a^{5/6}c^{2/3}} - \left(d - \frac{\sqrt{ae}}{\sqrt{c}}\right) \log\left(\frac{d + \sqrt{ae}}{\sqrt{c}}\right)$$

```
[Out] -((d - (Sqrt[a]*e)/Sqrt[c])*ArcTan[(a^(1/6) - 2*c^(1/6)*x)/(Sqrt[3]*a^(1/6)
)])/((2*Sqrt[3]*a^(5/6)*c^(1/6)) + ((Sqrt[c]*d + Sqrt[a]*e)*ArcTan[(a^(1/6)
+ 2*c^(1/6)*x)/(Sqrt[3]*a^(1/6))])/((2*Sqrt[3]*a^(5/6)*c^(2/3)) - ((Sqrt[c]*
d + Sqrt[a]*e)*Log[a^(1/6) - c^(1/6)*x])/(6*a^(5/6)*c^(2/3)) + ((d - (Sqrt[
a]*e)/Sqrt[c])*Log[a^(1/6) + c^(1/6)*x])/(6*a^(5/6)*c^(1/6)) - ((d - (Sqrt[
a]*e)/Sqrt[c])*Log[a^(1/3) - a^(1/6)*c^(1/6)*x + c^(1/3)*x^2])/(12*a^(5/6)*
c^(1/6)) + ((Sqrt[c]*d + Sqrt[a]*e)*Log[a^(1/3) + a^(1/6)*c^(1/6)*x + c^(1/
3)*x^2])/(12*a^(5/6)*c^(2/3))
```

---

**Rubi [A]** time = 0.188983, antiderivative size = 323, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 7, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$ , Rules used = {1417, 200, 31, 634, 617, 204, 628}

$$\frac{(\sqrt{ae} + \sqrt{cd}) \log(\sqrt[6]{a}\sqrt[6]{cx} + \sqrt[3]{a} + \sqrt[3]{cx^2})}{12a^{5/6}c^{2/3}} - \frac{(\sqrt{ae} + \sqrt{cd}) \log(\sqrt[6]{a} - \sqrt[6]{cx})}{6a^{5/6}c^{2/3}} + \frac{(\sqrt{ae} + \sqrt{cd}) \tan^{-1}\left(\frac{\sqrt[6]{a+2\sqrt[6]{cx}}}{\sqrt{3}\sqrt[6]{a}}\right)}{2\sqrt{3}a^{5/6}c^{2/3}} - \left(d - \frac{\sqrt{ae}}{\sqrt{c}}\right) \log\left(\frac{d + \sqrt{ae}}{\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^3)/(a - c*x^6), x]
```

```
[Out] -((d - (Sqrt[a]*e)/Sqrt[c])*ArcTan[(a^(1/6) - 2*c^(1/6)*x)/(Sqrt[3]*a^(1/6)
)])/((2*Sqrt[3]*a^(5/6)*c^(1/6)) + ((Sqrt[c]*d + Sqrt[a]*e)*ArcTan[(a^(1/6)
+ 2*c^(1/6)*x)/(Sqrt[3]*a^(1/6))])/((2*Sqrt[3]*a^(5/6)*c^(2/3)) - ((Sqrt[c]*
d + Sqrt[a]*e)*Log[a^(1/6) - c^(1/6)*x])/(6*a^(5/6)*c^(2/3)) + ((d - (Sqrt[
a]*e)/Sqrt[c])*Log[a^(1/6) + c^(1/6)*x])/(6*a^(5/6)*c^(1/6)) - ((d - (Sqrt[
a]*e)/Sqrt[c])*Log[a^(1/3) - a^(1/6)*c^(1/6)*x + c^(1/3)*x^2])/(12*a^(5/6)*
c^(1/6)) + ((Sqrt[c]*d + Sqrt[a]*e)*Log[a^(1/3) + a^(1/6)*c^(1/6)*x + c^(1/
3)*x^2])/(12*a^(5/6)*c^(2/3))
```

### Rule 1417

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (c_.)*(x_)^(n2_)), x_Symbol] := With[{
q = Rt[-(a/c), 2]}, Dist[(d + e*q)/2, Int[1/(a + c*q*x^n), x], x] + Dist[(d
```

- e\*q)/2, Int[1/(a - c\*q\*x^n), x], x] /; FreeQ[{a, c, d, e, n}, x] && EqQ[n2, 2\*n] && NeQ[c\*d^2 + a\*e^2, 0] && NegQ[a\*c] && IntegerQ[n]

### Rule 200

Int[((a\_) + (b\_)\*(x\_)^3)^(-1), x\_Symbol] := Dist[1/(3\*Rt[a, 3]^2), Int[1/(Rt[a, 3] + Rt[b, 3]\*x), x], x] + Dist[1/(3\*Rt[a, 3]^2), Int[(2\*Rt[a, 3] - Rt[b, 3]\*x)/(Rt[a, 3]^2 - Rt[a, 3]\*Rt[b, 3]\*x + Rt[b, 3]^2\*x^2), x], x] /; FreeQ[{a, b}, x]

### Rule 31

Int[((a\_) + (b\_)\*(x\_)^(-1), x\_Symbol] := Simp[Log[RemoveContent[a + b\*x, x]]/b, x] /; FreeQ[{a, b}, x]

### Rule 634

Int[((d\_) + (e\_)\*(x\_))/((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2), x\_Symbol] := Dist[(2\*c\*d - b\*e)/(2\*c), Int[1/(a + b\*x + c\*x^2), x], x] + Dist[e/(2\*c), Int[(b + 2\*c\*x)/(a + b\*x + c\*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2\*c\*d - b\*e, 0] && NeQ[b^2 - 4\*a\*c, 0] && !NiceSqrtQ[b^2 - 4\*a\*c]

### Rule 617

Int[((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(-1), x\_Symbol] := With[{q = 1 - 4\*Simplify[(a\*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2\*c\*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4\*a\*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

### Rule 204

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

### Rule 628

Int[((d\_) + (e\_)\*(x\_))/((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2), x\_Symbol] := Simp[(d\*Log[RemoveContent[a + b\*x + c\*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

### Rubi steps

$$\begin{aligned}
\int \frac{d+ex^3}{a-cx^6} dx &= \frac{1}{2} \left( d - \frac{\sqrt{ae}}{\sqrt{c}} \right) \int \frac{1}{a + \sqrt{a}\sqrt{cx^3}} dx + \frac{1}{2} \left( d + \frac{\sqrt{ae}}{\sqrt{c}} \right) \int \frac{1}{a - \sqrt{a}\sqrt{cx^3}} dx \\
&= \frac{\left( d - \frac{\sqrt{ae}}{\sqrt{c}} \right) \int \frac{1}{\sqrt[3]{a} + \sqrt[6]{a}\sqrt[6]{cx}} dx}{6a^{2/3}} + \frac{\left( d - \frac{\sqrt{ae}}{\sqrt{c}} \right) \int \frac{2\sqrt[3]{a} - \sqrt[6]{a}\sqrt[6]{cx}}{a^{2/3} - \sqrt{a}\sqrt[6]{cx} + \sqrt[3]{a}\sqrt[3]{cx^2}} dx}{6a^{2/3}} + \frac{\left( d + \frac{\sqrt{ae}}{\sqrt{c}} \right) \int \frac{1}{\sqrt[3]{a} - \sqrt[6]{a}\sqrt[6]{cx}} dx}{6a^{2/3}} + \frac{\left( d + \frac{\sqrt{ae}}{\sqrt{c}} \right) \int \frac{2\sqrt[3]{a} + \sqrt[6]{a}\sqrt[6]{cx}}{a^{2/3} + \sqrt{a}\sqrt[6]{cx} + \sqrt[3]{a}\sqrt[3]{cx^2}} dx}{6a^{2/3}} \\
&= -\frac{(\sqrt{cd} + \sqrt{ae}) \log(\sqrt[6]{a} - \sqrt[6]{cx})}{6a^{5/6}c^{2/3}} + \frac{\left( d - \frac{\sqrt{ae}}{\sqrt{c}} \right) \log(\sqrt[6]{a} + \sqrt[6]{cx})}{6a^{5/6}\sqrt[6]{c}} + \frac{(\sqrt{cd} + \sqrt{ae}) \int \frac{\sqrt{a}\sqrt[6]{c} + 2\sqrt[3]{a}\sqrt[3]{cx}}{a^{2/3} + \sqrt{a}\sqrt[6]{cx} + \sqrt[3]{a}\sqrt[3]{cx^2}} dx}{12a^{5/6}c^{2/3}} + \frac{\left( d + \frac{\sqrt{ae}}{\sqrt{c}} \right) \log(\sqrt[6]{a} - \sqrt[6]{cx})}{6a^{5/6}c^{2/3}} \\
&= -\frac{(\sqrt{cd} + \sqrt{ae}) \log(\sqrt[6]{a} - \sqrt[6]{cx})}{6a^{5/6}c^{2/3}} + \frac{\left( d - \frac{\sqrt{ae}}{\sqrt{c}} \right) \log(\sqrt[6]{a} + \sqrt[6]{cx})}{6a^{5/6}\sqrt[6]{c}} - \frac{\left( d - \frac{\sqrt{ae}}{\sqrt{c}} \right) \log(\sqrt[3]{a} - \sqrt[6]{a}\sqrt[6]{cx} + \sqrt[3]{cx^2})}{12a^{5/6}\sqrt[6]{c}} + \frac{(\sqrt{cd} + \sqrt{ae}) \log(\sqrt[6]{a} - \sqrt[6]{cx})}{6a^{5/6}c^{2/3}} \\
&= -\frac{\left( d - \frac{\sqrt{ae}}{\sqrt{c}} \right) \tan^{-1}\left(\frac{\sqrt[6]{a} - 2\sqrt[6]{cx}}{\sqrt{3}\sqrt[6]{a}}\right)}{2\sqrt{3}a^{5/6}\sqrt[6]{c}} + \frac{(\sqrt{cd} + \sqrt{ae}) \tan^{-1}\left(\frac{\sqrt[6]{a} + 2\sqrt[6]{cx}}{\sqrt{3}\sqrt[6]{a}}\right)}{2\sqrt{3}a^{5/6}c^{2/3}} - \frac{(\sqrt{cd} + \sqrt{ae}) \log(\sqrt[6]{a} - \sqrt[6]{cx})}{6a^{5/6}c^{2/3}} + \frac{\left( d - \frac{\sqrt{ae}}{\sqrt{c}} \right) \log(\sqrt[6]{a} + \sqrt[6]{cx})}{6a^{5/6}\sqrt[6]{c}}
\end{aligned}$$

**Mathematica [A]** time = 0.110284, size = 337, normalized size = 1.04

$$-2\sqrt{3}(\sqrt{cd} - \sqrt{ae}) \tan^{-1}\left(\frac{1 - \frac{2\sqrt[6]{cx}}{\sqrt[6]{a}}}{\sqrt{3}}\right) + 2\sqrt{3}(\sqrt{ae} + \sqrt{cd}) \tan^{-1}\left(\frac{\frac{2\sqrt[6]{cx}}{\sqrt[6]{a}} + 1}{\sqrt{3}}\right) - \sqrt{cd} \log(-\sqrt[6]{a}\sqrt[6]{cx} + \sqrt[3]{a} + \sqrt[3]{cx^2}) + \sqrt{cd} \log(\sqrt[6]{a} - \sqrt[6]{cx})$$

Antiderivative was successfully verified.

[In] Integrate[(d + e\*x^3)/(a - c\*x^6),x]

[Out] (-2\*Sqrt[3]\*(Sqrt[c]\*d - Sqrt[a]\*e)\*ArcTan[(1 - (2\*c^(1/6)\*x)/a^(1/6))/Sqrt[3]] + 2\*Sqrt[3]\*(Sqrt[c]\*d + Sqrt[a]\*e)\*ArcTan[(1 + (2\*c^(1/6)\*x)/a^(1/6))/Sqrt[3]] - 2\*Sqrt[c]\*d\*Log[a^(1/6) - c^(1/6)\*x] - 2\*Sqrt[a]\*e\*Log[a^(1/6) - c^(1/6)\*x] + 2\*Sqrt[c]\*d\*Log[a^(1/6) + c^(1/6)\*x] - 2\*Sqrt[a]\*e\*Log[a^(1/6) + c^(1/6)\*x] - Sqrt[c]\*d\*Log[a^(1/3) - a^(1/6)\*c^(1/6)\*x + c^(1/3)\*x^2] + Sqrt[a]\*e\*Log[a^(1/3) - a^(1/6)\*c^(1/6)\*x + c^(1/3)\*x^2] + Sqrt[c]\*d\*Log[a^(1/3) + a^(1/6)\*c^(1/6)\*x + c^(1/3)\*x^2] + Sqrt[a]\*e\*Log[a^(1/3) + a^(1/6)\*c^(1/6)\*x + c^(1/3)\*x^2])/(12\*a^(5/6)\*c^(2/3))

**Maple [A]** time = 0.12, size = 386, normalized size = 1.2

$$\frac{e}{12a} \left(\frac{a}{c}\right)^{\frac{2}{3}} \ln\left(\sqrt[6]{\frac{a}{c}}x - x^2 - \sqrt[3]{\frac{a}{c}}\right) - \frac{d}{12a} \sqrt[6]{\frac{a}{c}} \ln\left(\sqrt[6]{\frac{a}{c}}x - x^2 - \sqrt[3]{\frac{a}{c}}\right) - \frac{\sqrt{3}e}{6a} \left(\frac{a}{c}\right)^{\frac{2}{3}} \arctan\left(-\frac{\sqrt{3}}{3} + \frac{2x\sqrt{3}}{3} \frac{1}{\sqrt[6]{\frac{a}{c}}}\right) + \frac{\sqrt{3}d}{6a} \sqrt[6]{\frac{a}{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x^3+d)/(-c*x^6+a),x)`

[Out]  $\frac{1}{12} \left( \frac{a}{c} \right)^{\frac{2}{3}} \frac{1}{a} \ln \left( \left( \frac{a}{c} \right)^{\frac{1}{6}} x - x^2 - \left( \frac{a}{c} \right)^{\frac{1}{3}} \right) e - \frac{1}{12} \left( \frac{a}{c} \right)^{\frac{1}{6}} \frac{1}{a} \ln \left( \left( \frac{a}{c} \right)^{\frac{1}{6}} x - x^2 - \left( \frac{a}{c} \right)^{\frac{1}{3}} \right) d - \frac{1}{6} \left( \frac{a}{c} \right)^{\frac{2}{3}} \frac{1}{a} 3^{\frac{1}{2}} e \arctan \left( \frac{-1/3 \cdot 3^{\frac{1}{2}} + 2/3 x \cdot 3^{\frac{1}{2}}}{\left( \frac{a}{c} \right)^{\frac{1}{6}}} \right) + \frac{1}{6} \left( \frac{a}{c} \right)^{\frac{1}{6}} \frac{1}{a} 3^{\frac{1}{2}} d \arctan \left( \frac{-1/3 \cdot 3^{\frac{1}{2}} + 2/3 x \cdot 3^{\frac{1}{2}}}{\left( \frac{a}{c} \right)^{\frac{1}{6}}} \right) - \frac{1}{6} \frac{c}{\left( \frac{a}{c} \right)^{\frac{1}{3}}} \ln \left( x + \left( \frac{a}{c} \right)^{\frac{1}{6}} \right) e + \frac{1}{6} \frac{c}{\left( \frac{a}{c} \right)^{\frac{5}{6}}} \ln \left( x + \left( \frac{a}{c} \right)^{\frac{1}{6}} \right) d - \frac{1}{6} \frac{c}{\left( \frac{a}{c} \right)^{\frac{1}{3}}} \ln \left( -x + \left( \frac{a}{c} \right)^{\frac{1}{6}} \right) e - \frac{1}{6} \frac{c}{\left( \frac{a}{c} \right)^{\frac{5}{6}}} \ln \left( -x + \left( \frac{a}{c} \right)^{\frac{1}{6}} \right) d + \frac{1}{12} \frac{1}{a} \left( \frac{a}{c} \right)^{\frac{2}{3}} e \ln \left( x^2 + \left( \frac{a}{c} \right)^{\frac{1}{6}} x + \left( \frac{a}{c} \right)^{\frac{1}{3}} \right) + \frac{1}{6} \frac{1}{a} \left( \frac{a}{c} \right)^{\frac{2}{3}} e 3^{\frac{1}{2}} \arctan \left( \frac{2/3 x \cdot 3^{\frac{1}{2}}}{\left( \frac{a}{c} \right)^{\frac{1}{6}}} \right) + \frac{1}{3} 3^{\frac{1}{2}} + \frac{1}{12} \frac{1}{a} d \left( \frac{a}{c} \right)^{\frac{1}{6}} \ln \left( x^2 + \left( \frac{a}{c} \right)^{\frac{1}{6}} x + \left( \frac{a}{c} \right)^{\frac{1}{3}} \right) + \frac{1}{6} \frac{1}{a} d \left( \frac{a}{c} \right)^{\frac{1}{6}} 3^{\frac{1}{2}} \arctan \left( \frac{2/3 x \cdot 3^{\frac{1}{2}}}{\left( \frac{a}{c} \right)^{\frac{1}{6}}} \right) + \frac{1}{3} 3^{\frac{1}{2}} \right)$

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x^3+d)/(-c*x^6+a),x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [B]** time = 2.41328, size = 6541, normalized size = 20.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x^3+d)/(-c*x^6+a),x, algorithm="fricas")`

[Out]  $\frac{1}{3} \sqrt{3} \left( - \left( a^2 c^2 \sqrt{c^2 d^6 + 6 a c d^4 e^2 + 9 a^2 d^2 e^4} \right) / \left( a^5 c^3 \right) + 3 c d^2 e + a e^3 \right) / \left( a^2 c^2 \right)^{\frac{1}{3}} \arctan \left( \frac{1}{3} \left( 2 \left( \sqrt{3} \left( a^4 c^4 d^2 + a^5 c^3 e^2 \right) \sqrt{c^2 d^6 + 6 a c d^4 e^2 + 9 a^2 d^2 e^4} \right) / \left( a^5 c^3 \right) \right) - 2 \sqrt{3} \left( a^2 c^3 d^4 e + 3 a^3 c^2 d^2 e^3 \right) \sqrt{c^3 d^7 + a c^2 d^5 e^2 - 5 a^2 c d^3 e^4 + 3 a^3 d e^6} \right) x^2 - \left( 2 a^5 c^3 d e \sqrt{c^2 d^6 + 6 a c d^4 e^2 + 9 a^2 d^2 e^4} \right) / \left( a^5 c^3 \right) - a^2 c^3 d^5 - 4 a^3 c^2 d^3 e^2 - 3 a^4 c d e^4 \right) \left( - \left( a^2 c^2 \sqrt{c^2 d^6 + 6 a c d^4 e^2 + 9 a^2 d^2 e^4} \right) / \left( a^5 c^3 \right) + 3 c d^2 e + a e^3 \right) / \left( a^2 c^2 \right)^{\frac{1}{3}} \arctan \left( \frac{1}{3} \left( 2 \left( \sqrt{3} \left( a^4 c^4 d^2 + a^5 c^3 e^2 \right) \sqrt{c^2 d^6 + 6 a c d^4 e^2 + 9 a^2 d^2 e^4} \right) / \left( a^5 c^3 \right) \right) - 2 \sqrt{3} \left( a^2 c^3 d^4 e + 3 a^3 c^2 d^2 e^3 \right) \sqrt{c^3 d^7 + a c^2 d^5 e^2 - 5 a^2 c d^3 e^4 + 3 a^3 d e^6} \right) x^2 - \left( 2 a^5 c^3 d e \sqrt{c^2 d^6 + 6 a c d^4 e^2 + 9 a^2 d^2 e^4} \right) / \left( a^5 c^3 \right) - a^2 c^3 d^5 - 4 a^3 c^2 d^3 e^2 - 3 a^4 c d e^4 \right)$



$$\begin{aligned}
& + 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^5*c^3)) + (a*c^3*d^6 + 2*a^2*c^2*d^4*e^2 - 3*a^3*c*d^2*e^4)*x*((a^2*c^2*sqrt((c^2*d^6 + 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^5*c^3)) - 3*c*d^2*e - a*e^3)/(a^2*c^2))^(1/3)) + 1/6*(-(a^2*c^2*sqrt((c^2*d^6 + 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^5*c^3)) + 3*c*d^2*e + a*e^3)/(a^2*c^2))^(1/3)*log(-(c^2*d^5 + 2*a*c*d^3*e^2 - 3*a^2*d*e^4)*x + (a^4*c^2*e*sqrt((c^2*d^6 + 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^5*c^3)) - a*c^2*d^4 - 3*a^2*c*d^2*e^2)*(-(a^2*c^2*sqrt((c^2*d^6 + 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^5*c^3)) + 3*c*d^2*e + a*e^3)/(a^2*c^2))^(1/3)) + 1/6*((a^2*c^2*sqrt((c^2*d^6 + 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^5*c^3)) - 3*c*d^2*e - a*e^3)/(a^2*c^2))^(1/3)*log(-(c^2*d^5 + 2*a*c*d^3*e^2 - 3*a^2*d*e^4)*x - (a^4*c^2*e*sqrt((c^2*d^6 + 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^5*c^3)) + a*c^2*d^4 + 3*a^2*c*d^2*e^2)*((a^2*c^2*sqrt((c^2*d^6 + 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^5*c^3)) - 3*c*d^2*e - a*e^3)/(a^2*c^2))^(1/3))
\end{aligned}$$

**Sympy [A]** time = 2.18282, size = 168, normalized size = 0.52

$$-\text{RootSum}\left(46656t^6a^5c^4 + t^3(-432a^4c^2e^3 - 1296a^3c^3d^2e) + a^3e^6 - 3a^2cd^2e^4 + 3ac^2d^4e^2 - c^3d^6, \left(t \mapsto t \log\left(x + \frac{-1296t^3a^4c^2e^3 - 1296t^4a^3c^3d^2e + 6t^5a^2c^2d^4e^2 + 36t^6a^3c^2d^2e^2 + 6t^7a^4c^2d^4e}{3a^2d^2e^4 - 2a^3cd^3e^2 - c^2d^5}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x\*\*3+d)/(-c\*x\*\*6+a), x)

[Out] -RootSum(46656\*\_t\*\*6\*a\*\*5\*c\*\*4 + \_t\*\*3\*(-432\*a\*\*4\*c\*\*2\*e\*\*3 - 1296\*a\*\*3\*c\*\*3\*d\*\*2\*e) + a\*\*3\*e\*\*6 - 3\*a\*\*2\*c\*d\*\*2\*e\*\*4 + 3\*a\*c\*\*2\*d\*\*4\*e\*\*2 - c\*\*3\*d\*\*6, Lambda(\_t, \_t\*log(x + (-1296\*\_t\*\*4\*a\*\*4\*c\*\*2\*e + 6\*\_t\*a\*\*3\*e\*\*4 + 36\*\_t\*a\*\*2\*c\*d\*\*2\*e\*\*2 + 6\*\_t\*a\*c\*\*2\*d\*\*4)/(3\*a\*\*2\*d\*e\*\*4 - 2\*a\*c\*d\*\*3\*e\*\*2 - c\*\*2\*d\*\*5))))

**Giac [A]** time = 1.14158, size = 424, normalized size = 1.31

$$\frac{(-ac^5)^{\frac{1}{6}} d \arctan\left(\frac{x}{\left(-\frac{a}{c}\right)^{\frac{1}{6}}}\right)}{3ac} - \frac{(-ac^5)^{\frac{2}{3}} |c|e \log\left(x^2 + \left(-\frac{a}{c}\right)^{\frac{1}{3}}\right)}{6ac^5} + \frac{\left(\left(-ac^5\right)^{\frac{1}{6}} c^3d - \sqrt{3}\left(-ac^5\right)^{\frac{2}{3}} e\right) \arctan\left(\frac{2x + \sqrt{3}\left(-\frac{a}{c}\right)^{\frac{1}{6}}}{\left(-\frac{a}{c}\right)^{\frac{1}{6}}}\right)}{6ac^4} + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^3+d)/(-c\*x^6+a), x, algorithm="giac")

```
[Out] 1/3*(-a*c^5)^(1/6)*d*arctan(x/(-a/c)^(1/6))/(a*c) - 1/6*(-a*c^5)^(2/3)*abs(
c)*e*log(x^2 + (-a/c)^(1/3))/(a*c^5) + 1/6*((-a*c^5)^(1/6)*c^3*d - sqrt(3)*
(-a*c^5)^(2/3)*e)*arctan((2*x + sqrt(3)*(-a/c)^(1/6))/(-a/c)^(1/6))/(a*c^4)
+ 1/6*((-a*c^5)^(1/6)*c^3*d + sqrt(3)*(-a*c^5)^(2/3)*e)*arctan((2*x - sqrt
(3)*(-a/c)^(1/6))/(-a/c)^(1/6))/(a*c^4) + 1/12*(sqrt(3)*(-a*c^5)^(1/6)*c^3*
d + (-a*c^5)^(2/3)*e)*log(x^2 + sqrt(3)*x*(-a/c)^(1/6) + (-a/c)^(1/3))/(a*c
^4) - 1/12*(sqrt(3)*(-a*c^5)^(1/6)*c^3*d - (-a*c^5)^(2/3)*e)*log(x^2 - sqrt
(3)*x*(-a/c)^(1/6) + (-a/c)^(1/3))/(a*c^4)
```



### 3.3 $\int \frac{d+ex^4}{a+cx^8} dx$

**Optimal.** Leaf size=754

$$\frac{((1-\sqrt{2})\sqrt{cd}-\sqrt{ae})\log\left(-\sqrt{2-\sqrt{2}}\sqrt[8]{a}\sqrt[8]{cx}+\sqrt[4]{a}+\sqrt[4]{cx^2}\right)}{8\sqrt{2}(2-\sqrt{2})a^{7/8}c^{5/8}} - \frac{((1-\sqrt{2})\sqrt{cd}-\sqrt{ae})\log\left(\sqrt{2-\sqrt{2}}\sqrt[8]{a}\sqrt[8]{cx}+\sqrt[4]{a}+\sqrt[4]{cx^2}\right)}{8\sqrt{2}(2-\sqrt{2})a^{7/8}c^{5/8}}$$

[Out]  $-(\text{Sqrt}[2 - \text{Sqrt}[2]] * ((1 + \text{Sqrt}[2]) * \text{Sqrt}[c] * d - \text{Sqrt}[a] * e) * \text{ArcTan}[(\text{Sqrt}[2 - \text{Sqrt}[2]] * a^{(1/8)} - 2 * c^{(1/8)} * x) / (\text{Sqrt}[2 + \text{Sqrt}[2]] * a^{(1/8)})]) / (8 * a^{(7/8)} * c^{(5/8)}) + (\text{Sqrt}[2 + \text{Sqrt}[2]] * ((1 - \text{Sqrt}[2]) * \text{Sqrt}[c] * d - \text{Sqrt}[a] * e) * \text{ArcTan}[(\text{Sqrt}[2 + \text{Sqrt}[2]] * a^{(1/8)} - 2 * c^{(1/8)} * x) / (\text{Sqrt}[2 - \text{Sqrt}[2]] * a^{(1/8)})]) / (8 * a^{(7/8)} * c^{(5/8)}) + (\text{Sqrt}[2 - \text{Sqrt}[2]] * ((1 + \text{Sqrt}[2]) * \text{Sqrt}[c] * d - \text{Sqrt}[a] * e) * \text{ArcTan}[(\text{Sqrt}[2 - \text{Sqrt}[2]] * a^{(1/8)} + 2 * c^{(1/8)} * x) / (\text{Sqrt}[2 + \text{Sqrt}[2]] * a^{(1/8)})]) / (8 * a^{(7/8)} * c^{(5/8)}) - (\text{Sqrt}[2 + \text{Sqrt}[2]] * ((1 - \text{Sqrt}[2]) * \text{Sqrt}[c] * d - \text{Sqrt}[a] * e) * \text{ArcTan}[(\text{Sqrt}[2 + \text{Sqrt}[2]] * a^{(1/8)} + 2 * c^{(1/8)} * x) / (\text{Sqrt}[2 - \text{Sqrt}[2]] * a^{(1/8)})]) / (8 * a^{(7/8)} * c^{(5/8)}) + (((1 - \text{Sqrt}[2]) * \text{Sqrt}[c] * d - \text{Sqrt}[a] * e) * \text{Log}[a^{(1/4)} - \text{Sqrt}[2 - \text{Sqrt}[2]] * a^{(1/8)} * c^{(1/8)} * x + c^{(1/4)} * x^2]) / (8 * \text{Sqrt}[2 * (2 - \text{Sqrt}[2])] * a^{(7/8)} * c^{(5/8)}) - (((1 - \text{Sqrt}[2]) * \text{Sqrt}[c] * d - \text{Sqrt}[a] * e) * \text{Log}[a^{(1/4)} + \text{Sqrt}[2 - \text{Sqrt}[2]] * a^{(1/8)} * c^{(1/8)} * x + c^{(1/4)} * x^2]) / (8 * \text{Sqrt}[2 * (2 - \text{Sqrt}[2])] * a^{(7/8)} * c^{(5/8)}) - (((1 + \text{Sqrt}[2]) * \text{Sqrt}[c] * d - \text{Sqrt}[a] * e) * \text{Log}[a^{(1/4)} - \text{Sqrt}[2 + \text{Sqrt}[2]] * a^{(1/8)} * c^{(1/8)} * x + c^{(1/4)} * x^2]) / (8 * \text{Sqrt}[2 * (2 + \text{Sqrt}[2])] * a^{(7/8)} * c^{(5/8)}) + ((d + \text{Sqrt}[2] * d - (\text{Sqrt}[a] * e) / \text{Sqrt}[c]) * \text{Log}[a^{(1/4)} + \text{Sqrt}[2 + \text{Sqrt}[2]] * a^{(1/8)} * c^{(1/8)} * x + c^{(1/4)} * x^2]) / (8 * \text{Sqrt}[2 * (2 + \text{Sqrt}[2])] * a^{(7/8)} * c^{(1/8)})$

**Rubi [A]** time = 1.24683, antiderivative size = 754, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 6, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$ , Rules used = {1415, 1169, 634, 618, 204, 628}

$$\frac{((1-\sqrt{2})\sqrt{cd}-\sqrt{ae})\log\left(-\sqrt{2-\sqrt{2}}\sqrt[8]{a}\sqrt[8]{cx}+\sqrt[4]{a}+\sqrt[4]{cx^2}\right)}{8\sqrt{2}(2-\sqrt{2})a^{7/8}c^{5/8}} - \frac{((1-\sqrt{2})\sqrt{cd}-\sqrt{ae})\log\left(\sqrt{2-\sqrt{2}}\sqrt[8]{a}\sqrt[8]{cx}+\sqrt[4]{a}+\sqrt[4]{cx^2}\right)}{8\sqrt{2}(2-\sqrt{2})a^{7/8}c^{5/8}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(d + e * x^4) / (a + c * x^8), x]$

[Out]  $-(\text{Sqrt}[2 - \text{Sqrt}[2]] * ((1 + \text{Sqrt}[2]) * \text{Sqrt}[c] * d - \text{Sqrt}[a] * e) * \text{ArcTan}[(\text{Sqrt}[2 - \text{Sqrt}[2]] * a^{(1/8)} - 2 * c^{(1/8)} * x) / (\text{Sqrt}[2 + \text{Sqrt}[2]] * a^{(1/8)})]) / (8 * a^{(7/8)} * c^{(5/8)})$

$$\begin{aligned} & (5/8)) + (\text{Sqrt}[2 + \text{Sqrt}[2]]*((1 - \text{Sqrt}[2])*\text{Sqrt}[c]*d - \text{Sqrt}[a]*e)*\text{ArcTan}[(\text{Sqrt}[2 + \text{Sqrt}[2]]*a^{(1/8)} - 2*c^{(1/8)*x}/(\text{Sqrt}[2 - \text{Sqrt}[2]]*a^{(1/8)}))]/(8*a^{(7/8)*c^{(5/8)}}) + (\text{Sqrt}[2 - \text{Sqrt}[2]]*((1 + \text{Sqrt}[2])*\text{Sqrt}[c]*d - \text{Sqrt}[a]*e)*\text{ArcTan}[(\text{Sqrt}[2 - \text{Sqrt}[2]]*a^{(1/8)} + 2*c^{(1/8)*x}/(\text{Sqrt}[2 + \text{Sqrt}[2]]*a^{(1/8)}))]/(8*a^{(7/8)*c^{(5/8)}}) - (\text{Sqrt}[2 + \text{Sqrt}[2]]*((1 - \text{Sqrt}[2])*\text{Sqrt}[c]*d - \text{Sqrt}[a]*e)*\text{ArcTan}[(\text{Sqrt}[2 + \text{Sqrt}[2]]*a^{(1/8)} + 2*c^{(1/8)*x}/(\text{Sqrt}[2 - \text{Sqrt}[2]]*a^{(1/8)}))]/(8*a^{(7/8)*c^{(5/8)}}) + (((1 - \text{Sqrt}[2])*\text{Sqrt}[c]*d - \text{Sqrt}[a]*e)*\text{Log}[a^{(1/4)} - \text{Sqrt}[2 - \text{Sqrt}[2]]*a^{(1/8)*c^{(1/8)*x} + c^{(1/4)*x^2}]/(8*\text{Sqrt}[2*(2 - \text{Sqrt}[2]))]*a^{(7/8)*c^{(5/8)}}) - (((1 - \text{Sqrt}[2])*\text{Sqrt}[c]*d - \text{Sqrt}[a]*e)*\text{Log}[a^{(1/4)} + \text{Sqrt}[2 - \text{Sqrt}[2]]*a^{(1/8)*c^{(1/8)*x} + c^{(1/4)*x^2}]/(8*\text{Sqrt}[2*(2 - \text{Sqrt}[2]))]*a^{(7/8)*c^{(5/8)}}) - (((1 + \text{Sqrt}[2])*\text{Sqrt}[c]*d - \text{Sqrt}[a]*e)*\text{Log}[a^{(1/4)} - \text{Sqrt}[2 + \text{Sqrt}[2]]*a^{(1/8)*c^{(1/8)*x} + c^{(1/4)*x^2}]/(8*\text{Sqrt}[2*(2 + \text{Sqrt}[2]))]*a^{(7/8)*c^{(5/8)}}) + ((d + \text{Sqrt}[2]*d - (\text{Sqrt}[a]*e)/\text{Sqrt}[c])*\text{Log}[a^{(1/4)} + \text{Sqrt}[2 + \text{Sqrt}[2]]*a^{(1/8)*c^{(1/8)*x} + c^{(1/4)*x^2}]/(8*\text{Sqrt}[2*(2 + \text{Sqrt}[2]))]*a^{(7/8)*c^{(1/8)}}) \end{aligned}$$

### Rule 1415

$$\begin{aligned} & \text{Int}[\frac{(d_.) + (e_.)*(x_)^{(n_)}}{(a_.) + (c_.)*(x_)^{(n2_)}}, x\_Symbol] \text{ :> With}[\{q = \text{Rt}[a/c, 4]\}, \text{Dist}[1/(2*\text{Sqrt}[2]*c*q^3), \text{Int}[(\text{Sqrt}[2]*d*q - (d - e*q^2)*x^{(n/2)})/(q^2 - \text{Sqrt}[2]*q*x^{(n/2)} + x^n), x], x] + \text{Dist}[1/(2*\text{Sqrt}[2]*c*q^3), \text{Int}[(\text{Sqrt}[2]*d*q + (d - e*q^2)*x^{(n/2)})/(q^2 + \text{Sqrt}[2]*q*x^{(n/2)} + x^n), x], x]] /; \text{FreeQ}[\{a, c, d, e\}, x] \&\& \text{EqQ}[n2, 2*n] \&\& \text{NeQ}[c*d^2 + a*e^2, 0] \&\& \text{NeQ}[c*d^2 - a*e^2, 0] \&\& \text{IGtQ}[n/2, 0] \&\& \text{PosQ}[a*c] \end{aligned}$$

### Rule 1169

$$\begin{aligned} & \text{Int}[\frac{(d_.) + (e_.)*(x_)^2}{(a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4}, x\_Symbol] \text{ :> With}[\{q = \text{Rt}[a/c, 2]\}, \text{With}[\{r = \text{Rt}[2*q - b/c, 2]\}, \text{Dist}[1/(2*c*q*r), \text{Int}[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + \text{Dist}[1/(2*c*q*r), \text{Int}[(d*r + (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; \text{FreeQ}[\{a, b, c, d, e\}, x] \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& \text{NeQ}[c*d^2 - b*d*e + a*e^2, 0] \&\& \text{NegQ}[b^2 - 4*a*c] \end{aligned}$$

### Rule 634

$$\begin{aligned} & \text{Int}[\frac{(d_.) + (e_.)*(x_)}{(a_.) + (b_.)*(x_) + (c_.)*(x_)^2}, x\_Symbol] \text{ :> Dist}[(2*c*d - b*e)/(2*c), \text{Int}[1/(a + b*x + c*x^2), x], x] + \text{Dist}[e/(2*c), \text{Int}[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; \text{FreeQ}[\{a, b, c, d, e\}, x] \&\& \text{NeQ}[2*c*d - b*e, 0] \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& \text{!NiceSqrtQ}[b^2 - 4*a*c] \end{aligned}$$

### Rule 618

$$\begin{aligned} & \text{Int}[\frac{(a_.) + (b_.)*(x_) + (c_.)*(x_)^2}{(a_.) + (b_.)*(x_) + (c_.)*(x_)^2}^{-1}, x\_Symbol] \text{ :> Dist}[-2, \text{Subst}[\text{Int}[1/\text{Simp}[b^2 - 4*a*c - x^2, x], x], x], x, b + 2*c*x], x] /; \text{FreeQ}[\{a, b, c\}, \end{aligned}$$

x] && NeQ[b^2 - 4\*a\*c, 0]

### Rule 204

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

### Rule 628

Int[((d\_) + (e\_)\*(x\_))/((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2), x\_Symbol] := Simp[(d\*Log[RemoveContent[a + b\*x + c\*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

### Rubi steps

$$\begin{aligned}
 \int \frac{d + ex^4}{a + cx^8} dx &= \frac{\int \frac{\frac{\sqrt{2}\sqrt[4]{ad} + (-d + \frac{\sqrt{ae}}{\sqrt{c}})x^2}{\sqrt{c}}}{\frac{\sqrt{a}}{\sqrt{c}} - \frac{\sqrt{2}\sqrt[4]{ax^2}}{\sqrt{c}} + x^4} dx}{2\sqrt{2}a^{3/4}\sqrt[4]{c}} + \frac{\int \frac{\frac{\sqrt{2}\sqrt[4]{ad} + (d - \frac{\sqrt{ae}}{\sqrt{c}})x^2}{\sqrt{c}}}{\frac{\sqrt{a}}{\sqrt{c}} + \frac{\sqrt{2}\sqrt[4]{ax^2}}{\sqrt{c}} + x^4} dx}{2\sqrt{2}a^{3/4}\sqrt[4]{c}} \\
 &= \frac{\sqrt[8]{c} \int \frac{\frac{\sqrt{2(2-\sqrt{2})}a^{3/8}d}{c^{3/8}} - \left(\frac{\sqrt{2}\sqrt[4]{ad}}{\sqrt[4]{c}} - \frac{\sqrt[4]{a}(d - \frac{\sqrt{ae}}{\sqrt{c}})}{\sqrt[4]{c}}\right)x}{\frac{\sqrt[4]{a}}{\sqrt[4]{c}} - \frac{\sqrt{2-\sqrt{2}}\sqrt[8]{ax}}{\sqrt[8]{c}} + x^2} dx}{4\sqrt{2(2-\sqrt{2})}a^{9/8}} + \frac{\sqrt[8]{c} \int \frac{\frac{\sqrt{2(2-\sqrt{2})}a^{3/8}d}{c^{3/8}} + \left(\frac{\sqrt{2}\sqrt[4]{ad}}{\sqrt[4]{c}} - \frac{\sqrt[4]{a}(d - \frac{\sqrt{ae}}{\sqrt{c}})}{\sqrt[4]{c}}\right)x}{\frac{\sqrt[4]{a}}{\sqrt[4]{c}} + \frac{\sqrt{2-\sqrt{2}}\sqrt[8]{ax}}{\sqrt[8]{c}} + x^2} dx}{4\sqrt{2(2-\sqrt{2})}a^{9/8}} + \frac{\sqrt[8]{c} \int \frac{\frac{\sqrt{2(2+\sqrt{2})}a^{3/8}d}{c^{3/8}} - \left(\frac{\sqrt{2}\sqrt[4]{ad}}{\sqrt[4]{c}} - \frac{\sqrt[4]{a}(d - \frac{\sqrt{ae}}{\sqrt{c}})}{\sqrt[4]{c}}\right)x}{\frac{\sqrt[4]{a}}{\sqrt[4]{c}} - \frac{\sqrt{2+\sqrt{2}}\sqrt[8]{ax}}{\sqrt[8]{c}} + x^2} dx}{4\sqrt{2(2+\sqrt{2})}a^{9/8}} \\
 &= \frac{((1-\sqrt{2})\sqrt{cd} - \sqrt{ae}) \int \frac{1}{\frac{\sqrt[4]{a}}{\sqrt[4]{c}} - \frac{\sqrt{2+\sqrt{2}}\sqrt[8]{ax}}{\sqrt[8]{c}} + x^2} dx}{8\sqrt{2}a^{3/4}c^{3/4}} - \frac{((1-\sqrt{2})\sqrt{cd} - \sqrt{ae}) \int \frac{1}{\frac{\sqrt[4]{a}}{\sqrt[4]{c}} + \frac{\sqrt{2+\sqrt{2}}\sqrt[8]{ax}}{\sqrt[8]{c}} + x^2} dx}{8\sqrt{2}a^{3/4}c^{3/4}} + \frac{((1-\sqrt{2})\sqrt{cd} - \sqrt{ae}) \int \frac{1}{\frac{\sqrt[4]{a}}{\sqrt[4]{c}} - \frac{\sqrt{2-\sqrt{2}}\sqrt[8]{ax}}{\sqrt[8]{c}} + x^2} dx}{8\sqrt{2}a^{3/4}c^{3/4}} \\
 &= \frac{((1-\sqrt{2})\sqrt{cd} - \sqrt{ae}) \log\left(\frac{\sqrt[4]{a} - \sqrt{2-\sqrt{2}}\sqrt[8]{a}\sqrt[8]{cx} + \sqrt[4]{cx^2}}{\sqrt[4]{a} + \sqrt{2-\sqrt{2}}\sqrt[8]{a}\sqrt[8]{cx} + \sqrt[4]{cx^2}}\right)}{8\sqrt{2(2-\sqrt{2})}a^{7/8}c^{5/8}} - \frac{((1-\sqrt{2})\sqrt{cd} - \sqrt{ae}) \log\left(\frac{\sqrt[4]{a} + \sqrt{2-\sqrt{2}}\sqrt[8]{a}\sqrt[8]{cx} + \sqrt[4]{cx^2}}{\sqrt[4]{a} - \sqrt{2-\sqrt{2}}\sqrt[8]{a}\sqrt[8]{cx} + \sqrt[4]{cx^2}}\right)}{8\sqrt{2(2-\sqrt{2})}a^{7/8}c^{5/8}} \\
 &= \frac{((1+\sqrt{2})\sqrt{cd} - \sqrt{ae}) \tan^{-1}\left(\frac{\sqrt{2-\sqrt{2}}\sqrt[8]{a-2}\sqrt[8]{cx}}{\sqrt{2+\sqrt{2}}\sqrt[8]{a}}\right)}{4\sqrt{2(2+\sqrt{2})}a^{7/8}c^{5/8}} + \frac{((1-\sqrt{2})\sqrt{cd} - \sqrt{ae}) \tan^{-1}\left(\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{a-2}\sqrt[8]{cx}}{\sqrt{2-\sqrt{2}}\sqrt[8]{a}}\right)}{4\sqrt{2(2-\sqrt{2})}a^{7/8}c^{5/8}} + \frac{((1-\sqrt{2})\sqrt{cd} - \sqrt{ae}) \tan^{-1}\left(\frac{\sqrt{2-\sqrt{2}}\sqrt[8]{a}\sqrt[8]{cx}}{\sqrt{2+\sqrt{2}}\sqrt[8]{a}}\right)}{4\sqrt{2(2-\sqrt{2})}a^{7/8}c^{5/8}}
 \end{aligned}$$

**Mathematica [A]** time = 0.623322, size = 534, normalized size = 0.71

$$2 \tan^{-1} \left( \frac{\sqrt[8]{cx} \sec\left(\frac{\pi}{8}\right)}{\sqrt[8]{a}} - \tan\left(\frac{\pi}{8}\right) \right) \left( \sqrt[8]{a} \sqrt{cd} \cos\left(\frac{\pi}{8}\right) - a^{5/8} e \sin\left(\frac{\pi}{8}\right) \right) + 2 \tan^{-1} \left( \frac{\sqrt[8]{cx} \sec\left(\frac{\pi}{8}\right)}{\sqrt[8]{a}} + \tan\left(\frac{\pi}{8}\right) \right) \left( \sqrt[8]{a} \sqrt{cd} \cos\left(\frac{\pi}{8}\right) - a^{5/8} e \right)$$

Antiderivative was successfully verified.

[In] Integrate[(d + e\*x^4)/(a + c\*x^8), x]

[Out]  $(-2*a^{(1/8)}*ArcTan[Cot[Pi/8] - (c^{(1/8)}*x*Csc[Pi/8])/a^{(1/8)}]*(Sqrt[a]*e*Cos[Pi/8] + Sqrt[c]*d*Sin[Pi/8]) + 2*a^{(1/8)}*ArcTan[Cot[Pi/8] + (c^{(1/8)}*x*Csc[Pi/8])/a^{(1/8)}]*(Sqrt[a]*e*Cos[Pi/8] + Sqrt[c]*d*Sin[Pi/8]) - a^{(1/8)}*Log[a^{(1/4)} + c^{(1/4)}*x^2 - 2*a^{(1/8)}*c^{(1/8)}*x*Sin[Pi/8]]*(Sqrt[a]*e*Cos[Pi/8] + Sqrt[c]*d*Sin[Pi/8]) + a^{(1/8)}*Log[a^{(1/4)} + c^{(1/4)}*x^2 + 2*a^{(1/8)}*c^{(1/8)}*x*Sin[Pi/8]]*(Sqrt[a]*e*Cos[Pi/8] + Sqrt[c]*d*Sin[Pi/8]) + a^{(1/8)}*Log[a^{(1/4)} + c^{(1/4)}*x^2 - 2*a^{(1/8)}*c^{(1/8)}*x*Cos[Pi/8]]*(-(Sqrt[c]*d*Cos[Pi/8]) + Sqrt[a]*e*Sin[Pi/8]) - a^{(1/8)}*Log[a^{(1/4)} + c^{(1/4)}*x^2 + 2*a^{(1/8)}*c^{(1/8)}*x*Cos[Pi/8]]*(-(Sqrt[c]*d*Cos[Pi/8]) + Sqrt[a]*e*Sin[Pi/8]) + 2*ArcTan[(c^{(1/8)}*x*Sec[Pi/8])/a^{(1/8)} - Tan[Pi/8]]*(a^{(1/8)}*Sqrt[c]*d*Cos[Pi/8] - a^{(5/8)}*e*Sin[Pi/8]) + 2*ArcTan[(c^{(1/8)}*x*Sec[Pi/8])/a^{(1/8)} + Tan[Pi/8]]*(a^{(1/8)}*Sqrt[c]*d*Cos[Pi/8] - a^{(5/8)}*e*Sin[Pi/8]))/(8*a*c^{(5/8)})$

**Maple [C]** time = 0.023, size = 34, normalized size = 0.1

$$\frac{1}{8c} \sum_{_R=\text{RootOf}(c_Z^8+a)} \frac{(_R^4 e + d) \ln(x - _R)}{_R^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x^4+d)/(c\*x^8+a), x)

[Out] 1/8/c\*sum((\_R^4\*e+d)/\_R^7\*ln(x-\_R), \_R=RootOf(\_Z^8\*c+a))

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{ex^4 + d}{cx^8 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^4+d)/(c*x^8+a),x, algorithm="maxima")
```

```
[Out] integrate((e*x^4 + d)/(c*x^8 + a), x)
```

**Fricas [B]** time = 3.36605, size = 6978, normalized size = 9.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^4+d)/(c*x^8+a),x, algorithm="fricas")
```

```
[Out] -1/2*((a^3*c^2*sqrt(-(c^4*d^8 - 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*
a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)) - 4*c*d^3*e + 4*a*d*e^3)/(a^3*c^2))^(1/
4)*arctan(-((3*a^3*c^5*d^6*e - 19*a^4*c^4*d^4*e^3 + 9*a^5*c^3*d^2*e^5 - a^6
*c^2*e^7 + (a^6*c^6*d^3 - 3*a^7*c^5*d*e^2)*sqrt(-(c^4*d^8 - 12*a*c^3*d^6*e^
2 + 38*a^2*c^2*d^4*e^4 - 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)))*sqrt(((c^4
*d^8 - 4*a*c^3*d^6*e^2 - 10*a^2*c^2*d^4*e^4 - 4*a^3*c*d^2*e^6 + a^4*e^8)*x^
2 - (2*a^6*c^4*d*e*sqrt(-(c^4*d^8 - 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 -
12*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)) - a^2*c^4*d^6 + 7*a^3*c^3*d^4*e^2 -
7*a^4*c^2*d^2*e^4 + a^5*c*e^6)*sqrt((a^3*c^2*sqrt(-(c^4*d^8 - 12*a*c^3*d^6
*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)) - 4*c*d^
3*e + 4*a*d*e^3)/(a^3*c^2)))/(c^4*d^8 - 4*a*c^3*d^6*e^2 - 10*a^2*c^2*d^4*e^
4 - 4*a^3*c*d^2*e^6 + a^4*e^8))*sqrt((a^3*c^2*sqrt(-(c^4*d^8 - 12*a*c^3*d^6
*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)) - 4*c*d^
3*e + 4*a*d*e^3)/(a^3*c^2)) - ((a^6*c^6*d^3 - 3*a^7*c^5*d*e^2)*x*sqrt(-(c^4
*d^8 - 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a^3*c*d^2*e^6 + a^4*e^8)/
(a^7*c^5)) + (3*a^3*c^5*d^6*e - 19*a^4*c^4*d^4*e^3 + 9*a^5*c^3*d^2*e^5 - a^
6*c^2*e^7)*x)*sqrt((a^3*c^2*sqrt(-(c^4*d^8 - 12*a*c^3*d^6*e^2 + 38*a^2*c^2*
d^4*e^4 - 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)) - 4*c*d^3*e + 4*a*d*e^3)/(
a^3*c^2)))*((a^3*c^2*sqrt(-(c^4*d^8 - 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4
- 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)) - 4*c*d^3*e + 4*a*d*e^3)/(a^3*c^2
))^1/4)/(c^5*d^10 - 3*a*c^4*d^8*e^2 - 14*a^2*c^3*d^6*e^4 - 14*a^3*c^2*d^4*
e^6 - 3*a^4*c*d^2*e^8 + a^5*e^10)) + 1/2*(-(a^3*c^2*sqrt(-(c^4*d^8 - 12*a*c
^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)) +
4*c*d^3*e - 4*a*d*e^3)/(a^3*c^2))^1/4)*arctan(((3*a^3*c^5*d^6*e - 19*a^4*c
^4*d^4*e^3 + 9*a^5*c^3*d^2*e^5 - a^6*c^2*e^7 - (a^6*c^6*d^3 - 3*a^7*c^5*d*e
^2)*sqrt(-(c^4*d^8 - 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a^3*c*d^2*e
^6 + a^4*e^8)/(a^7*c^5)))*sqrt(((c^4*d^8 - 4*a*c^3*d^6*e^2 - 10*a^2*c^2*d^4
*e^4 - 4*a^3*c*d^2*e^6 + a^4*e^8)*x^2 + (2*a^6*c^4*d*e*sqrt(-(c^4*d^8 - 12*
a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5))
+ a^2*c^4*d^6 - 7*a^3*c^3*d^4*e^2 + 7*a^4*c^2*d^2*e^4 - a^5*c*e^6)*sqrt(-(
```

$$\begin{aligned}
& a^3c^2\sqrt{-(c^4d^8 - 12a^3c^3d^6e^2 + 38a^2c^2d^4e^4 - 12a^3c^3d^2e^6 + a^4e^8)/(a^7c^5)} + 4c^3d^3e - 4a^3d^3e^3/(a^3c^2)) / (c^4d^8 - 4a^3c^3d^6e^2 - 10a^2c^2d^4e^4 - 4a^3c^3d^2e^6 + a^4e^8)) * (- (a^3c^2\sqrt{-(c^4d^8 - 12a^3c^3d^6e^2 + 38a^2c^2d^4e^4 - 12a^3c^3d^2e^6 + a^4e^8)/(a^7c^5)} + 4c^3d^3e - 4a^3d^3e^3/(a^3c^2))^{3/4} + ((a^6c^6d^3 - 3a^7c^5d^2e^2) * x * \sqrt{-(c^4d^8 - 12a^3c^3d^6e^2 + 38a^2c^2d^4e^4 - 12a^3c^3d^2e^6 + a^4e^8)/(a^7c^5)} - (3a^3c^5d^6e - 19a^4c^4d^4e^3 + 9a^5c^3d^2e^5 - a^6c^2e^7) * x) * (- (a^3c^2\sqrt{-(c^4d^8 - 12a^3c^3d^6e^2 + 38a^2c^2d^4e^4 - 12a^3c^3d^2e^6 + a^4e^8)/(a^7c^5)} + 4c^3d^3e - 4a^3d^3e^3/(a^3c^2))^{3/4}) / (c^5d^{10} - 3a^3c^4d^8e^2 - 14a^2c^3d^6e^4 - 14a^3c^2d^4e^6 - 3a^4c^2d^2e^8 + a^5e^{10})) + 1/8 * (- (a^3c^2\sqrt{-(c^4d^8 - 12a^3c^3d^6e^2 + 38a^2c^2d^4e^4 - 12a^3c^3d^2e^6 + a^4e^8)/(a^7c^5)} + 4c^3d^3e - 4a^3d^3e^3/(a^3c^2))^{1/4} * \log((c^3d^6 - 5a^3c^2d^4e^2 - 5a^2c^2d^2e^4 + a^3e^6) * x + (a^5c^3e * \sqrt{-(c^4d^8 - 12a^3c^3d^6e^2 + 38a^2c^2d^4e^4 - 12a^3c^3d^2e^6 + a^4e^8)/(a^7c^5)} + a^3c^3d^5 - 6a^2c^2d^3e^2 + a^3c^3d^3e^4) * (- (a^3c^2\sqrt{-(c^4d^8 - 12a^3c^3d^6e^2 + 38a^2c^2d^4e^4 - 12a^3c^3d^2e^6 + a^4e^8)/(a^7c^5)} + 4c^3d^3e - 4a^3d^3e^3/(a^3c^2))^{1/4}) - 1/8 * (- (a^3c^2\sqrt{-(c^4d^8 - 12a^3c^3d^6e^2 + 38a^2c^2d^4e^4 - 12a^3c^3d^2e^6 + a^4e^8)/(a^7c^5)} + 4c^3d^3e - 4a^3d^3e^3/(a^3c^2))^{1/4} * \log((c^3d^6 - 5a^3c^2d^4e^2 - 5a^2c^2d^2e^4 + a^3e^6) * x - (a^5c^3e * \sqrt{-(c^4d^8 - 12a^3c^3d^6e^2 + 38a^2c^2d^4e^4 - 12a^3c^3d^2e^6 + a^4e^8)/(a^7c^5)} + a^3c^3d^5 - 6a^2c^2d^3e^2 + a^3c^3d^3e^4) * (- (a^3c^2\sqrt{-(c^4d^8 - 12a^3c^3d^6e^2 + 38a^2c^2d^4e^4 - 12a^3c^3d^2e^6 + a^4e^8)/(a^7c^5)} + 4c^3d^3e - 4a^3d^3e^3/(a^3c^2))^{1/4}) - 1/8 * ((a^3c^2\sqrt{-(c^4d^8 - 12a^3c^3d^6e^2 + 38a^2c^2d^4e^4 - 12a^3c^3d^2e^6 + a^4e^8)/(a^7c^5)} - 4c^3d^3e + 4a^3d^3e^3/(a^3c^2))^{1/4} * \log((c^3d^6 - 5a^3c^2d^4e^2 - 5a^2c^2d^2e^4 + a^3e^6) * x + (a^5c^3e * \sqrt{-(c^4d^8 - 12a^3c^3d^6e^2 + 38a^2c^2d^4e^4 - 12a^3c^3d^2e^6 + a^4e^8)/(a^7c^5)} - a^3c^3d^5 + 6a^2c^2d^3e^2 - a^3c^3d^3e^4) * ((a^3c^2\sqrt{-(c^4d^8 - 12a^3c^3d^6e^2 + 38a^2c^2d^4e^4 - 12a^3c^3d^2e^6 + a^4e^8)/(a^7c^5)} - 4c^3d^3e + 4a^3d^3e^3/(a^3c^2))^{1/4}) + 1/8 * ((a^3c^2\sqrt{-(c^4d^8 - 12a^3c^3d^6e^2 + 38a^2c^2d^4e^4 - 12a^3c^3d^2e^6 + a^4e^8)/(a^7c^5)} - 4c^3d^3e + 4a^3d^3e^3/(a^3c^2))^{1/4} * \log((c^3d^6 - 5a^3c^2d^4e^2 - 5a^2c^2d^2e^4 + a^3e^6) * x - (a^5c^3e * \sqrt{-(c^4d^8 - 12a^3c^3d^6e^2 + 38a^2c^2d^4e^4 - 12a^3c^3d^2e^6 + a^4e^8)/(a^7c^5)} - a^3c^3d^5 + 6a^2c^2d^3e^2 - a^3c^3d^3e^4) * ((a^3c^2\sqrt{-(c^4d^8 - 12a^3c^3d^6e^2 + 38a^2c^2d^4e^4 - 12a^3c^3d^2e^6 + a^4e^8)/(a^7c^5)} - 4c^3d^3e + 4a^3d^3e^3/(a^3c^2))^{1/4}))
\end{aligned}$$


---

**Sympy [A]** time = 12.6218, size = 199, normalized size = 0.26

$$\text{RootSum}\left(16777216t^8a^7c^5 + t^4(-32768a^5c^3de^3 + 32768a^4c^4d^3e) + a^4e^8 + 4a^3cd^2e^6 + 6a^2c^2d^4e^4 + 4ac^3d^6e^2 + c^4d^8, t\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x\*\*4+d)/(c\*x\*\*8+a),x)

[Out] RootSum(16777216\*\_t\*\*8\*a\*\*7\*c\*\*5 + \_t\*\*4\*(-32768\*a\*\*5\*c\*\*3\*d\*e\*\*3 + 32768\*a\*\*4\*c\*\*4\*d\*\*3\*e) + a\*\*4\*e\*\*8 + 4\*a\*\*3\*c\*d\*\*2\*e\*\*6 + 6\*a\*\*2\*c\*\*2\*d\*\*4\*e\*\*4 + 4\*a\*c\*\*3\*d\*\*6\*e\*\*2 + c\*\*4\*d\*\*8, Lambda(\_t, \_t\*log(x + (-32768\*\_t\*\*5\*a\*\*5\*c\*\*3\*e + 40\*\_t\*a\*\*3\*c\*d\*e\*\*4 - 80\*\_t\*a\*\*2\*c\*\*2\*d\*\*3\*e\*\*2 + 8\*\_t\*a\*c\*\*3\*d\*\*5)/(a\*\*3\*e\*\*6 - 5\*a\*\*2\*c\*d\*\*2\*e\*\*4 - 5\*a\*c\*\*2\*d\*\*4\*e\*\*2 + c\*\*3\*d\*\*6))))

**Giac [A]** time = 1.1892, size = 811, normalized size = 1.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^4+d)/(c\*x^8+a),x, algorithm="giac")

[Out] 
$$\begin{aligned} & -1/8*(\sqrt{-\sqrt{2} + 2}*(a/c)^{(5/8)}*e - d*\sqrt{\sqrt{2} + 2}*(a/c)^{(1/8)})*a \\ & \text{rctan}((2*x + \sqrt{-\sqrt{2} + 2}*(a/c)^{(1/8)})/(\sqrt{\sqrt{2} + 2}*(a/c)^{(1/8)})) \\ & )/a - 1/8*(\sqrt{-\sqrt{2} + 2}*(a/c)^{(5/8)}*e - d*\sqrt{\sqrt{2} + 2}*(a/c)^{(1/8)}) \\ & * \arctan((2*x - \sqrt{-\sqrt{2} + 2}*(a/c)^{(1/8)})/(\sqrt{\sqrt{2} + 2}*(a/c)^{(1/8)})) \\ & )/a + 1/8*(\sqrt{\sqrt{2} + 2}*(a/c)^{(5/8)}*e + d*\sqrt{-\sqrt{2} + 2}*(a/c)^{(1/8)}) \\ & * \arctan((2*x + \sqrt{\sqrt{2} + 2}*(a/c)^{(1/8)})/(\sqrt{-\sqrt{2} + 2}*(a/c)^{(1/8)})) \\ & )/a + 1/8*(\sqrt{\sqrt{2} + 2}*(a/c)^{(5/8)}*e + d*\sqrt{-\sqrt{2} + 2}*(a/c)^{(1/8)}) \\ & * \arctan((2*x - \sqrt{\sqrt{2} + 2}*(a/c)^{(1/8)})/(\sqrt{-\sqrt{2} + 2}*(a/c)^{(1/8)})) \\ & )/a - 1/16*(\sqrt{-\sqrt{2} + 2}*(a/c)^{(5/8)}*e - d*\sqrt{\sqrt{2} + 2}*(a/c)^{(1/8)}) \\ & * \log(x^2 + x*\sqrt{\sqrt{2} + 2}*(a/c)^{(1/8)} + (a/c)^{(1/4)})/a + 1/16*(\sqrt{-\sqrt{2} + 2}*(a/c)^{(5/8)}*e - d*\sqrt{\sqrt{2} + 2}*(a/c)^{(1/8)}) \\ & * \log(x^2 - x*\sqrt{\sqrt{2} + 2}*(a/c)^{(1/8)} + (a/c)^{(1/4)})/a + 1/16*(\sqrt{\sqrt{2} + 2}*(a/c)^{(5/8)}*e + d*\sqrt{-\sqrt{2} + 2}*(a/c)^{(1/8)}) \\ & * \log(x^2 + x*\sqrt{-\sqrt{2} + 2}*(a/c)^{(1/8)} + (a/c)^{(1/4)})/a - 1/16*(\sqrt{\sqrt{2} + 2}*(a/c)^{(5/8)}*e + d*\sqrt{-\sqrt{2} + 2}*(a/c)^{(1/8)}) \\ & * \log(x^2 - x*\sqrt{-\sqrt{2} + 2}*(a/c)^{(1/8)} + (a/c)^{(1/4)})/a \end{aligned}$$

### 3.4 $\int \frac{d+ex^4}{a-cx^8} dx$

**Optimal.** Leaf size=329

$$\frac{(\sqrt{ae} + \sqrt{cd}) \tan^{-1}\left(\frac{\sqrt[8]{cx}}{\sqrt[8]{a}}\right)}{4a^{7/8}c^{5/8}} + \frac{(\sqrt{ae} + \sqrt{cd}) \tanh^{-1}\left(\frac{\sqrt[8]{cx}}{\sqrt[8]{a}}\right)}{4a^{7/8}c^{5/8}} - \frac{\left(d - \frac{\sqrt{ae}}{\sqrt{c}}\right) \log\left(-\sqrt{2}\sqrt[8]{a}\sqrt[8]{cx} + \sqrt[4]{a} + \sqrt[4]{cx^2}\right)}{8\sqrt{2}a^{7/8}\sqrt[8]{c}} + \frac{\left(d - \frac{\sqrt{ae}}{\sqrt{c}}\right) \log\left(\sqrt{2}\sqrt[8]{a}\sqrt[8]{cx} + \sqrt[4]{a} + \sqrt[4]{cx^2}\right)}{8\sqrt{2}a^{7/8}\sqrt[8]{c}}$$

[Out] ((Sqrt[c]\*d + Sqrt[a]\*e)\*ArcTan[(c^(1/8)\*x)/a^(1/8)]/(4\*a^(7/8)\*c^(5/8)) - ((d - (Sqrt[a]\*e)/Sqrt[c])\*ArcTan[1 - (Sqrt[2]\*c^(1/8)\*x)/a^(1/8)]/(4\*Sqrt[2]\*a^(7/8)\*c^(1/8)) + ((d - (Sqrt[a]\*e)/Sqrt[c])\*ArcTan[1 + (Sqrt[2]\*c^(1/8)\*x)/a^(1/8)]/(4\*Sqrt[2]\*a^(7/8)\*c^(1/8)) + ((Sqrt[c]\*d + Sqrt[a]\*e)\*ArcTanH[(c^(1/8)\*x)/a^(1/8)]/(4\*a^(7/8)\*c^(5/8)) - ((d - (Sqrt[a]\*e)/Sqrt[c])\*Log[a^(1/4) - Sqrt[2]\*a^(1/8)\*c^(1/8)\*x + c^(1/4)\*x^2])/(8\*Sqrt[2]\*a^(7/8)\*c^(1/8)) + ((d - (Sqrt[a]\*e)/Sqrt[c])\*Log[a^(1/4) + Sqrt[2]\*a^(1/8)\*c^(1/8)\*x + c^(1/4)\*x^2])/(8\*Sqrt[2]\*a^(7/8)\*c^(1/8))

**Rubi [A]** time = 0.209223, antiderivative size = 329, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$ , Rules used = {1417, 212, 208, 205, 211, 1165, 628, 1162, 617, 204}

$$\frac{(\sqrt{ae} + \sqrt{cd}) \tan^{-1}\left(\frac{\sqrt[8]{cx}}{\sqrt[8]{a}}\right)}{4a^{7/8}c^{5/8}} + \frac{(\sqrt{ae} + \sqrt{cd}) \tanh^{-1}\left(\frac{\sqrt[8]{cx}}{\sqrt[8]{a}}\right)}{4a^{7/8}c^{5/8}} - \frac{\left(d - \frac{\sqrt{ae}}{\sqrt{c}}\right) \log\left(-\sqrt{2}\sqrt[8]{a}\sqrt[8]{cx} + \sqrt[4]{a} + \sqrt[4]{cx^2}\right)}{8\sqrt{2}a^{7/8}\sqrt[8]{c}} + \frac{\left(d - \frac{\sqrt{ae}}{\sqrt{c}}\right) \log\left(\sqrt{2}\sqrt[8]{a}\sqrt[8]{cx} + \sqrt[4]{a} + \sqrt[4]{cx^2}\right)}{8\sqrt{2}a^{7/8}\sqrt[8]{c}}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x^4)/(a - c\*x^8), x]

[Out] ((Sqrt[c]\*d + Sqrt[a]\*e)\*ArcTan[(c^(1/8)\*x)/a^(1/8)]/(4\*a^(7/8)\*c^(5/8)) - ((d - (Sqrt[a]\*e)/Sqrt[c])\*ArcTan[1 - (Sqrt[2]\*c^(1/8)\*x)/a^(1/8)]/(4\*Sqrt[2]\*a^(7/8)\*c^(1/8)) + ((d - (Sqrt[a]\*e)/Sqrt[c])\*ArcTan[1 + (Sqrt[2]\*c^(1/8)\*x)/a^(1/8)]/(4\*Sqrt[2]\*a^(7/8)\*c^(1/8)) + ((Sqrt[c]\*d + Sqrt[a]\*e)\*ArcTanH[(c^(1/8)\*x)/a^(1/8)]/(4\*a^(7/8)\*c^(5/8)) - ((d - (Sqrt[a]\*e)/Sqrt[c])\*Log[a^(1/4) - Sqrt[2]\*a^(1/8)\*c^(1/8)\*x + c^(1/4)\*x^2])/(8\*Sqrt[2]\*a^(7/8)\*c^(1/8)) + ((d - (Sqrt[a]\*e)/Sqrt[c])\*Log[a^(1/4) + Sqrt[2]\*a^(1/8)\*c^(1/8)\*x + c^(1/4)\*x^2])/(8\*Sqrt[2]\*a^(7/8)\*c^(1/8))

#### Rule 1417

Int[((d\_) + (e\_.)\*(x\_)^(n\_))/((a\_) + (c\_.)\*(x\_)^(n2\_)), x\_Symbol] := With[{q = Rt[-(a/c), 2]}, Dist[(d + e\*q)/2, Int[1/(a + c\*q\*x^n), x], x] + Dist[(d



- e\*q)/2, Int[1/(a - c\*q\*x^n), x], x]] /; FreeQ[{a, c, d, e, n}, x] && EqQ[n2, 2\*n] && NeQ[c\*d^2 + a\*e^2, 0] && NegQ[a\*c] && IntegerQ[n]

### Rule 212

Int[((a\_) + (b\_)\*(x\_)^4)^(-1), x\_Symbol] := With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2\*a), Int[1/(r - s\*x^2), x], x] + Dist[r/(2\*a), Int[1/(r + s\*x^2), x], x]] /; FreeQ[{a, b}, x] && !GtQ[a/b, 0]

### Rule 208

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[-(a/b), 2]\*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

### Rule 205

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

### Rule 211

Int[((a\_) + (b\_)\*(x\_)^4)^(-1), x\_Symbol] := With[{r = Numerator[Rt[a/b, 2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2\*r), Int[(r - s\*x^2)/(a + b\*x^4), x], x] + Dist[1/(2\*r), Int[(r + s\*x^2)/(a + b\*x^4), x], x]] /; FreeQ[{a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] && AtomQ[SplitProduct[SumBaseQ, b]]))

### Rule 1165

Int[((d\_) + (e\_)\*(x\_)^2)/((a\_) + (c\_)\*(x\_)^4), x\_Symbol] := With[{q = Rt[(-2\*d)/e, 2]}, Dist[e/(2\*c\*q), Int[(q - 2\*x)/Simp[d/e + q\*x - x^2, x], x], x] + Dist[e/(2\*c\*q), Int[(q + 2\*x)/Simp[d/e - q\*x - x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && NegQ[d\*e]

### Rule 628

Int[((d\_) + (e\_)\*(x\_))/((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2), x\_Symbol] := Simp[(d\*Log[RemoveContent[a + b\*x + c\*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

### Rule 1162

Int[((d\_) + (e\_)\*(x\_)^2)/((a\_) + (c\_)\*(x\_)^4), x\_Symbol] := With[{q = Rt[(2\*d)/e, 2]}, Dist[e/(2\*c), Int[1/Simp[d/e + q\*x + x^2, x], x], x] + Dist[e

/(2\*c), Int[1/Simp[d/e - q\*x + x^2, x], x] /; FreeQ[{a, c, d, e}, x] &  
& EqQ[c\*d^2 - a\*e^2, 0] && PosQ[d\*e]

### Rule 617

Int[((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := With[{q = 1 - 4\*S  
implify[(a\*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2\*c\*x)/b  
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4\*a\*c))] /; Free  
Q[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[  
-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[  
a, 0] || LtQ[b, 0])

### Rubi steps

$$\begin{aligned}
 \int \frac{d + ex^4}{a - cx^8} dx &= \frac{1}{2} \left( d - \frac{\sqrt{ae}}{\sqrt{c}} \right) \int \frac{1}{a + \sqrt{a}\sqrt{cx^4}} dx + \frac{1}{2} \left( d + \frac{\sqrt{ae}}{\sqrt{c}} \right) \int \frac{1}{a - \sqrt{a}\sqrt{cx^4}} dx \\
 &= \frac{\left( d - \frac{\sqrt{ae}}{\sqrt{c}} \right) \int \frac{\sqrt[4]{a - \sqrt[4]{cx^2}}}{a + \sqrt{a}\sqrt{cx^4}} dx}{4\sqrt[4]{a}} + \frac{\left( d - \frac{\sqrt{ae}}{\sqrt{c}} \right) \int \frac{\sqrt[4]{a + \sqrt[4]{cx^2}}}{a + \sqrt{a}\sqrt{cx^4}} dx}{4\sqrt[4]{a}} + \frac{\left( d + \frac{\sqrt{ae}}{\sqrt{c}} \right) \int \frac{1}{\sqrt[4]{a - \sqrt[4]{cx^2}}}}{4a^{3/4}} + \frac{\left( d + \frac{\sqrt{ae}}{\sqrt{c}} \right) \int \frac{1}{\sqrt[4]{a + \sqrt[4]{cx^2}}}}{4a^{3/4}} dx \\
 &= \frac{(\sqrt{cd} + \sqrt{ae}) \tan^{-1} \left( \frac{\sqrt[8]{cx}}{\sqrt[8]{a}} \right)}{4a^{7/8}c^{5/8}} + \frac{(\sqrt{cd} + \sqrt{ae}) \tanh^{-1} \left( \frac{\sqrt[8]{cx}}{\sqrt[8]{a}} \right)}{4a^{7/8}c^{5/8}} + \frac{\left( d - \frac{\sqrt{ae}}{\sqrt{c}} \right) \int \frac{1}{\frac{\sqrt[4]{a}}{\sqrt[4]{c}} - \frac{\sqrt{2}\sqrt[8]{ax}}{\sqrt[4]{c}} + x^2} dx}{8a^{3/4}\sqrt[4]{c}} + \frac{\left( d - \frac{\sqrt{ae}}{\sqrt{c}} \right) \int \frac{1}{\frac{\sqrt[4]{a}}{\sqrt[4]{c}} + \frac{\sqrt{2}\sqrt[8]{ax}}{\sqrt[4]{c}} + x^2} dx}{8a^{3/4}\sqrt[4]{c}} \\
 &= \frac{(\sqrt{cd} + \sqrt{ae}) \tan^{-1} \left( \frac{\sqrt[8]{cx}}{\sqrt[8]{a}} \right)}{4a^{7/8}c^{5/8}} + \frac{(\sqrt{cd} + \sqrt{ae}) \tanh^{-1} \left( \frac{\sqrt[8]{cx}}{\sqrt[8]{a}} \right)}{4a^{7/8}c^{5/8}} - \frac{\left( d - \frac{\sqrt{ae}}{\sqrt{c}} \right) \log \left( \sqrt[4]{a} - \sqrt{2}\sqrt[8]{a}\sqrt[8]{cx} + \sqrt[4]{cx^2} \right)}{8\sqrt{2}a^{7/8}\sqrt[8]{c}} + \frac{\left( d - \frac{\sqrt{ae}}{\sqrt{c}} \right) \log \left( \sqrt[4]{a} + \sqrt{2}\sqrt[8]{a}\sqrt[8]{cx} + \sqrt[4]{cx^2} \right)}{8\sqrt{2}a^{7/8}\sqrt[8]{c}} \\
 &= \frac{(\sqrt{cd} + \sqrt{ae}) \tan^{-1} \left( \frac{\sqrt[8]{cx}}{\sqrt[8]{a}} \right)}{4a^{7/8}c^{5/8}} - \frac{\left( d - \frac{\sqrt{ae}}{\sqrt{c}} \right) \tan^{-1} \left( 1 - \frac{\sqrt{2}\sqrt[8]{cx}}{\sqrt[8]{a}} \right)}{4\sqrt{2}a^{7/8}\sqrt[8]{c}} + \frac{\left( d - \frac{\sqrt{ae}}{\sqrt{c}} \right) \tan^{-1} \left( 1 + \frac{\sqrt{2}\sqrt[8]{cx}}{\sqrt[8]{a}} \right)}{4\sqrt{2}a^{7/8}\sqrt[8]{c}} + \frac{(\sqrt{cd} + \sqrt{ae}) \log \left( \sqrt[4]{a} - \sqrt{2}\sqrt[8]{a}\sqrt[8]{cx} + \sqrt[4]{cx^2} \right)}{8a^{5/8}\sqrt{2}ac^{5/8}}
 \end{aligned}$$

**Mathematica [A]** time = 0.136066, size = 425, normalized size = 1.29

$$\frac{(a^{5/8}e - \sqrt[8]{a}\sqrt{cd}) \log \left( -\sqrt{2}\sqrt[8]{a}\sqrt[8]{cx} + \sqrt[4]{a} + \sqrt[4]{cx^2} \right)}{8\sqrt{2}ac^{5/8}} - \frac{(a^{5/8}e - \sqrt[8]{a}\sqrt{cd}) \log \left( \sqrt{2}\sqrt[8]{a}\sqrt[8]{cx} + \sqrt[4]{a} + \sqrt[4]{cx^2} \right)}{8\sqrt{2}ac^{5/8}} - \frac{(a^{5/8}e + \sqrt[8]{a}\sqrt{cd}) \log \left( \sqrt[4]{a} - \sqrt{2}\sqrt[8]{a}\sqrt[8]{cx} + \sqrt[4]{cx^2} \right)}{8ac^{5/8}}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e\*x^4)/(a - c\*x^8),x]

[Out] 
$$\begin{aligned} & ((a^{1/8} \sqrt{c} d + a^{5/8} e) \operatorname{ArcTan}[c^{1/8} x / a^{1/8}]) / (4 a c^{5/8}) \\ & - ((-a^{1/8} \sqrt{c} d + a^{5/8} e) \operatorname{ArcTan}[(-\sqrt{2} a^{1/8}) + 2 c^{1/8} x] / (\sqrt{2} a^{1/8})) / (4 \sqrt{2} a c^{5/8}) \\ & - ((-a^{1/8} \sqrt{c} d + a^{5/8} e) \operatorname{ArcTan}[\sqrt{2} a^{1/8} + 2 c^{1/8} x] / (\sqrt{2} a^{1/8})) / (4 \sqrt{2} a c^{5/8}) \\ & - ((a^{1/8} \sqrt{c} d + a^{5/8} e) \operatorname{Log}[a^{1/8} - c^{1/8} x]) / (8 a c^{5/8}) \\ & - ((-a^{1/8} \sqrt{c} d - a^{5/8} e) \operatorname{Log}[a^{1/8} + c^{1/8} x]) / (8 a c^{5/8}) \\ & + ((-a^{1/8} \sqrt{c} d + a^{5/8} e) \operatorname{Log}[a^{1/4} - \sqrt{2} a^{1/8} c^{1/8} x + c^{1/4} x^2]) / (8 \sqrt{2} a c^{5/8}) \\ & - ((-a^{1/8} \sqrt{c} d + a^{5/8} e) \operatorname{Log}[a^{1/4} + \sqrt{2} a^{1/8} c^{1/8} x + c^{1/4} x^2]) / (8 \sqrt{2} a c^{5/8}) \end{aligned}$$

**Maple [C]** time = 0.015, size = 39, normalized size = 0.1

$$\frac{1}{8c} \sum_{_R=\operatorname{RootOf}(c_Z^8-a)} \frac{(-_R^4 e - d) \ln(x - _R)}{-_R^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x^4+d)/(-c\*x^8+a),x)

[Out] 1/8/c\*sum((-\_R^4\*e-d)/\_R^7\*ln(x-\_R),\_R=RootOf(\_Z^8\*c-a))

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$- \int \frac{ex^4 + d}{cx^8 - a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^4+d)/(-c\*x^8+a),x, algorithm="maxima")

[Out] -integrate((e\*x^4 + d)/(c\*x^8 - a), x)

**Fricas [B]** time = 3.30455, size = 6946, normalized size = 21.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^4+d)/(-c\*x^8+a),x, algorithm="fricas")

[Out] 
$$\frac{1}{2} \left( (a^3 c^2 \sqrt{(c^4 d^8 + 12 a^2 c^3 d^6 e^2 + 38 a^2 c^2 d^4 e^4 + 12 a^3 c d^2 e^6 + a^4 e^8)} / (a^7 c^5)) + 4 c^3 d^3 e + 4 a d e^3 \right) / (a^3 c^2)^{1/4} \arctan \left( \frac{(3 a^3 c^5 d^6 e + 19 a^4 c^4 d^4 e^3 + 9 a^5 c^3 d^2 e^5 + a^6 c^2 e^7 - (a^6 c^6 d^3 + 3 a^7 c^5 d e^2) \sqrt{(c^4 d^8 + 12 a^2 c^3 d^6 e^2 + 38 a^2 c^2 d^4 e^4 + 12 a^3 c d^2 e^6 + a^4 e^8)} / (a^7 c^5))}{(c^4 d^8 + 4 a^2 c^3 d^6 e^2 - 10 a^2 c^2 d^4 e^4 + 4 a^3 c d^2 e^6 + a^4 e^8) x^2 - (2 a^6 c^4 d e \sqrt{(c^4 d^8 + 12 a^2 c^3 d^6 e^2 + 38 a^2 c^2 d^4 e^4 + 12 a^3 c d^2 e^6 + a^4 e^8)} / (a^7 c^5)) - a^2 c^4 d^6 - 7 a^3 c^3 d^4 e^2 - 7 a^4 c^2 d^2 e^4 - a^5 c e^6) \sqrt{(a^3 c^2 \sqrt{(c^4 d^8 + 12 a^2 c^3 d^6 e^2 + 38 a^2 c^2 d^4 e^4 + 12 a^3 c d^2 e^6 + a^4 e^8)} / (a^7 c^5)) + 4 c^3 d^3 e + 4 a d e^3} / (a^3 c^2)} \right) / (c^4 d^8 + 4 a^2 c^3 d^6 e^2 - 10 a^2 c^2 d^4 e^4 + 4 a^3 c d^2 e^6 + a^4 e^8) \sqrt{(a^3 c^2 \sqrt{(c^4 d^8 + 12 a^2 c^3 d^6 e^2 + 38 a^2 c^2 d^4 e^4 + 12 a^3 c d^2 e^6 + a^4 e^8)} / (a^7 c^5)) + 4 c^3 d^3 e + 4 a d e^3} / (a^3 c^2)} + ((a^6 c^6 d^3 + 3 a^7 c^5 d e^2) x \sqrt{(c^4 d^8 + 12 a^2 c^3 d^6 e^2 + 38 a^2 c^2 d^4 e^4 + 12 a^3 c d^2 e^6 + a^4 e^8)} / (a^7 c^5)) - (3 a^3 c^5 d^6 e + 19 a^4 c^4 d^4 e^3 + 9 a^5 c^3 d^2 e^5 + a^6 c^2 e^7) x \sqrt{(a^3 c^2 \sqrt{(c^4 d^8 + 12 a^2 c^3 d^6 e^2 + 38 a^2 c^2 d^4 e^4 + 12 a^3 c d^2 e^6 + a^4 e^8)} / (a^7 c^5)) + 4 c^3 d^3 e + 4 a d e^3} / (a^3 c^2)} \right) \left( (a^3 c^2 \sqrt{(c^4 d^8 + 12 a^2 c^3 d^6 e^2 + 38 a^2 c^2 d^4 e^4 + 12 a^3 c d^2 e^6 + a^4 e^8)} / (a^7 c^5)) + 4 c^3 d^3 e + 4 a d e^3 \right) / (a^3 c^2)^{1/4} / (c^5 d^{10} + 3 a^2 c^4 d^8 e^2 - 14 a^2 c^3 d^6 e^4 + 14 a^3 c^2 d^4 e^6 - 3 a^4 c d^2 e^8 - a^5 e^{10}) - \frac{1}{2} \left( - (a^3 c^2 \sqrt{(c^4 d^8 + 12 a^2 c^3 d^6 e^2 + 38 a^2 c^2 d^4 e^4 + 12 a^3 c d^2 e^6 + a^4 e^8)} / (a^7 c^5)) - 4 c^3 d^3 e - 4 a d e^3 \right) / (a^3 c^2)^{1/4} \arctan \left( - \frac{(3 a^3 c^5 d^6 e + 19 a^4 c^4 d^4 e^3 + 9 a^5 c^3 d^2 e^5 + a^6 c^2 e^7 + (a^6 c^6 d^3 + 3 a^7 c^5 d e^2) \sqrt{(c^4 d^8 + 12 a^2 c^3 d^6 e^2 + 38 a^2 c^2 d^4 e^4 + 12 a^3 c d^2 e^6 + a^4 e^8)} / (a^7 c^5))}{(c^4 d^8 + 4 a^2 c^3 d^6 e^2 - 10 a^2 c^2 d^4 e^4 + 4 a^3 c d^2 e^6 + a^4 e^8) x^2 + (2 a^6 c^4 d e \sqrt{(c^4 d^8 + 12 a^2 c^3 d^6 e^2 + 38 a^2 c^2 d^4 e^4 + 12 a^3 c d^2 e^6 + a^4 e^8)} / (a^7 c^5)) + a^2 c^4 d^6 + 7 a^3 c^3 d^4 e^2 + 7 a^4 c^2 d^2 e^4 + a^5 c e^6) \sqrt{-(a^3 c^2 \sqrt{(c^4 d^8 + 12 a^2 c^3 d^6 e^2 + 38 a^2 c^2 d^4 e^4 + 12 a^3 c d^2 e^6 + a^4 e^8)} / (a^7 c^5)) - 4 c^3 d^3 e - 4 a d e^3} / (a^3 c^2)} \right) / (c^4 d^8 + 4 a^2 c^3 d^6 e^2 - 10 a^2 c^2 d^4 e^4 + 4 a^3 c d^2 e^6 + a^4 e^8) \left( - (a^3 c^2 \sqrt{(c^4 d^8 + 12 a^2 c^3 d^6 e^2 + 38 a^2 c^2 d^4 e^4 + 12 a^3 c d^2 e^6 + a^4 e^8)} / (a^7 c^5)) - 4 c^3 d^3 e - 4 a d e^3 \right) / (a^3 c^2)^{3/4} - ((a^6 c^6 d^3 + 3 a^7 c^5 d e^2) x \sqrt{(c^4 d^8 + 12 a^2 c^3 d^6 e^2 + 38 a^2 c^2 d^4 e^4 + 12 a^3 c d^2 e^6 + a^4 e^8)} / (a^7 c^5)) + (3 a^3 c^5 d^6 e + 19 a^4 c^4 d^4 e^3 + 9 a^5 c^3 d^2 e^5 + a^6 c^2 e^7) x \left( - (a^3 c^2 \sqrt{(c^4 d^8 + 12 a^2 c^3 d^6 e^2 + 38 a^2 c^2 d^4 e^4 + 12 a^3 c d^2 e^6 + a^4 e^8)} / (a^7 c^5)) - 4 c^3 d^3 e - 4 a d e^3 \right) / (a^3 c^2)^{3/4} \right) / (c^5 d^{10} + 3 a^2 c^4 d^8 e^2 - 14 a^2 c^3 d^6 e^4 - 14 a^3 c^2 d^4 e^6 - 3 a^4 c d^2 e^8 - a^5 e^{10})$$

$$\begin{aligned}
& c^3 d^6 e^4 + 14 a^3 c^2 d^4 e^6 - 3 a^4 c d^2 e^8 - a^5 e^{10} \Big) + 1/8 \Big( (a^3 c^2 \sqrt{(c^4 d^8 + 12 a^3 c^3 d^6 e^2 + 38 a^2 c^2 d^4 e^4 + 12 a^3 c d^2 e^6 + a^4 e^8)} / (a^7 c^5)) + 4 c d^3 e + 4 a d e^3 \Big) / (a^3 c^2) \Big)^{1/4} \log(- (c^3 d^6 + 5 a^2 c^2 d^4 e^2 - 5 a^2 c d^2 e^4 - a^3 e^6) x + (a^5 c^3 e \sqrt{(c^4 d^8 + 12 a^3 c^3 d^6 e^2 + 38 a^2 c^2 d^4 e^4 + 12 a^3 c d^2 e^6 + a^4 e^8)} / (a^7 c^5)) - a c^3 d^5 - 6 a^2 c^2 d^3 e^2 - a^3 c d e^4) \Big) \Big( (a^3 c^2 \sqrt{(c^4 d^8 + 12 a^3 c^3 d^6 e^2 + 38 a^2 c^2 d^4 e^4 + 12 a^3 c d^2 e^6 + a^4 e^8)} / (a^7 c^5)) + 4 c d^3 e + 4 a d e^3 \Big) / (a^3 c^2) \Big)^{1/4} \Big) - 1/8 \Big( (a^3 c^2 \sqrt{(c^4 d^8 + 12 a^3 c^3 d^6 e^2 + 38 a^2 c^2 d^4 e^4 + 12 a^3 c d^2 e^6 + a^4 e^8)} / (a^7 c^5)) + 4 c d^3 e + 4 a d e^3 \Big) / (a^3 c^2) \Big)^{1/4} \log(- (c^3 d^6 + 5 a^2 c^2 d^4 e^2 - 5 a^2 c d^2 e^4 - a^3 e^6) x - (a^5 c^3 e \sqrt{(c^4 d^8 + 12 a^3 c^3 d^6 e^2 + 38 a^2 c^2 d^4 e^4 + 12 a^3 c d^2 e^6 + a^4 e^8)} / (a^7 c^5)) - a c^3 d^5 - 6 a^2 c^2 d^3 e^2 - a^3 c d e^4) \Big) \Big( (a^3 c^2 \sqrt{(c^4 d^8 + 12 a^3 c^3 d^6 e^2 + 38 a^2 c^2 d^4 e^4 + 12 a^3 c d^2 e^6 + a^4 e^8)} / (a^7 c^5)) + 4 c d^3 e + 4 a d e^3 \Big) / (a^3 c^2) \Big)^{1/4} \Big) - 1/8 \Big( - (a^3 c^2 \sqrt{(c^4 d^8 + 12 a^3 c^3 d^6 e^2 + 38 a^2 c^2 d^4 e^4 + 12 a^3 c d^2 e^6 + a^4 e^8)} / (a^7 c^5)) - 4 c d^3 e - 4 a d e^3 \Big) / (a^3 c^2) \Big)^{1/4} \log(- (c^3 d^6 + 5 a^2 c^2 d^4 e^2 - 5 a^2 c d^2 e^4 - a^3 e^6) x + (a^5 c^3 e \sqrt{(c^4 d^8 + 12 a^3 c^3 d^6 e^2 + 38 a^2 c^2 d^4 e^4 + 12 a^3 c d^2 e^6 + a^4 e^8)} / (a^7 c^5)) + a c^3 d^5 + 6 a^2 c^2 d^3 e^2 + a^3 c d e^4) \Big) \Big( - (a^3 c^2 \sqrt{(c^4 d^8 + 12 a^3 c^3 d^6 e^2 + 38 a^2 c^2 d^4 e^4 + 12 a^3 c d^2 e^6 + a^4 e^8)} / (a^7 c^5)) - 4 c d^3 e - 4 a d e^3 \Big) / (a^3 c^2) \Big)^{1/4} \Big) + 1/8 \Big( - (a^3 c^2 \sqrt{(c^4 d^8 + 12 a^3 c^3 d^6 e^2 + 38 a^2 c^2 d^4 e^4 + 12 a^3 c d^2 e^6 + a^4 e^8)} / (a^7 c^5)) - 4 c d^3 e - 4 a d e^3 \Big) / (a^3 c^2) \Big)^{1/4} \log(- (c^3 d^6 + 5 a^2 c^2 d^4 e^2 - 5 a^2 c d^2 e^4 - a^3 e^6) x - (a^5 c^3 e \sqrt{(c^4 d^8 + 12 a^3 c^3 d^6 e^2 + 38 a^2 c^2 d^4 e^4 + 12 a^3 c d^2 e^6 + a^4 e^8)} / (a^7 c^5)) + a c^3 d^5 + 6 a^2 c^2 d^3 e^2 + a^3 c d e^4) \Big) \Big( - (a^3 c^2 \sqrt{(c^4 d^8 + 12 a^3 c^3 d^6 e^2 + 38 a^2 c^2 d^4 e^4 + 12 a^3 c d^2 e^6 + a^4 e^8)} / (a^7 c^5)) - 4 c d^3 e - 4 a d e^3 \Big) / (a^3 c^2) \Big)^{1/4} \Big)
\end{aligned}$$

---

**Sympy [A]** time = 12.4586, size = 202, normalized size = 0.61

$$- \text{RootSum} \left( 16777216 t^8 a^7 c^5 + t^4 \left( -32768 a^5 c^3 d e^3 - 32768 a^4 c^4 d^3 e \right) - a^4 e^8 + 4 a^3 c d^2 e^6 - 6 a^2 c^2 d^4 e^4 + 4 a c^3 d^6 e^2 - c^4 d^8, \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x\*\*4+d)/(-c\*x\*\*8+a),x)

[Out] -RootSum(16777216\*\_t\*\*8\*a\*\*7\*c\*\*5 + \_t\*\*4\*(-32768\*a\*\*5\*c\*\*3\*d\*e\*\*3 - 32768\*a\*\*4\*c\*\*4\*d\*\*3\*e) - a\*\*4\*e\*\*8 + 4\*a\*\*3\*c\*d\*\*2\*e\*\*6 - 6\*a\*\*2\*c\*\*2\*d\*\*4\*e\*\*4 + 4\*a\*c\*\*3\*d\*\*6\*e\*\*2 - c\*\*4\*d\*\*8, Lambda(\_t, \_t\*log(x + (-32768\*\_t\*\*5\*a\*\*5\*

$$\frac{c^{**3}e + 40*_t*a^{**3}*c*d*e^{**4} + 80*_t*a^{**2}*c^{**2}*d^{**3}e^{**2} + 8*_t*a*c^{**3}*d^{**5}}{(a^{**3}e^{**6} + 5*a^{**2}*c*d^{**2}e^{**4} - 5*a*c^{**2}*d^{**4}e^{**2} - c^{**3}*d^{**6}))}$$

**Giac [B]** time = 1.22088, size = 855, normalized size = 2.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^4+d)/(-c\*x^8+a),x, algorithm="giac")

[Out] 
$$\begin{aligned} & -1/8*(\sqrt{-\sqrt{2} + 2})*(-a/c)^{(5/8)}*e - d*\sqrt{\sqrt{2} + 2})*(-a/c)^{(1/8)}) \\ & * \arctan((2*x + \sqrt{-\sqrt{2} + 2})*(-a/c)^{(1/8)})/(\sqrt{\sqrt{2} + 2})*(-a/c)^{(1/8)})) / a - 1/8*(\sqrt{-\sqrt{2} + 2})*(-a/c)^{(5/8)}*e - d*\sqrt{\sqrt{2} + 2})*(-a/c)^{(1/8)}) \\ & * \arctan((2*x - \sqrt{-\sqrt{2} + 2})*(-a/c)^{(1/8)})/(\sqrt{\sqrt{2} + 2})*(-a/c)^{(1/8)})) / a + 1/8*(\sqrt{\sqrt{2} + 2})*(-a/c)^{(5/8)}*e + d*\sqrt{-\sqrt{2} + 2})*(-a/c)^{(1/8)}) \\ & * \arctan((2*x + \sqrt{\sqrt{2} + 2})*(-a/c)^{(1/8)})/(\sqrt{-\sqrt{2} + 2})*(-a/c)^{(1/8)})) / a + 1/8*(\sqrt{\sqrt{2} + 2})*(-a/c)^{(5/8)}*e + d*\sqrt{-\sqrt{2} + 2})*(-a/c)^{(1/8)}) \\ & * \arctan((2*x - \sqrt{\sqrt{2} + 2})*(-a/c)^{(1/8)})/(\sqrt{-\sqrt{2} + 2})*(-a/c)^{(1/8)})) / a - 1/16*(\sqrt{-\sqrt{2} + 2})*(-a/c)^{(5/8)}*e - d*\sqrt{\sqrt{2} + 2})*(-a/c)^{(1/8)}) \\ & * \log(x^2 + x*\sqrt{\sqrt{2} + 2})*(-a/c)^{(1/8)} + (-a/c)^{(1/4)}) / a + 1/16*(\sqrt{-\sqrt{2} + 2})*(-a/c)^{(5/8)}*e - d*\sqrt{\sqrt{2} + 2})*(-a/c)^{(1/8)}) \\ & * \log(x^2 - x*\sqrt{\sqrt{2} + 2})*(-a/c)^{(1/8)} + (-a/c)^{(1/4)}) / a + 1/16*(\sqrt{\sqrt{2} + 2})*(-a/c)^{(5/8)}*e + d*\sqrt{-\sqrt{2} + 2})*(-a/c)^{(1/8)}) \\ & * \log(x^2 + x*\sqrt{-\sqrt{2} + 2})*(-a/c)^{(1/8)} + (-a/c)^{(1/4)}) / a - 1/16*(\sqrt{\sqrt{2} + 2})*(-a/c)^{(5/8)}*e + d*\sqrt{-\sqrt{2} + 2})*(-a/c)^{(1/8)}) \\ & * \log(x^2 - x*\sqrt{-\sqrt{2} + 2})*(-a/c)^{(1/8)} + (-a/c)^{(1/4)}) / a \end{aligned}$$

$$3.5 \quad \int \frac{d+ex^4}{d^2+bx^4+e^2x^8} dx$$

**Optimal.** Leaf size=791

$$\frac{\log\left(-x\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-b}}+\sqrt{d}+\sqrt{ex^2}\right)}{8\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-b}}} + \frac{\log\left(x\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-b}}+\sqrt{d}+\sqrt{ex^2}\right)}{8\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-b}}} - \frac{\log\left(-x\sqrt{\sqrt{2de-b}+2\sqrt{d}}\right)}{8\sqrt{d}\sqrt{\sqrt{2de-b}+2\sqrt{d}}}$$

```
[Out] -ArcTan[(Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[-b + 2*d*e]] - 2*Sqrt[e]*x)/Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[-b + 2*d*e]]]/(4*Sqrt[d]*Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[-b + 2*d*e]]) - ArcTan[(Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[-b + 2*d*e]] - 2*Sqrt[e]*x)/Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[-b + 2*d*e]]]/(4*Sqrt[d]*Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[-b + 2*d*e]]) + ArcTan[(Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[-b + 2*d*e]] + 2*Sqrt[e]*x)/Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[-b + 2*d*e]]]/(4*Sqrt[d]*Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[-b + 2*d*e]]) + ArcTan[(Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[-b + 2*d*e]] + 2*Sqrt[e]*x)/Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[-b + 2*d*e]]]/(4*Sqrt[d]*Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[-b + 2*d*e]]) - Log[Sqrt[d] - Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[-b + 2*d*e]]*x + Sqrt[e]*x^2]/(8*Sqrt[d]*Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[-b + 2*d*e]]) + Log[Sqrt[d] + Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[-b + 2*d*e]]*x + Sqrt[e]*x^2]/(8*Sqrt[d]*Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[-b + 2*d*e]]) - Log[Sqrt[d] - Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[-b + 2*d*e]]*x + Sqrt[e]*x^2]/(8*Sqrt[d]*Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[-b + 2*d*e]]) + Log[Sqrt[d] + Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[-b + 2*d*e]]*x + Sqrt[e]*x^2]/(8*Sqrt[d]*Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[-b + 2*d*e]])
```

**Rubi [A]** time = 0.862735, antiderivative size = 791, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 6, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$ , Rules used = {1419, 1094, 634, 618, 204, 628}

$$\frac{\log\left(-x\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-b}}+\sqrt{d}+\sqrt{ex^2}\right)}{8\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-b}}} + \frac{\log\left(x\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-b}}+\sqrt{d}+\sqrt{ex^2}\right)}{8\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-b}}} - \frac{\log\left(-x\sqrt{\sqrt{2de-b}+2\sqrt{d}}\right)}{8\sqrt{d}\sqrt{\sqrt{2de-b}+2\sqrt{d}}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^4)/(d^2 + b*x^4 + e^2*x^8), x]
```

```
[Out] -ArcTan[(Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[-b + 2*d*e]] - 2*Sqrt[e]*x)/Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[-b + 2*d*e]]]/(4*Sqrt[d]*Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[-b + 2*d*e]]) - ArcTan[(Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[-b + 2*d*e]] - 2*Sqrt[e]*x)/Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[-b + 2*d*e]]]/(4*Sqrt[d]*Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[-b + 2*d*e]]) + ArcTan[(Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[-b + 2*d*e]] + 2*Sqrt[e]*x)/Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[-b + 2*d*e]]]/(4*Sqrt[d]*Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[-b + 2*d*e]]) + ArcTan[(Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[-b + 2*d*e]] + 2*Sqrt[e]*x)/Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[-b + 2*d*e]]]/(4*Sqrt[d]*Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[-b + 2*d*e]]) - Log[Sqrt[d] - Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[-b + 2*d*e]]*x + Sqrt[e]*x^2]/(8*Sqrt[d]*Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[-b + 2*d*e]]) + Log[Sqrt[d] + Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[-b + 2*d*e]]*x + Sqrt[e]*x^2]/(8*Sqrt[d]*Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[-b + 2*d*e]]) - Log[Sqrt[d] - Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[-b + 2*d*e]]*x + Sqrt[e]*x^2]/(8*Sqrt[d]*Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[-b + 2*d*e]]) + Log[Sqrt[d] + Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[-b + 2*d*e]]*x + Sqrt[e]*x^2]/(8*Sqrt[d]*Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[-b + 2*d*e]])
```

```

t[-b + 2*d*e]]) - ArcTan[(Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[-b + 2*d*e]] - 2*Sqrt[e]*x)/Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[-b + 2*d*e]]]/(4*Sqrt[d]*Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[-b + 2*d*e]]) + ArcTan[(Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[-b + 2*d*e]] + 2*Sqrt[e]*x)/Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[-b + 2*d*e]]]/(4*Sqrt[d]*Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[-b + 2*d*e]]) + ArcTan[(Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[-b + 2*d*e]] + 2*Sqrt[e]*x)/Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[-b + 2*d*e]]]/(4*Sqrt[d]*Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[-b + 2*d*e]]) - Log[Sqrt[d] - Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[-b + 2*d*e]]*x + Sqrt[e]*x^2]/(8*Sqrt[d]*Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[-b + 2*d*e]]) + Log[Sqrt[d] + Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[-b + 2*d*e]]*x + Sqrt[e]*x^2]/(8*Sqrt[d]*Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[-b + 2*d*e]]) - Log[Sqrt[d] - Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[-b + 2*d*e]]*x + Sqrt[e]*x^2]/(8*Sqrt[d]*Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[-b + 2*d*e]]) + Log[Sqrt[d] + Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[-b + 2*d*e]]*x + Sqrt[e]*x^2]/(8*Sqrt[d]*Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[-b + 2*d*e]])

```

### Rule 1419

```

Int[((d_) + (e_)*(x_)^(n_))/((a_) + (b_)*(x_)^(n_) + (c_)*(x_)^(n2_)), x_Symbol] := With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x^(n/2) + x^n, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x^(n/2) + x^n, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && IGtQ[n/2, 0] && (GtQ[(2*d)/e - b/c, 0] || (!LtQ[(2*d)/e - b/c, 0] && EqQ[d, e*Rt[a/c, 2]]))

```

### Rule 1094

```

Int[((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(-1), x_Symbol] := With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(r - x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(r + x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]

```

### Rule 634

```

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

```

### Rule 618

```

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

```



Rule 204

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 628

Int[((d\_) + (e\_)\*(x\_))/((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2), x\_Symbol] := Simp[(d\*Log[RemoveContent[a + b\*x + c\*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

Rubi steps

$$\begin{aligned}
 \int \frac{d + ex^4}{d^2 + bx^4 + e^2x^8} dx &= \frac{\int \frac{1}{\frac{d}{e} - \frac{\sqrt{-b+2dex^2}}{e} + x^4} dx}{2e} + \frac{\int \frac{1}{\frac{d}{e} + \frac{\sqrt{-b+2dex^2}}{e} + x^4} dx}{2e} \\
 &= \frac{\int \frac{\frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{-b+2de}}}{\sqrt{e}} - x}{\frac{\sqrt{d}}{\sqrt{e}} - \frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{-b+2dex}}}{\sqrt{e}} + x^2} dx}{4\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{-b+2de}}} + \frac{\int \frac{\frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{-b+2de}}}{\sqrt{e}} + x}{\frac{\sqrt{d}}{\sqrt{e}} + \frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{-b+2dex}}}{\sqrt{e}} + x^2} dx}{4\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{-b+2de}}} + \frac{\int \frac{\frac{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{-b+2de}}}{\sqrt{e}} - x}{\frac{\sqrt{d}}{\sqrt{e}} - \frac{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{-b+2dex}}}{\sqrt{e}} + x^2} dx}{4\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{-b+2de}}} + \frac{\int \frac{\frac{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{-b+2de}}}{\sqrt{e}} + x}{\frac{\sqrt{d}}{\sqrt{e}} + \frac{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{-b+2dex}}}{\sqrt{e}} + x^2} dx}{4\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{-b+2de}}} \\
 &= \frac{\int \frac{1}{\frac{\sqrt{d}}{\sqrt{e}} - \frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{-b+2dex}}}{\sqrt{e}} + x^2} dx}{8\sqrt{d}\sqrt{e}} + \frac{\int \frac{1}{\frac{\sqrt{d}}{\sqrt{e}} + \frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{-b+2dex}}}{\sqrt{e}} + x^2} dx}{8\sqrt{d}\sqrt{e}} + \frac{\int \frac{1}{\frac{\sqrt{d}}{\sqrt{e}} - \frac{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{-b+2dex}}}{\sqrt{e}} + x^2} dx}{8\sqrt{d}\sqrt{e}} + \frac{\int \frac{1}{\frac{\sqrt{d}}{\sqrt{e}} + \frac{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{-b+2dex}}}{\sqrt{e}} + x^2} dx}{8\sqrt{d}\sqrt{e}} \\
 &= -\frac{\log\left(\sqrt{d} - \sqrt{2\sqrt{d}\sqrt{e}-\sqrt{-b+2dex}} + \sqrt{ex^2}\right)}{8\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{-b+2de}}} + \frac{\log\left(\sqrt{d} + \sqrt{2\sqrt{d}\sqrt{e}-\sqrt{-b+2dex}} + \sqrt{ex^2}\right)}{8\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{-b+2de}}} \\
 &= -\frac{\tan^{-1}\left(\frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{-b+2de}}-2\sqrt{ex}}{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{-b+2de}}}\right)}{4\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{-b+2de}}} - \frac{\tan^{-1}\left(\frac{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{-b+2de}}-2\sqrt{ex}}{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{-b+2de}}}\right)}{4\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{-b+2de}}} + \frac{\tan^{-1}\left(\frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{-b+2de}}+2\sqrt{ex}}{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{-b+2de}}}\right)}{4\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{-b+2de}}}
 \end{aligned}$$

**Mathematica [C]** time = 0.0449239, size = 67, normalized size = 0.08

$$\frac{1}{4} \text{RootSum} \left[ \#1^4 b + \#1^8 e^2 + d^2 \&, \frac{\#1^4 e \log(x - \#1) + d \log(x - \#1)}{\#1^3 b + 2\#1^7 e^2} \& \right]$$

Antiderivative was successfully verified.

[In] Integrate[(d + e\*x^4)/(d^2 + b\*x^4 + e^2\*x^8), x]

[Out] RootSum[d^2 + b\*#1^4 + e^2\*#1^8 & , (d\*Log[x - #1] + e\*Log[x - #1]\*#1^4)/(b\*#1^3 + 2\*e^2\*#1^7) & ]/4

**Maple [C]** time = 0.059, size = 53, normalized size = 0.1

$$\frac{1}{4} \sum_{_R=\text{RootOf}(e^2_Z^8+b_Z^4+d^2)} \frac{(-_R^4 e + d) \ln(x - _R)}{2\_R^7 e^2 + \_R^3 b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x^4+d)/(e^2\*x^8+b\*x^4+d^2), x)

[Out] 1/4\*sum((\\_R^4\*e+d)/(2\*\\_R^7\*e^2+\\_R^3\*b)\*ln(x-\\_R), \\_R=RootOf(\\_Z^8\*e^2+\\_Z^4\*b+d^2))

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{ex^4 + d}{e^2x^8 + bx^4 + d^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^4+d)/(e^2\*x^8+b\*x^4+d^2), x, algorithm="maxima")

[Out] integrate((e\*x^4 + d)/(e^2\*x^8 + b\*x^4 + d^2), x)

**Fricas [B]** time = 1.94518, size = 6279, normalized size = 7.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.



$$\begin{aligned}
& 3 + 12*b*d^6*e^2 + 6*b^2*d^5*e + b^3*d^4)) + b)/(4*d^4*e^2 + 4*b*d^3*e + b^2*d^2)) - 1/4*\sqrt{\sqrt{1/2}*\sqrt{-((4*d^4*e^2 + 4*b*d^3*e + b^2*d^2)*\sqrt{-((2*d*e - b)/(8*d^7*e^3 + 12*b*d^6*e^2 + 6*b^2*d^5*e + b^3*d^4)) + b)/(4*d^4*e^2 + 4*b*d^3*e + b^2*d^2))}}*\log(e*x - 1/2*(2*d*e - (4*d^4*e^2 + 4*b*d^3*e + b^2*d^2)*\sqrt{-((2*d*e - b)/(8*d^7*e^3 + 12*b*d^6*e^2 + 6*b^2*d^5*e + b^3*d^4)) + b}*\sqrt{\sqrt{1/2}*\sqrt{-((4*d^4*e^2 + 4*b*d^3*e + b^2*d^2)*\sqrt{-((2*d*e - b)/(8*d^7*e^3 + 12*b*d^6*e^2 + 6*b^2*d^5*e + b^3*d^4)) + b)/(4*d^4*e^2 + 4*b*d^3*e + b^2*d^2))}})) + 1/4*\sqrt{\sqrt{1/2}*\sqrt{((4*d^4*e^2 + 4*b*d^3*e + b^2*d^2)*\sqrt{-((2*d*e - b)/(8*d^7*e^3 + 12*b*d^6*e^2 + 6*b^2*d^5*e + b^3*d^4)) - b)/(4*d^4*e^2 + 4*b*d^3*e + b^2*d^2))}}*\log(e*x + 1/2*(2*d*e + (4*d^4*e^2 + 4*b*d^3*e + b^2*d^2)*\sqrt{-((2*d*e - b)/(8*d^7*e^3 + 12*b*d^6*e^2 + 6*b^2*d^5*e + b^3*d^4)) + b}*\sqrt{\sqrt{1/2}*\sqrt{((4*d^4*e^2 + 4*b*d^3*e + b^2*d^2)*\sqrt{-((2*d*e - b)/(8*d^7*e^3 + 12*b*d^6*e^2 + 6*b^2*d^5*e + b^3*d^4)) - b)/(4*d^4*e^2 + 4*b*d^3*e + b^2*d^2))}})) - 1/4*\sqrt{\sqrt{1/2}*\sqrt{((4*d^4*e^2 + 4*b*d^3*e + b^2*d^2)*\sqrt{-((2*d*e - b)/(8*d^7*e^3 + 12*b*d^6*e^2 + 6*b^2*d^5*e + b^3*d^4)) - b)/(4*d^4*e^2 + 4*b*d^3*e + b^2*d^2))}}*\log(e*x - 1/2*(2*d*e + (4*d^4*e^2 + 4*b*d^3*e + b^2*d^2)*\sqrt{-((2*d*e - b)/(8*d^7*e^3 + 12*b*d^6*e^2 + 6*b^2*d^5*e + b^3*d^4)) + b}*\sqrt{\sqrt{1/2}*\sqrt{((4*d^4*e^2 + 4*b*d^3*e + b^2*d^2)*\sqrt{-((2*d*e - b)/(8*d^7*e^3 + 12*b*d^6*e^2 + 6*b^2*d^5*e + b^3*d^4)) - b)/(4*d^4*e^2 + 4*b*d^3*e + b^2*d^2))}}))}
\end{aligned}$$

**Sympy [A]** time = 5.78472, size = 136, normalized size = 0.17

$$\text{RootSum}\left(t^8 (65536b^4d^2 + 524288b^3d^3e + 1572864b^2d^4e^2 + 2097152bd^5e^3 + 1048576d^6e^4) + t^4 (256b^3 + 1024b^2de + 1024b^2d^2e^2 + 1024b^2d^3e^2 + 1024b^2d^4e^2 + 1024b^2d^5e^2 + 1024b^2d^6e^2) + e^2, \text{Lambda}(t, t*\log(x + (1024*_t**5*b**2*d**2 + 4096*_t**5*b*d**3*e + 4096*_t**5*d**4*e**2 + 4*_t*b + 4*_t*d*e)/e))\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x\*\*4+d)/(e\*\*2\*x\*\*8+b\*x\*\*4+d\*\*2),x)

[Out] RootSum(\_t\*\*8\*(65536\*b\*\*4\*d\*\*2 + 524288\*b\*\*3\*d\*\*3\*e + 1572864\*b\*\*2\*d\*\*4\*e\*\*2 + 2097152\*b\*d\*\*5\*e\*\*3 + 1048576\*d\*\*6\*e\*\*4) + \_t\*\*4\*(256\*b\*\*3 + 1024\*b\*\*2\*d\*e + 1024\*b\*d\*\*2\*e\*\*2) + e\*\*2, Lambda(\_t, \_t\*log(x + (1024\*\_t\*\*5\*b\*\*2\*d\*\*2 + 4096\*\_t\*\*5\*b\*d\*\*3\*e + 4096\*\_t\*\*5\*d\*\*4\*e\*\*2 + 4\*\_t\*b + 4\*\_t\*d\*e)/e)))

**Giac [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^4+d)/(e^2*x^8+b*x^4+d^2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

### 3.6 $\int \frac{d+ex^4}{d^2+fx^4+e^2x^8} dx$

**Optimal.** Leaf size=791

$$\frac{\log\left(-x\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}+\sqrt{d}+\sqrt{ex^2}\right)}{8\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}} + \frac{\log\left(x\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}+\sqrt{d}+\sqrt{ex^2}\right)}{8\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}} - \frac{\log\left(-x\sqrt{\sqrt{2de-f}+2\sqrt{d}}\right)}{8\sqrt{d}\sqrt{\sqrt{2de-f}}}$$

```
[Out] -ArcTan[(Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[2*d*e - f]] - 2*Sqrt[e]*x)/Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[2*d*e - f]]]/(4*Sqrt[d]*Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[2*d*e - f]]) - ArcTan[(Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[2*d*e - f]] - 2*Sqrt[e]*x)/Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[2*d*e - f]]]/(4*Sqrt[d]*Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[2*d*e - f]]) + ArcTan[(Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[2*d*e - f]] + 2*Sqrt[e]*x)/Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[2*d*e - f]]]/(4*Sqrt[d]*Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[2*d*e - f]]) + ArcTan[(Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[2*d*e - f]] + 2*Sqrt[e]*x)/Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[2*d*e - f]]]/(4*Sqrt[d]*Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[2*d*e - f]]) - Log[Sqrt[d] - Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[2*d*e - f]]*x + Sqrt[e]*x^2]/(8*Sqrt[d]*Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[2*d*e - f]]) + Log[Sqrt[d] + Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[2*d*e - f]]*x + Sqrt[e]*x^2]/(8*Sqrt[d]*Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[2*d*e - f]]) - Log[Sqrt[d] - Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[2*d*e - f]]*x + Sqrt[e]*x^2]/(8*Sqrt[d]*Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[2*d*e - f]]) + Log[Sqrt[d] + Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[2*d*e - f]]*x + Sqrt[e]*x^2]/(8*Sqrt[d]*Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[2*d*e - f]])
```

**Rubi [A]** time = 0.805457, antiderivative size = 791, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 6, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$ , Rules used = {1419, 1094, 634, 618, 204, 628}

$$\frac{\log\left(-x\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}+\sqrt{d}+\sqrt{ex^2}\right)}{8\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}} + \frac{\log\left(x\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}+\sqrt{d}+\sqrt{ex^2}\right)}{8\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}} - \frac{\log\left(-x\sqrt{\sqrt{2de-f}+2\sqrt{d}}\right)}{8\sqrt{d}\sqrt{\sqrt{2de-f}}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^4)/(d^2 + f*x^4 + e^2*x^8), x]
```

```
[Out] -ArcTan[(Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[2*d*e - f]] - 2*Sqrt[e]*x)/Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[2*d*e - f]]]/(4*Sqrt[d]*Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[2*d*e - f]]) - ArcTan[(Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[2*d*e - f]] - 2*Sqrt[e]*x)/Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[2*d*e - f]]]/(4*Sqrt[d]*Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[2*d*e - f]]) + ArcTan[(Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[2*d*e - f]] + 2*Sqrt[e]*x)/Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[2*d*e - f]]]/(4*Sqrt[d]*Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[2*d*e - f]]) + ArcTan[(Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[2*d*e - f]] + 2*Sqrt[e]*x)/Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[2*d*e - f]]]/(4*Sqrt[d]*Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[2*d*e - f]]) - Log[Sqrt[d] - Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[2*d*e - f]]*x + Sqrt[e]*x^2]/(8*Sqrt[d]*Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[2*d*e - f]]) + Log[Sqrt[d] + Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[2*d*e - f]]*x + Sqrt[e]*x^2]/(8*Sqrt[d]*Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[2*d*e - f]]) - Log[Sqrt[d] - Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[2*d*e - f]]*x + Sqrt[e]*x^2]/(8*Sqrt[d]*Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[2*d*e - f]]) + Log[Sqrt[d] + Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[2*d*e - f]]*x + Sqrt[e]*x^2]/(8*Sqrt[d]*Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[2*d*e - f]])
```

```

2*d*e - f]]) - ArcTan[(Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[2*d*e - f]] - 2*Sqrt[e]
]*x)/Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[2*d*e - f]]]/(4*Sqrt[d]*Sqrt[2*Sqrt[d]*S
qrt[e] - Sqrt[2*d*e - f]]) + ArcTan[(Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[2*d*e -
f]] + 2*Sqrt[e]*x)/Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[2*d*e - f]]]/(4*Sqrt[d]*Sq
rt[2*Sqrt[d]*Sqrt[e] + Sqrt[2*d*e - f]]) + ArcTan[(Sqrt[2*Sqrt[d]*Sqrt[e] +
Sqrt[2*d*e - f]] + 2*Sqrt[e]*x)/Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[2*d*e - f]]]
/(4*Sqrt[d]*Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[2*d*e - f]]) - Log[Sqrt[d] - Sqrt
[2*Sqrt[d]*Sqrt[e] - Sqrt[2*d*e - f]]*x + Sqrt[e]*x^2]/(8*Sqrt[d]*Sqrt[2*Sq
rt[d]*Sqrt[e] - Sqrt[2*d*e - f]]) + Log[Sqrt[d] + Sqrt[2*Sqrt[d]*Sqrt[e] -
Sqrt[2*d*e - f]]*x + Sqrt[e]*x^2]/(8*Sqrt[d]*Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[
2*d*e - f]]) - Log[Sqrt[d] - Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[2*d*e - f]]*x +
Sqrt[e]*x^2]/(8*Sqrt[d]*Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[2*d*e - f]]) + Log[Sq
rt[d] + Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[2*d*e - f]]*x + Sqrt[e]*x^2]/(8*Sqrt[
d]*Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[2*d*e - f]])

```

### Rule 1419

```

Int[((d_) + (e_)*(x_)^(n_))/((a_) + (b_)*(x_)^(n_) + (c_)*(x_)^(n2_)), x
_Symbol] := With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e +
q*x^(n/2) + x^n, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x^(n/2) + x
^n, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4
*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && IGtQ[n/2, 0] && (GtQ[(2*d)/e - b/c, 0]
|| (!LtQ[(2*d)/e - b/c, 0] && EqQ[d, e*Rt[a/c, 2]]))

```

### Rule 1094

```

Int[((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(-1), x_Symbol] := With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(r - x)/(q - r*x
+ x^2), x], x] + Dist[1/(2*c*q*r), Int[(r + x)/(q + r*x + x^2), x], x]]] /
; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]

```

### Rule 634

```

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

```

### Rule 618

```

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int
[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]

```

Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 628

Int[((d\_) + (e\_.)\*(x\_))/((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Simp[(d\*Log[RemoveContent[a + b\*x + c\*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

Rubi steps

$$\begin{aligned}
\int \frac{d + ex^4}{d^2 + fx^4 + e^2x^8} dx &= \frac{\int \frac{1}{\frac{d}{e} - \frac{\sqrt{2de-fx^2}}{e} + x^4} dx}{2e} + \frac{\int \frac{1}{\frac{d}{e} + \frac{\sqrt{2de-fx^2}}{e} + x^4} dx}{2e} \\
&= \frac{\int \frac{\frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}}{\sqrt{e}} - x}{\frac{\sqrt{d}}{\sqrt{e}} - \frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-fx}}}{\sqrt{e}} + x^2} dx}{4\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}} + \frac{\int \frac{\frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}}{\sqrt{e}} + x}{\frac{\sqrt{d}}{\sqrt{e}} + \frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-fx}}}{\sqrt{e}} + x^2} dx}{4\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}} + \frac{\int \frac{\frac{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de-f}}}{\sqrt{e}} - x}{\frac{\sqrt{d}}{\sqrt{e}} - \frac{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de-fx}}}{\sqrt{e}} + x^2} dx}{4\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de-f}}} + \frac{\int \frac{\frac{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de-f}}}{\sqrt{e}} + x}{\frac{\sqrt{d}}{\sqrt{e}} + \frac{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de-fx}}}{\sqrt{e}} + x^2} dx}{4\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de-f}}} \\
&= \frac{\int \frac{1}{\frac{\sqrt{d}}{\sqrt{e}} - \frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-fx}}}{\sqrt{e}} + x^2} dx}{8\sqrt{d}\sqrt{e}} + \frac{\int \frac{1}{\frac{\sqrt{d}}{\sqrt{e}} + \frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-fx}}}{\sqrt{e}} + x^2} dx}{8\sqrt{d}\sqrt{e}} + \frac{\int \frac{1}{\frac{\sqrt{d}}{\sqrt{e}} - \frac{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de-fx}}}{\sqrt{e}} + x^2} dx}{8\sqrt{d}\sqrt{e}} + \frac{\int \frac{1}{\frac{\sqrt{d}}{\sqrt{e}} + \frac{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de-fx}}}{\sqrt{e}} + x^2} dx}{8\sqrt{d}\sqrt{e}} \\
&= -\frac{\log\left(\sqrt{d} - \sqrt{2\sqrt{d}\sqrt{e} - \sqrt{2de-f}}x + \sqrt{ex^2}\right)}{8\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e} - \sqrt{2de-f}}} + \frac{\log\left(\sqrt{d} + \sqrt{2\sqrt{d}\sqrt{e} - \sqrt{2de-f}}x + \sqrt{ex^2}\right)}{8\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e} - \sqrt{2de-f}}} - \frac{\log\left(\sqrt{d} - \sqrt{2\sqrt{d}\sqrt{e} + \sqrt{2de-f}}x + \sqrt{ex^2}\right)}{8\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e} + \sqrt{2de-f}}} + \frac{\log\left(\sqrt{d} + \sqrt{2\sqrt{d}\sqrt{e} + \sqrt{2de-f}}x + \sqrt{ex^2}\right)}{8\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e} + \sqrt{2de-f}}} \\
&= -\frac{\tan^{-1}\left(\frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}-2\sqrt{ex}}{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de-f}}}\right)}{4\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de-f}}} - \frac{\tan^{-1}\left(\frac{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de-f}}-2\sqrt{ex}}{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}}\right)}{4\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}} + \frac{\tan^{-1}\left(\frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}+2\sqrt{ex}}{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de-f}}}\right)}{4\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de-f}}} + \frac{\tan^{-1}\left(\frac{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de-f}}+2\sqrt{ex}}{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}}\right)}{4\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}}
\end{aligned}$$

**Mathematica [C]** time = 0.0363693, size = 67, normalized size = 0.08

$$\frac{1}{4}\text{RootSum}\left[\#1^8e^2 + \#1^4f + d^2\&, \frac{\#1^4e \log(x - \#1) + d \log(x - \#1)}{2\#1^7e^2 + \#1^3f}\&\right]$$



Antiderivative was successfully verified.

[In] Integrate[(d + e\*x^4)/(d^2 + f\*x^4 + e^2\*x^8), x]

[Out] RootSum[d^2 + f\*#1^4 + e^2\*#1^8 & , (d\*Log[x - #1] + e\*Log[x - #1]\*#1^4)/(f\*#1^3 + 2\*e^2\*#1^7) & ]/4

**Maple [C]** time = 0.049, size = 53, normalized size = 0.1

$$\frac{1}{4} \sum_{_R=\text{RootOf}(e^2_Z^8+f_Z^4+d^2)} \frac{(_R^4 e + d) \ln(x - _R)}{2\_R^7 e^2 + \_R^3 f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x^4+d)/(e^2\*x^8+f\*x^4+d^2), x)

[Out] 1/4\*sum((\_R^4\*e+d)/(2\*\_R^7\*e^2+\_R^3\*f)\*ln(x-\_R), \_R=RootOf(\_Z^8\*e^2+\_Z^4\*f+d^2))

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{ex^4 + d}{e^2x^8 + fx^4 + d^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^4+d)/(e^2\*x^8+f\*x^4+d^2), x, algorithm="maxima")

[Out] integrate((e\*x^4 + d)/(e^2\*x^8 + f\*x^4 + d^2), x)

**Fricas [B]** time = 1.85567, size = 6279, normalized size = 7.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^4+d)/(e^2\*x^8+f\*x^4+d^2),x, algorithm="fricas")

[Out] 
$$\begin{aligned} & -\sqrt{\sqrt{1/2}*\sqrt{-((4*d^4*e^2 + 4*d^3*e*f + d^2*f^2)*\sqrt{-(2*d*e - f)/}} \\ & (8*d^7*e^3 + 12*d^6*e^2*f + 6*d^5*e*f^2 + d^4*f^3)) + f)/(4*d^4*e^2 + 4*d^3 \\ & *e*f + d^2*f^2)))*\arctan(-1/4*(2*\sqrt{1/2})*((8*d^5*e^3 + 12*d^4*e^2*f + 6*d \\ & ^3*e*f^2 + d^2*f^3)*x*\sqrt{-(2*d*e - f)/(8*d^7*e^3 + 12*d^6*e^2*f + 6*d^5*e \\ & *f^2 + d^4*f^3)) - (4*d^2*e^2 + 4*d*e*f + f^2)*x)*\sqrt{-((4*d^4*e^2 + 4*d^3 \\ & *e*f + d^2*f^2)*\sqrt{-(2*d*e - f)/(8*d^7*e^3 + 12*d^6*e^2*f + 6*d^5*e*f^2 + \\ & d^4*f^3)) + f)/(4*d^4*e^2 + 4*d^3*e*f + d^2*f^2)) + (4*d^2*e^2 + 4*d*e*f + \\ & f^2 - (8*d^5*e^3 + 12*d^4*e^2*f + 6*d^3*e*f^2 + d^2*f^3)*\sqrt{-(2*d*e - f)} \\ & / (8*d^7*e^3 + 12*d^6*e^2*f + 6*d^5*e*f^2 + d^4*f^3)))*\sqrt{-((4*d^4*e^2 + 4 \\ & *d^3*e*f + d^2*f^2)*\sqrt{-(2*d*e - f)/(8*d^7*e^3 + 12*d^6*e^2*f + 6*d^5*e*f \\ & ^2 + d^4*f^3)) + f)/(4*d^4*e^2 + 4*d^3*e*f + d^2*f^2)))*\sqrt{(2*e^2*x^2 + sq \\ & rt(1/2)*(2*d*e*f + f^2 - (8*d^5*e^3 + 12*d^4*e^2*f + 6*d^3*e*f^2 + d^2*f^3) \\ & *\sqrt{-(2*d*e - f)/(8*d^7*e^3 + 12*d^6*e^2*f + 6*d^5*e*f^2 + d^4*f^3)))*\sqrt{ \\ & t(-((4*d^4*e^2 + 4*d^3*e*f + d^2*f^2)*\sqrt{-(2*d*e - f)/(8*d^7*e^3 + 12*d^6 \\ & *e^2*f + 6*d^5*e*f^2 + d^4*f^3)) + f)/(4*d^4*e^2 + 4*d^3*e*f + d^2*f^2)))/e \\ & ^2))*\sqrt{\sqrt{1/2}*\sqrt{-((4*d^4*e^2 + 4*d^3*e*f + d^2*f^2)*\sqrt{-(2*d*e - \\ & f)/(8*d^7*e^3 + 12*d^6*e^2*f + 6*d^5*e*f^2 + d^4*f^3)) + f)/(4*d^4*e^2 + 4 \\ & *d^3*e*f + d^2*f^2)))/e} + \sqrt{\sqrt{1/2}*\sqrt{((4*d^4*e^2 + 4*d^3*e*f + d^ \\ & 2*f^2)*\sqrt{-(2*d*e - f)/(8*d^7*e^3 + 12*d^6*e^2*f + 6*d^5*e*f^2 + d^4*f^3)} \\ & ) - f)/(4*d^4*e^2 + 4*d^3*e*f + d^2*f^2)))*\arctan(-1/4*(2*\sqrt{1/2})*((8*d^5 \\ & *e^3 + 12*d^4*e^2*f + 6*d^3*e*f^2 + d^2*f^3)*x*\sqrt{-(2*d*e - f)/(8*d^7*e^3 \\ & + 12*d^6*e^2*f + 6*d^5*e*f^2 + d^4*f^3)) + (4*d^2*e^2 + 4*d*e*f + f^2)*x)* \\ & \sqrt{\sqrt{1/2}*\sqrt{((4*d^4*e^2 + 4*d^3*e*f + d^2*f^2)*\sqrt{-(2*d*e - f)/(8 \\ & *d^7*e^3 + 12*d^6*e^2*f + 6*d^5*e*f^2 + d^4*f^3)) - f)/(4*d^4*e^2 + 4*d^3*e \\ & *f + d^2*f^2)))*\sqrt{((4*d^4*e^2 + 4*d^3*e*f + d^2*f^2)*\sqrt{-(2*d*e - f)/( \\ & 8*d^7*e^3 + 12*d^6*e^2*f + 6*d^5*e*f^2 + d^4*f^3)) - f)/(4*d^4*e^2 + 4*d^3* \\ & e*f + d^2*f^2)) - (4*d^2*e^2 + 4*d*e*f + f^2 + (8*d^5*e^3 + 12*d^4*e^2*f + \\ & 6*d^3*e*f^2 + d^2*f^3)*\sqrt{-(2*d*e - f)/(8*d^7*e^3 + 12*d^6*e^2*f + 6*d^5* \\ & e*f^2 + d^4*f^3)))*\sqrt{\sqrt{1/2}*\sqrt{((4*d^4*e^2 + 4*d^3*e*f + d^2*f^2)*s \\ & qrt(-2*d*e - f)/(8*d^7*e^3 + 12*d^6*e^2*f + 6*d^5*e*f^2 + d^4*f^3)) - f)/( \\ & 4*d^4*e^2 + 4*d^3*e*f + d^2*f^2)))*\sqrt{((4*d^4*e^2 + 4*d^3*e*f + d^2*f^2)* \\ & sqrt(-2*d*e - f)/(8*d^7*e^3 + 12*d^6*e^2*f + 6*d^5*e*f^2 + d^4*f^3)) - f)/( \\ & (4*d^4*e^2 + 4*d^3*e*f + d^2*f^2))*\sqrt{(2*e^2*x^2 + \sqrt{1/2}*(2*d*e*f + f \\ & ^2 + (8*d^5*e^3 + 12*d^4*e^2*f + 6*d^3*e*f^2 + d^2*f^3)*\sqrt{-(2*d*e - f)/( \\ & 8*d^7*e^3 + 12*d^6*e^2*f + 6*d^5*e*f^2 + d^4*f^3)))*\sqrt{((4*d^4*e^2 + 4*d^ \\ & 3*e*f + d^2*f^2)*\sqrt{-(2*d*e - f)/(8*d^7*e^3 + 12*d^6*e^2*f + 6*d^5*e*f^2 \\ & + d^4*f^3)) - f)/(4*d^4*e^2 + 4*d^3*e*f + d^2*f^2)))/e^2)/e} + 1/4*\sqrt{\sqrt{ \\ & rt(1/2)*\sqrt{-((4*d^4*e^2 + 4*d^3*e*f + d^2*f^2)*\sqrt{-(2*d*e - f)/(8*d^7*e \\ & ^3 + 12*d^6*e^2*f + 6*d^5*e*f^2 + d^4*f^3)) + f)/(4*d^4*e^2 + 4*d^3*e*f + d \\ & ^2*f^2)))*\log(e*x + 1/2*(2*d*e - (4*d^4*e^2 + 4*d^3*e*f + d^2*f^2)*\sqrt{-(2 \\ & *d*e - f)/(8*d^7*e^3 + 12*d^6*e^2*f + 6*d^5*e*f^2 + d^4*f^3)) + f)*\sqrt{\sqrt{ \\ & t(1/2)*\sqrt{-((4*d^4*e^2 + 4*d^3*e*f + d^2*f^2)*\sqrt{-(2*d*e - f)/(8*d^7*e^} \end{aligned}$$



```
[In] integrate((e*x^4+d)/(e^2*x^8+f*x^4+d^2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

$$3.7 \quad \int \frac{d+ex^4}{d^2-bx^4+e^2x^8} dx$$

**Optimal.** Leaf size=349

$$\frac{\sqrt{e} \tan^{-1}\left(\frac{\sqrt{2}\sqrt{ex}}{\sqrt{\sqrt{b-2de}-\sqrt{b+2de}}}\right)}{\sqrt{2}\sqrt{b-2de}\sqrt{\sqrt{b-2de}-\sqrt{b+2de}}} - \frac{\sqrt{e} \tan^{-1}\left(\frac{\sqrt{2}\sqrt{ex}}{\sqrt{\sqrt{b-2de}+\sqrt{b+2de}}}\right)}{\sqrt{2}\sqrt{b-2de}\sqrt{\sqrt{b-2de}+\sqrt{b+2de}}} - \frac{\sqrt{e} \tanh^{-1}\left(\frac{\sqrt{2}\sqrt{ex}}{\sqrt{\sqrt{b-2de}-\sqrt{b+2de}}}\right)}{\sqrt{2}\sqrt{b-2de}\sqrt{\sqrt{b-2de}-\sqrt{b+2de}}} - \frac{\sqrt{e} \tanh^{-1}\left(\frac{\sqrt{2}\sqrt{ex}}{\sqrt{\sqrt{b-2de}+\sqrt{b+2de}}}\right)}{\sqrt{2}\sqrt{b-2de}\sqrt{\sqrt{b-2de}+\sqrt{b+2de}}}$$

[Out] -((Sqrt[e]\*ArcTan[(Sqrt[2]\*Sqrt[e]\*x)/Sqrt[Sqrt[b-2\*d\*e]-Sqrt[b+2\*d\*e]]])/(Sqrt[2]\*Sqrt[b-2\*d\*e]\*Sqrt[Sqrt[b-2\*d\*e]-Sqrt[b+2\*d\*e]])) - (Sqrt[e]\*ArcTan[(Sqrt[2]\*Sqrt[e]\*x)/Sqrt[Sqrt[b-2\*d\*e]+Sqrt[b+2\*d\*e]]])/(Sqrt[2]\*Sqrt[b-2\*d\*e]\*Sqrt[Sqrt[b-2\*d\*e]+Sqrt[b+2\*d\*e]]) - (Sqrt[e]\*ArcTanh[(Sqrt[2]\*Sqrt[e]\*x)/Sqrt[Sqrt[b-2\*d\*e]-Sqrt[b+2\*d\*e]]])/(Sqrt[2]\*Sqrt[b-2\*d\*e]\*Sqrt[Sqrt[b-2\*d\*e]-Sqrt[b+2\*d\*e]]) - (Sqrt[e]\*ArcTanh[(Sqrt[2]\*Sqrt[e]\*x)/Sqrt[Sqrt[b-2\*d\*e]+Sqrt[b+2\*d\*e]]])/(Sqrt[2]\*Sqrt[b-2\*d\*e]\*Sqrt[Sqrt[b-2\*d\*e]+Sqrt[b+2\*d\*e]])

**Rubi [A]** time = 0.421594, antiderivative size = 349, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$ , Rules used = {1419, 1093, 207, 203}

$$\frac{\sqrt{e} \tan^{-1}\left(\frac{\sqrt{2}\sqrt{ex}}{\sqrt{\sqrt{b-2de}-\sqrt{b+2de}}}\right)}{\sqrt{2}\sqrt{b-2de}\sqrt{\sqrt{b-2de}-\sqrt{b+2de}}} - \frac{\sqrt{e} \tan^{-1}\left(\frac{\sqrt{2}\sqrt{ex}}{\sqrt{\sqrt{b-2de}+\sqrt{b+2de}}}\right)}{\sqrt{2}\sqrt{b-2de}\sqrt{\sqrt{b-2de}+\sqrt{b+2de}}} - \frac{\sqrt{e} \tanh^{-1}\left(\frac{\sqrt{2}\sqrt{ex}}{\sqrt{\sqrt{b-2de}-\sqrt{b+2de}}}\right)}{\sqrt{2}\sqrt{b-2de}\sqrt{\sqrt{b-2de}-\sqrt{b+2de}}} - \frac{\sqrt{e} \tanh^{-1}\left(\frac{\sqrt{2}\sqrt{ex}}{\sqrt{\sqrt{b-2de}+\sqrt{b+2de}}}\right)}{\sqrt{2}\sqrt{b-2de}\sqrt{\sqrt{b-2de}+\sqrt{b+2de}}}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x^4)/(d^2 - b\*x^4 + e^2\*x^8), x]

[Out] -((Sqrt[e]\*ArcTan[(Sqrt[2]\*Sqrt[e]\*x)/Sqrt[Sqrt[b-2\*d\*e]-Sqrt[b+2\*d\*e]]])/(Sqrt[2]\*Sqrt[b-2\*d\*e]\*Sqrt[Sqrt[b-2\*d\*e]-Sqrt[b+2\*d\*e]])) - (Sqrt[e]\*ArcTan[(Sqrt[2]\*Sqrt[e]\*x)/Sqrt[Sqrt[b-2\*d\*e]+Sqrt[b+2\*d\*e]]])/(Sqrt[2]\*Sqrt[b-2\*d\*e]\*Sqrt[Sqrt[b-2\*d\*e]+Sqrt[b+2\*d\*e]]) - (Sqrt[e]\*ArcTanh[(Sqrt[2]\*Sqrt[e]\*x)/Sqrt[Sqrt[b-2\*d\*e]-Sqrt[b+2\*d\*e]]])/(Sqrt[2]\*Sqrt[b-2\*d\*e]\*Sqrt[Sqrt[b-2\*d\*e]-Sqrt[b+2\*d\*e]]) - (Sqrt[e]\*ArcTanh[(Sqrt[2]\*Sqrt[e]\*x)/Sqrt[Sqrt[b-2\*d\*e]+Sqrt[b+2\*d\*e]]])/(Sqrt[2]\*Sqrt[b-2\*d\*e]\*Sqrt[Sqrt[b-2\*d\*e]+Sqrt[b+2\*d\*e]])

Rule 1419

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] := With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e +
q*x^(n/2) + x^n, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x^(n/2) + x
^n, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4
*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && IGtQ[n/2, 0] && (GtQ[(2*d)/e - b/c, 0]
|| (!LtQ[(2*d)/e - b/c, 0] && EqQ[d, e*Rt[a/c, 2]]))
```

### Rule 1093

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] := With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

### Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

### Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

### Rubi steps

$$\begin{aligned} \int \frac{d + ex^4}{d^2 - bx^4 + e^2x^8} dx &= \frac{\int \frac{1}{\frac{d}{e} - \frac{\sqrt{b+2de}x^2}{e} + x^4} dx}{2e} + \frac{\int \frac{1}{\frac{d}{e} + \frac{\sqrt{b+2de}x^2}{e} + x^4} dx}{2e} \\ &= \frac{\int \frac{1}{-\frac{\sqrt{b-2de}}{2e} - \frac{\sqrt{b+2de}}{2e} + x^2} dx}{2\sqrt{b-2de}} - \frac{\int \frac{1}{\frac{\sqrt{b-2de}}{2e} - \frac{\sqrt{b+2de}}{2e} + x^2} dx}{2\sqrt{b-2de}} + \frac{\int \frac{1}{-\frac{\sqrt{b-2de}}{2e} + \frac{\sqrt{b+2de}}{2e} + x^2} dx}{2\sqrt{b-2de}} - \frac{\int \frac{1}{\frac{\sqrt{b-2de}}{2e} + \frac{\sqrt{b+2de}}{2e} + x^2} dx}{2\sqrt{b-2de}} \\ &= -\frac{\sqrt{e} \tan^{-1}\left(\frac{\sqrt{2}\sqrt{ex}}{\sqrt{\sqrt{b-2de}-\sqrt{b+2de}}}\right)}{\sqrt{2}\sqrt{b-2de}\sqrt{\sqrt{b-2de}-\sqrt{b+2de}}} - \frac{\sqrt{e} \tan^{-1}\left(\frac{\sqrt{2}\sqrt{ex}}{\sqrt{\sqrt{b-2de}+\sqrt{b+2de}}}\right)}{\sqrt{2}\sqrt{b-2de}\sqrt{\sqrt{b-2de}+\sqrt{b+2de}}} - \frac{\sqrt{e} \tanh^{-1}\left(\frac{\sqrt{2}\sqrt{ex}}{\sqrt{\sqrt{b-2de}-\sqrt{b+2de}}}\right)}{\sqrt{2}\sqrt{b-2de}\sqrt{\sqrt{b-2de}-\sqrt{b+2de}}} + \frac{\sqrt{e} \tanh^{-1}\left(\frac{\sqrt{2}\sqrt{ex}}{\sqrt{\sqrt{b-2de}+\sqrt{b+2de}}}\right)}{\sqrt{2}\sqrt{b-2de}\sqrt{\sqrt{b-2de}+\sqrt{b+2de}}} \end{aligned}$$

**Mathematica [C]** time = 0.0449291, size = 69, normalized size = 0.2

$$\frac{1}{4} \text{RootSum} \left[ -\#1^4 b + \#1^8 e^2 + d^2 \&, \frac{\#1^4 e \log(x - \#1) + d \log(x - \#1)}{2\#1^7 e^2 - \#1^3 b} \& \right]$$

Antiderivative was successfully verified.

[In] Integrate[(d + e\*x^4)/(d^2 - b\*x^4 + e^2\*x^8), x]

[Out] RootSum[d^2 - b\*#1^4 + e^2\*#1^8 & , (d\*Log[x - #1] + e\*Log[x - #1]\*#1^4)/(- (b\*#1^3) + 2\*e^2\*#1^7) & ]/4

**Maple [C]** time = 0.031, size = 55, normalized size = 0.2

$$\frac{1}{4} \sum_{_R=\text{RootOf}(e^2 Z^8 - b Z^4 + d^2)} \frac{(_R^4 e + d) \ln(x - _R)}{2 _R^7 e^2 - _R^3 b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x^4+d)/(e^2\*x^8-b\*x^4+d^2), x)

[Out] 1/4\*sum((\_R^4\*e+d)/(2\*\_R^7\*e^2-\_R^3\*b)\*ln(x-\_R), \_R=RootOf(\_Z^8\*e^2-\_Z^4\*b+d^2))

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{ex^4 + d}{e^2x^8 - bx^4 + d^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^4+d)/(e^2\*x^8-b\*x^4+d^2), x, algorithm="maxima")

[Out] integrate((e\*x^4 + d)/(e^2\*x^8 - b\*x^4 + d^2), x)





$$\begin{aligned}
& - b^3 d^4) + b) / (4 d^4 e^2 - 4 b d^3 e + b^2 d^2) / e^2) / e) + 1/4 \sqrt{\text{sqrt}(1/2) \sqrt{\text{sqrt}(((4 d^4 e^2 - 4 b d^3 e + b^2 d^2) \sqrt{-(2 d e + b) / (8 d^7 e^3 - 12 b d^6 e^2 + 6 b^2 d^5 e - b^3 d^4)) + b) / (4 d^4 e^2 - 4 b d^3 e + b^2 d^2))} \log(e x + 1/2 (2 d e + (4 d^4 e^2 - 4 b d^3 e + b^2 d^2) \sqrt{-(2 d e + b) / (8 d^7 e^3 - 12 b d^6 e^2 + 6 b^2 d^5 e - b^3 d^4)) - b) \sqrt{\text{sqrt}(1/2) \sqrt{\text{sqrt}(((4 d^4 e^2 - 4 b d^3 e + b^2 d^2) \sqrt{-(2 d e + b) / (8 d^7 e^3 - 12 b d^6 e^2 + 6 b^2 d^5 e - b^3 d^4)) + b) / (4 d^4 e^2 - 4 b d^3 e + b^2 d^2))})} - 1/4 \sqrt{\text{sqrt}(1/2) \sqrt{\text{sqrt}(((4 d^4 e^2 - 4 b d^3 e + b^2 d^2) \sqrt{-(2 d e + b) / (8 d^7 e^3 - 12 b d^6 e^2 + 6 b^2 d^5 e - b^3 d^4)) + b) / (4 d^4 e^2 - 4 b d^3 e + b^2 d^2))} \log(e x - 1/2 (2 d e + (4 d^4 e^2 - 4 b d^3 e + b^2 d^2) \sqrt{-(2 d e + b) / (8 d^7 e^3 - 12 b d^6 e^2 + 6 b^2 d^5 e - b^3 d^4)) - b) \sqrt{\text{sqrt}(1/2) \sqrt{\text{sqrt}(((4 d^4 e^2 - 4 b d^3 e + b^2 d^2) \sqrt{-(2 d e + b) / (8 d^7 e^3 - 12 b d^6 e^2 + 6 b^2 d^5 e - b^3 d^4)) + b) / (4 d^4 e^2 - 4 b d^3 e + b^2 d^2))})} + 1/4 \sqrt{\text{sqrt}(1/2) \sqrt{\text{sqrt}(-(4 d^4 e^2 - 4 b d^3 e + b^2 d^2) \sqrt{-(2 d e + b) / (8 d^7 e^3 - 12 b d^6 e^2 + 6 b^2 d^5 e - b^3 d^4)) - b) / (4 d^4 e^2 - 4 b d^3 e + b^2 d^2))} \log(e x + 1/2 (2 d e - (4 d^4 e^2 - 4 b d^3 e + b^2 d^2) \sqrt{-(2 d e + b) / (8 d^7 e^3 - 12 b d^6 e^2 + 6 b^2 d^5 e - b^3 d^4)) - b) \sqrt{\text{sqrt}(1/2) \sqrt{\text{sqrt}(-(4 d^4 e^2 - 4 b d^3 e + b^2 d^2) \sqrt{-(2 d e + b) / (8 d^7 e^3 - 12 b d^6 e^2 + 6 b^2 d^5 e - b^3 d^4)) - b) / (4 d^4 e^2 - 4 b d^3 e + b^2 d^2))})} - 1/4 \sqrt{\text{sqrt}(1/2) \sqrt{\text{sqrt}(-(4 d^4 e^2 - 4 b d^3 e + b^2 d^2) \sqrt{-(2 d e + b) / (8 d^7 e^3 - 12 b d^6 e^2 + 6 b^2 d^5 e - b^3 d^4)) - b) / (4 d^4 e^2 - 4 b d^3 e + b^2 d^2))} \log(e x - 1/2 (2 d e - (4 d^4 e^2 - 4 b d^3 e + b^2 d^2) \sqrt{-(2 d e + b) / (8 d^7 e^3 - 12 b d^6 e^2 + 6 b^2 d^5 e - b^3 d^4)) - b) \sqrt{\text{sqrt}(1/2) \sqrt{\text{sqrt}(-(4 d^4 e^2 - 4 b d^3 e + b^2 d^2) \sqrt{-(2 d e + b) / (8 d^7 e^3 - 12 b d^6 e^2 + 6 b^2 d^5 e - b^3 d^4)) - b) / (4 d^4 e^2 - 4 b d^3 e + b^2 d^2))})}
\end{aligned}$$

**Sympy [A]** time = 5.86887, size = 136, normalized size = 0.39

$$\text{RootSum}\left(t^8 (65536 b^4 d^2 - 524288 b^3 d^3 e + 1572864 b^2 d^4 e^2 - 2097152 b d^5 e^3 + 1048576 d^6 e^4) + t^4 (-256 b^3 + 1024 b^2 d e - 1024 b d^2 e^2 + e^3)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x\*\*4+d)/(e\*\*2\*x\*\*8-b\*x\*\*4+d\*\*2),x)

[Out] RootSum(\_t\*\*8\*(65536\*b\*\*4\*d\*\*2 - 524288\*b\*\*3\*d\*\*3\*e + 1572864\*b\*\*2\*d\*\*4\*e\*\*2 - 2097152\*b\*d\*\*5\*e\*\*3 + 1048576\*d\*\*6\*e\*\*4) + \_t\*\*4\*(-256\*b\*\*3 + 1024\*b\*\*2\*d\*e - 1024\*b\*d\*\*2\*e\*\*2) + e\*\*2, Lambda(\_t, \_t\*log(x + (1024\*\_t\*\*5\*b\*\*2\*d\*\*2 - 4096\*\_t\*\*5\*b\*d\*\*3\*e + 4096\*\_t\*\*5\*d\*\*4\*e\*\*2 - 4\*\_t\*b + 4\*\_t\*d\*e)/e)))

**Giac [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^4+d)/(e^2*x^8-b*x^4+d^2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

$$3.8 \quad \int \frac{d+ex^4}{d^2-fx^4+e^2x^8} dx$$

**Optimal.** Leaf size=751

$$\frac{\log\left(-x\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}+\sqrt{d}+\sqrt{ex^2}\right)}{8\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}} + \frac{\log\left(x\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}+\sqrt{d}+\sqrt{ex^2}\right)}{8\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}} - \frac{\log\left(-x\sqrt{\sqrt{2de+f}+2\sqrt{d}\sqrt{e}}\right)}{8\sqrt{d}\sqrt{\sqrt{2de+f}+2\sqrt{d}\sqrt{e}}}$$

```
[Out] -ArcTan[(Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[2*d*e + f]] - 2*Sqrt[e]*x)/Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[2*d*e + f]]/(4*Sqrt[d]*Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[2*d*e + f]]) - ArcTan[(Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[2*d*e + f]] - 2*Sqrt[e]*x)/Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[2*d*e + f]]/(4*Sqrt[d]*Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[2*d*e + f]]) + ArcTan[(Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[2*d*e + f]] + 2*Sqrt[e]*x)/Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[2*d*e + f]]/(4*Sqrt[d]*Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[2*d*e + f]]) + ArcTan[(Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[2*d*e + f]] + 2*Sqrt[e]*x)/Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[2*d*e + f]]/(4*Sqrt[d]*Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[2*d*e + f]]) - Log[Sqrt[d] - Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[2*d*e + f]]*x + Sqrt[e]*x^2]/(8*Sqrt[d]*Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[2*d*e + f]]) + Log[Sqrt[d] + Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[2*d*e + f]]*x + Sqrt[e]*x^2]/(8*Sqrt[d]*Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[2*d*e + f]]) - Log[Sqrt[d] - Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[2*d*e + f]]*x + Sqrt[e]*x^2]/(8*Sqrt[d]*Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[2*d*e + f]]) + Log[Sqrt[d] + Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[2*d*e + f]]*x + Sqrt[e]*x^2]/(8*Sqrt[d]*Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[2*d*e + f]])
```

**Rubi [A]** time = 0.924504, antiderivative size = 751, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 6, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$ , Rules used = {1419, 1094, 634, 618, 204, 628}

$$\frac{\log\left(-x\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}+\sqrt{d}+\sqrt{ex^2}\right)}{8\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}} + \frac{\log\left(x\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}+\sqrt{d}+\sqrt{ex^2}\right)}{8\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}} - \frac{\log\left(-x\sqrt{\sqrt{2de+f}+2\sqrt{d}\sqrt{e}}\right)}{8\sqrt{d}\sqrt{\sqrt{2de+f}+2\sqrt{d}\sqrt{e}}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^4)/(d^2 - f*x^4 + e^2*x^8), x]
```

```
[Out] -ArcTan[(Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[2*d*e + f]] - 2*Sqrt[e]*x)/Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[2*d*e + f]]/(4*Sqrt[d]*Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[2*d*e + f]]) - ArcTan[(Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[2*d*e + f]] - 2*Sqrt[e]*x)/Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[2*d*e + f]]/(4*Sqrt[d]*Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[2*d*e + f]]) + ArcTan[(Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[2*d*e + f]] + 2*Sqrt[e]*x)/Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[2*d*e + f]]/(4*Sqrt[d]*Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[2*d*e + f]]) + ArcTan[(Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[2*d*e + f]] + 2*Sqrt[e]*x)/Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[2*d*e + f]]/(4*Sqrt[d]*Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[2*d*e + f]]) - Log[Sqrt[d] - Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[2*d*e + f]]*x + Sqrt[e]*x^2]/(8*Sqrt[d]*Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[2*d*e + f]]) + Log[Sqrt[d] + Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[2*d*e + f]]*x + Sqrt[e]*x^2]/(8*Sqrt[d]*Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[2*d*e + f]]) - Log[Sqrt[d] - Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[2*d*e + f]]*x + Sqrt[e]*x^2]/(8*Sqrt[d]*Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[2*d*e + f]]) + Log[Sqrt[d] + Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[2*d*e + f]]*x + Sqrt[e]*x^2]/(8*Sqrt[d]*Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[2*d*e + f]])
```

```

2*d*e + f]]) - ArcTan[(Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[2*d*e + f]] - 2*Sqrt[e
]*x)/Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[2*d*e + f]]]/(4*Sqrt[d]*Sqrt[2*Sqrt[d]*S
qrt[e] - Sqrt[2*d*e + f]]) + ArcTan[(Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[2*d*e +
f]] + 2*Sqrt[e]*x)/Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[2*d*e + f]]]/(4*Sqrt[d]*Sq
rt[2*Sqrt[d]*Sqrt[e] + Sqrt[2*d*e + f]]) + ArcTan[(Sqrt[2*Sqrt[d]*Sqrt[e] +
Sqrt[2*d*e + f]] + 2*Sqrt[e]*x)/Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[2*d*e + f]]]
/(4*Sqrt[d]*Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[2*d*e + f]]) - Log[Sqrt[d] - Sqrt
[2*Sqrt[d]*Sqrt[e] - Sqrt[2*d*e + f]]*x + Sqrt[e]*x^2]/(8*Sqrt[d]*Sqrt[2*Sq
rt[d]*Sqrt[e] - Sqrt[2*d*e + f]]) + Log[Sqrt[d] + Sqrt[2*Sqrt[d]*Sqrt[e] -
Sqrt[2*d*e + f]]*x + Sqrt[e]*x^2]/(8*Sqrt[d]*Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[
2*d*e + f]]) - Log[Sqrt[d] - Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[2*d*e + f]]*x +
Sqrt[e]*x^2]/(8*Sqrt[d]*Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[2*d*e + f]]) + Log[Sq
rt[d] + Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[2*d*e + f]]*x + Sqrt[e]*x^2]/(8*Sqrt[
d]*Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[2*d*e + f]])

```

### Rule 1419

```

Int[((d_) + (e_)*(x_)^(n_))/((a_) + (b_)*(x_)^(n_) + (c_)*(x_)^(n2_)), x
_Symbol] := With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e +
q*x^(n/2) + x^n, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x^(n/2) + x
^n, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4
*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && IGtQ[n/2, 0] && (GtQ[(2*d)/e - b/c, 0]
|| (!LtQ[(2*d)/e - b/c, 0] && EqQ[d, e*Rt[a/c, 2]]))

```

### Rule 1094

```

Int[((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(-1), x_Symbol] := With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(r - x)/(q - r*x
+ x^2), x], x] + Dist[1/(2*c*q*r), Int[(r + x)/(q + r*x + x^2), x], x]]] /
; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]

```

### Rule 634

```

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

```

### Rule 618

```

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[In
t[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]

```

Rule 204

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 628

Int[((d\_) + (e\_)\*(x\_))/((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2), x\_Symbol] := Simp[(d\*Log[RemoveContent[a + b\*x + c\*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

Rubi steps

$$\begin{aligned}
 \int \frac{d + ex^4}{d^2 - fx^4 + e^2x^8} dx &= \frac{\int \frac{1}{\frac{d}{e} - \frac{\sqrt{2de+fx^2}}{e} + x^4} dx}{2e} + \frac{\int \frac{1}{\frac{d}{e} + \frac{\sqrt{2de+fx^2}}{e} + x^4} dx}{2e} \\
 &= \frac{\int \frac{\frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}}{\sqrt{e}} - x}{\frac{\sqrt{d}}{\sqrt{e}} - \frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+fx}}}{\sqrt{e}} + x^2} dx}{4\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}} + \frac{\int \frac{\frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}}{\sqrt{e}} + x}{\frac{\sqrt{d}}{\sqrt{e}} + \frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+fx}}}{\sqrt{e}} + x^2} dx}{4\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}} + \frac{\int \frac{\frac{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de+f}}}{\sqrt{e}} - x}{\frac{\sqrt{d}}{\sqrt{e}} - \frac{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de+fx}}}{\sqrt{e}} + x^2} dx}{4\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de+f}}} + \frac{\int \frac{\frac{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de+f}}}{\sqrt{e}} + x}{\frac{\sqrt{d}}{\sqrt{e}} + \frac{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de+fx}}}{\sqrt{e}} + x^2} dx}{4\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de+f}}} \\
 &= \frac{\int \frac{1}{\frac{\sqrt{d}}{\sqrt{e}} - \frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+fx}}}{\sqrt{e}} + x^2} dx}{8\sqrt{d}\sqrt{e}} + \frac{\int \frac{1}{\frac{\sqrt{d}}{\sqrt{e}} + \frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+fx}}}{\sqrt{e}} + x^2} dx}{8\sqrt{d}\sqrt{e}} + \frac{\int \frac{1}{\frac{\sqrt{d}}{\sqrt{e}} - \frac{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de+fx}}}{\sqrt{e}} + x^2} dx}{8\sqrt{d}\sqrt{e}} + \frac{\int \frac{1}{\frac{\sqrt{d}}{\sqrt{e}} + \frac{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de+fx}}}{\sqrt{e}} + x^2} dx}{8\sqrt{d}\sqrt{e}} \\
 &= -\frac{\log\left(\sqrt{d} - \sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+fx}} + \sqrt{ex^2}\right)}{8\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}} + \frac{\log\left(\sqrt{d} + \sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+fx}} + \sqrt{ex^2}\right)}{8\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}} - \frac{\log\left(\sqrt{d} - \sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de+fx}} + \sqrt{ex^2}\right)}{8\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de+f}}} + \frac{\log\left(\sqrt{d} + \sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de+fx}} + \sqrt{ex^2}\right)}{8\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de+f}}} \\
 &= -\frac{\tan^{-1}\left(\frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}-2\sqrt{ex}}{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de+f}}}\right)}{4\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de+f}}} - \frac{\tan^{-1}\left(\frac{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de+f}}-2\sqrt{ex}}{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}}\right)}{4\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}} + \frac{\tan^{-1}\left(\frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}+2\sqrt{ex}}{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de+f}}}\right)}{4\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de+f}}} - \frac{\tan^{-1}\left(\frac{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de+f}}+2\sqrt{ex}}{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}}\right)}{4\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}}
 \end{aligned}$$

**Mathematica [C]** time = 0.0402587, size = 69, normalized size = 0.09

$$\frac{1}{4} \text{RootSum} \left[ \#1^8 e^2 - \#1^4 f + d^2 \&, \frac{\#1^4 e \log(x - \#1) + d \log(x - \#1)}{2\#1^7 e^2 - \#1^3 f} \& \right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^4)/(d^2 - f*x^4 + e^2*x^8),x]
```

```
[Out] RootSum[d^2 - f*#1^4 + e^2*#1^8 & , (d*Log[x - #1] + e*Log[x - #1]*#1^4)/(-
(f*#1^3) + 2*e^2*#1^7) & ]/4
```

**Maple [C]** time = 0.034, size = 55, normalized size = 0.1

$$\frac{1}{4} \sum_{_R=\text{RootOf}(e^2_Z^8-f_Z^4+d^2)} \frac{(-_R^4 e + d) \ln(x - _R)}{2\_R^7 e^2 - \_R^3 f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^4+d)/(e^2*x^8-f*x^4+d^2),x)
```

```
[Out] 1/4*sum((_R^4*e+d)/(2*_R^7*e^2-_R^3*f)*ln(x-_R),_R=RootOf(_Z^8*e^2-_Z^4*f+d
^2))
```

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{ex^4 + d}{e^2x^8 - fx^4 + d^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^4+d)/(e^2*x^8-f*x^4+d^2),x, algorithm="maxima")
```

```
[Out] integrate((e*x^4 + d)/(e^2*x^8 - f*x^4 + d^2), x)
```

**Fricas [B]** time = 1.94836, size = 6278, normalized size = 8.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^4+d)/(e^2\*x^8-f\*x^4+d^2),x, algorithm="fricas")

[Out] 
$$-\sqrt{\sqrt{1/2}}\sqrt{\left(\left(4d^4e^2 - 4d^3ef + d^2f^2\right)\sqrt{-(2de + f)}\right) / \left(8d^7e^3 - 12d^6e^2f + 6d^5ef^2 - d^4f^3\right) + f} / \left(4d^4e^2 - 4d^3ef + d^2f^2\right) \arctan\left(\frac{1}{4} \left(2\sqrt{1/2}\right) \left(\left(8d^5e^3 - 12d^4e^2f + 6d^3ef^2 - d^2f^3\right) x \sqrt{-(2de + f)} / \left(8d^7e^3 - 12d^6e^2f + 6d^5ef^2 - d^4f^3\right) + \left(4d^2e^2 - 4d^2ef + f^2\right) x\right) \sqrt{\left(\left(4d^4e^2 - 4d^3ef + d^2f^2\right)\sqrt{-(2de + f)}\right) / \left(8d^7e^3 - 12d^6e^2f + 6d^5ef^2 - d^4f^3\right) + f} / \left(4d^4e^2 - 4d^3ef + d^2f^2\right) - \left(4d^2e^2 - 4d^2ef + f^2 + \left(8d^5e^3 - 12d^4e^2f + 6d^3ef^2 - d^2f^3\right) \sqrt{-(2de + f)} / \left(8d^7e^3 - 12d^6e^2f + 6d^5ef^2 - d^4f^3\right)\right) \sqrt{\left(\left(4d^4e^2 - 4d^3ef + d^2f^2\right)\sqrt{-(2de + f)}\right) / \left(8d^7e^3 - 12d^6e^2f + 6d^5ef^2 - d^4f^3\right) + f} / \left(4d^4e^2 - 4d^3ef + d^2f^2\right) \sqrt{\left(2e^2x^2 - \sqrt{1/2}\right) \left(2de + f - f^2 - \left(8d^5e^3 - 12d^4e^2f + 6d^3ef^2 - d^2f^3\right) \sqrt{-(2de + f)} / \left(8d^7e^3 - 12d^6e^2f + 6d^5ef^2 - d^4f^3\right)\right) \sqrt{\left(\left(4d^4e^2 - 4d^3ef + d^2f^2\right)\sqrt{-(2de + f)}\right) / \left(8d^7e^3 - 12d^6e^2f + 6d^5ef^2 - d^4f^3\right) + f} / \left(4d^4e^2 - 4d^3ef + d^2f^2\right)} / e^2) \sqrt{\sqrt{1/2}}\sqrt{\left(\left(4d^4e^2 - 4d^3ef + d^2f^2\right)\sqrt{-(2de + f)}\right) / \left(8d^7e^3 - 12d^6e^2f + 6d^5ef^2 - d^4f^3\right) + f} / \left(4d^4e^2 - 4d^3ef + d^2f^2\right)} / e + \sqrt{\sqrt{1/2}}\sqrt{-\left(\left(4d^4e^2 - 4d^3ef + d^2f^2\right)\sqrt{-(2de + f)}\right) / \left(8d^7e^3 - 12d^6e^2f + 6d^5ef^2 - d^4f^3\right) - f} / \left(4d^4e^2 - 4d^3ef + d^2f^2\right) \arctan\left(\frac{1}{4} \left(2\sqrt{1/2}\right) \left(\left(8d^5e^3 - 12d^4e^2f + 6d^3ef^2 - d^2f^3\right) x \sqrt{-(2de + f)} / \left(8d^7e^3 - 12d^6e^2f + 6d^5ef^2 - d^4f^3\right) - \left(4d^2e^2 - 4d^2ef + f^2\right) x\right) \sqrt{\left(\left(4d^4e^2 - 4d^3ef + d^2f^2\right)\sqrt{-(2de + f)}\right) / \left(8d^7e^3 - 12d^6e^2f + 6d^5ef^2 - d^4f^3\right) - f} / \left(4d^4e^2 - 4d^3ef + d^2f^2\right) \sqrt{-\left(\left(4d^4e^2 - 4d^3ef + d^2f^2\right)\sqrt{-(2de + f)}\right) / \left(8d^7e^3 - 12d^6e^2f + 6d^5ef^2 - d^4f^3\right) - f} / \left(4d^4e^2 - 4d^3ef + d^2f^2\right) + \left(4d^2e^2 - 4d^2ef + f^2 - \left(8d^5e^3 - 12d^4e^2f + 6d^3ef^2 - d^2f^3\right) \sqrt{-(2de + f)} / \left(8d^7e^3 - 12d^6e^2f + 6d^5ef^2 - d^4f^3\right)\right) \sqrt{\left(\left(4d^4e^2 - 4d^3ef + d^2f^2\right)\sqrt{-(2de + f)}\right) / \left(8d^7e^3 - 12d^6e^2f + 6d^5ef^2 - d^4f^3\right) - f} / \left(4d^4e^2 - 4d^3ef + d^2f^2\right) \sqrt{\left(2e^2x^2 - \sqrt{1/2}\right) \left(2de + f - f^2 + \left(8d^5e^3 - 12d^4e^2f + 6d^3ef^2 - d^2f^3\right) \sqrt{-(2de + f)} / \left(8d^7e^3 - 12d^6e^2f + 6d^5ef^2 - d^4f^3\right)\right) \sqrt{\left(\left(4d^4e^2 - 4d^3ef + d^2f^2\right)\sqrt{-(2de + f)}\right) / \left(8d^7e^3 - 12d^6e^2f + 6d^5ef^2 - d^4f^3\right) + f} / \left(4d^4e^2 - 4d^3ef + d^2f^2\right)} / e + \frac{1}{4} \sqrt{\sqrt{1/2}}\sqrt{\left(\left(4d^4e^2 - 4d^3ef + d^2f^2\right)\sqrt{-(2de + f)}\right) / \left(8d^7e^3 - 12d^6e^2f + 6d^5ef^2 - d^4f^3\right) + f} / \left(4d^4e^2 - 4d^3ef + d^2f^2\right) \log\left(\frac{ex + \frac{1}{2}(2de + (4d^4e^2 - 4d^3ef + d^2f^2)\sqrt{-(2de + f)} / (8d^7e^3 - 12d^6e^2f + 6d^5ef^2 - d^4f^3) - f) \sqrt{\sqrt{1/2}}\sqrt{\left(\left(4d^4e^2 - 4d^3ef + d^2f^2\right)\sqrt{-(2de + f)}\right) / \left(8d^7e^3 - 12d^6e^2f + 6d^5ef^2 - d^4f^3\right) + f} / \left(4d^4e^2 - 4d^3ef + d^2f^2\right)}}{ex + \frac{1}{2}(2de + (4d^4e^2 - 4d^3ef + d^2f^2)\sqrt{-(2de + f)} / (8d^7e^3 - 12d^6e^2f + 6d^5ef^2 - d^4f^3) - f) \sqrt{\sqrt{1/2}}\sqrt{\left(\left(4d^4e^2 - 4d^3ef + d^2f^2\right)\sqrt{-(2de + f)}\right) / \left(8d^7e^3 - 12d^6e^2f + 6d^5ef^2 - d^4f^3\right) + f} / \left(4d^4e^2 - 4d^3ef + d^2f^2\right)}}\right)$$





```
[In] integrate((e*x^4+d)/(e^2*x^8-f*x^4+d^2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

### 3.9 $\int \frac{1+x^4}{1+bx^4+x^8} dx$

**Optimal.** Leaf size=411

$$\frac{\log\left(-\sqrt{2-\sqrt{2-bx}}+x^2+1\right)}{8\sqrt{2-\sqrt{2-b}}} + \frac{\log\left(\sqrt{2-\sqrt{2-bx}}+x^2+1\right)}{8\sqrt{2-\sqrt{2-b}}} - \frac{\log\left(-\sqrt{\sqrt{2-b}+2x}+x^2+1\right)}{8\sqrt{\sqrt{2-b}+2}} + \frac{\log\left(\sqrt{\sqrt{2-b}+2x}+x^2+1\right)}{8\sqrt{\sqrt{2-b}+2}}$$

```
[Out] -ArcTan[(Sqrt[2 - Sqrt[2 - b]] - 2*x)/Sqrt[2 + Sqrt[2 - b]]]/(4*Sqrt[2 + Sqrt[2 - b]]) - ArcTan[(Sqrt[2 + Sqrt[2 - b]] - 2*x)/Sqrt[2 - Sqrt[2 - b]]]/(4*Sqrt[2 - Sqrt[2 - b]]) + ArcTan[(Sqrt[2 - Sqrt[2 - b]] + 2*x)/Sqrt[2 + Sqrt[2 - b]]]/(4*Sqrt[2 + Sqrt[2 - b]]) + ArcTan[(Sqrt[2 + Sqrt[2 - b]] + 2*x)/Sqrt[2 - Sqrt[2 - b]]]/(4*Sqrt[2 - Sqrt[2 - b]]) - Log[1 - Sqrt[2 - Sqrt[2 - b]]*x + x^2]/(8*Sqrt[2 - Sqrt[2 - b]]) + Log[1 + Sqrt[2 - Sqrt[2 - b]]*x + x^2]/(8*Sqrt[2 - Sqrt[2 - b]]) - Log[1 - Sqrt[2 + Sqrt[2 - b]]*x + x^2]/(8*Sqrt[2 + Sqrt[2 - b]]) + Log[1 + Sqrt[2 + Sqrt[2 - b]]*x + x^2]/(8*Sqrt[2 + Sqrt[2 - b]])
```

---

**Rubi [A]** time = 0.291718, antiderivative size = 411, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 6, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {1419, 1094, 634, 618, 204, 628}

$$\frac{\log\left(-\sqrt{2-\sqrt{2-bx}}+x^2+1\right)}{8\sqrt{2-\sqrt{2-b}}} + \frac{\log\left(\sqrt{2-\sqrt{2-bx}}+x^2+1\right)}{8\sqrt{2-\sqrt{2-b}}} - \frac{\log\left(-\sqrt{\sqrt{2-b}+2x}+x^2+1\right)}{8\sqrt{\sqrt{2-b}+2}} + \frac{\log\left(\sqrt{\sqrt{2-b}+2x}+x^2+1\right)}{8\sqrt{\sqrt{2-b}+2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^4)/(1 + b*x^4 + x^8), x]
```

```
[Out] -ArcTan[(Sqrt[2 - Sqrt[2 - b]] - 2*x)/Sqrt[2 + Sqrt[2 - b]]]/(4*Sqrt[2 + Sqrt[2 - b]]) - ArcTan[(Sqrt[2 + Sqrt[2 - b]] - 2*x)/Sqrt[2 - Sqrt[2 - b]]]/(4*Sqrt[2 - Sqrt[2 - b]]) + ArcTan[(Sqrt[2 - Sqrt[2 - b]] + 2*x)/Sqrt[2 + Sqrt[2 - b]]]/(4*Sqrt[2 + Sqrt[2 - b]]) + ArcTan[(Sqrt[2 + Sqrt[2 - b]] + 2*x)/Sqrt[2 - Sqrt[2 - b]]]/(4*Sqrt[2 - Sqrt[2 - b]]) - Log[1 - Sqrt[2 - Sqrt[2 - b]]*x + x^2]/(8*Sqrt[2 - Sqrt[2 - b]]) + Log[1 + Sqrt[2 - Sqrt[2 - b]]*x + x^2]/(8*Sqrt[2 - Sqrt[2 - b]]) - Log[1 - Sqrt[2 + Sqrt[2 - b]]*x + x^2]/(8*Sqrt[2 + Sqrt[2 - b]]) + Log[1 + Sqrt[2 + Sqrt[2 - b]]*x + x^2]/(8*Sqrt[2 + Sqrt[2 - b]])
```

[2 + Sqrt[2 - b]])

### Rule 1419

```
Int[((d_) + (e_)*(x_)^(n_))/((a_) + (b_)*(x_)^(n_) + (c_)*(x_)^(n2_)), x
_Symbol] := With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e +
q*x^(n/2) + x^n, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x^(n/2) + x
^n, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4
*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && IGtQ[n/2, 0] && (GtQ[(2*d)/e - b/c, 0]
|| (!LtQ[(2*d)/e - b/c, 0] && EqQ[d, e*Rt[a/c, 2]]))
```

### Rule 1094

```
Int[((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(-1), x_Symbol] := With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(r - x)/(q - r*x
+ x^2), x], x] + Dist[1/(2*c*q*r), Int[(r + x)/(q + r*x + x^2), x], x]]] /
; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

### Rule 634

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

### Rule 618

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

### Rule 204

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

### Rule 628

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

### Rubi steps

$$\begin{aligned}
\int \frac{1+x^4}{1+bx^4+x^8} dx &= \frac{1}{2} \int \frac{1}{1-\sqrt{2-bx^2+x^4}} dx + \frac{1}{2} \int \frac{1}{1+\sqrt{2-bx^2+x^4}} dx \\
&= \frac{\int \frac{\sqrt{2-\sqrt{2-b}-x}}{1-\sqrt{2-\sqrt{2-b}+x^2}} dx}{4\sqrt{2-\sqrt{2-b}}} + \frac{\int \frac{\sqrt{2-\sqrt{2-b}+x}}{1+\sqrt{2-\sqrt{2-b}+x^2}} dx}{4\sqrt{2-\sqrt{2-b}}} + \frac{\int \frac{\sqrt{2+\sqrt{2-b}-x}}{1-\sqrt{2+\sqrt{2-b}+x^2}} dx}{4\sqrt{2+\sqrt{2-b}}} + \frac{\int \frac{\sqrt{2+\sqrt{2-b}+x}}{1+\sqrt{2+\sqrt{2-b}+x^2}} dx}{4\sqrt{2+\sqrt{2-b}}} \\
&= \frac{1}{8} \int \frac{1}{1-\sqrt{2-\sqrt{2-b}+x^2}} dx + \frac{1}{8} \int \frac{1}{1+\sqrt{2-\sqrt{2-b}+x^2}} dx + \frac{1}{8} \int \frac{1}{1-\sqrt{2+\sqrt{2-b}+x^2}} dx + \frac{1}{8} \int \frac{1}{1+\sqrt{2+\sqrt{2-b}+x^2}} dx \\
&= -\frac{\log\left(1-\sqrt{2-\sqrt{2-b}+x^2}\right)}{8\sqrt{2-\sqrt{2-b}}} + \frac{\log\left(1+\sqrt{2-\sqrt{2-b}+x^2}\right)}{8\sqrt{2-\sqrt{2-b}}} - \frac{\log\left(1-\sqrt{2+\sqrt{2-b}+x^2}\right)}{8\sqrt{2+\sqrt{2-b}}} + \frac{\log\left(1+\sqrt{2+\sqrt{2-b}+x^2}\right)}{8\sqrt{2+\sqrt{2-b}}} \\
&= -\frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{2-b}-2x}}{\sqrt{2+\sqrt{2-b}}}\right)}{4\sqrt{2+\sqrt{2-b}}} - \frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{2-b}-2x}}{\sqrt{2-\sqrt{2-b}}}\right)}{4\sqrt{2-\sqrt{2-b}}} + \frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{2-b}+2x}}{\sqrt{2+\sqrt{2-b}}}\right)}{4\sqrt{2+\sqrt{2-b}}} + \frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{2-b}+2x}}{\sqrt{2-\sqrt{2-b}}}\right)}{4\sqrt{2-\sqrt{2-b}}} - \frac{\log\left(\dots\right)}{4}
\end{aligned}$$

**Mathematica [C]** time = 0.027104, size = 55, normalized size = 0.13

$$\frac{1}{4} \text{RootSum}\left[\#1^4 b + \#1^8 + 1 \&, \frac{\#1^4 \log(x - \#1) + \log(x - \#1)}{\#1^3 b + 2\#1^7} \&\right]$$

Antiderivative was successfully verified.

[In] Integrate[(1 + x^4)/(1 + b\*x^4 + x^8), x]

[Out] RootSum[1 + b\*#1^4 + #1^8 & , (Log[x - #1] + Log[x - #1]\*#1^4)/(b\*#1^3 + 2\*#1^7) & ]/4

**Maple [C]** time = 0.052, size = 42, normalized size = 0.1

$$\frac{1}{4} \sum_{_R=\text{RootOf}(_Z^8+b_Z^4+1)} \frac{(-_R^4+1) \ln(x-_R)}{2\_R^7+_R^3 b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^4+1)/(x^8+b*x^4+1),x)`

[Out] `1/4*sum((_R^4+1)/(2*_R^7+_R^3*b)*ln(x-_R),_R=RootOf(_Z^8+_Z^4*b+1))`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4 + 1}{x^8 + bx^4 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^4+1)/(x^8+b*x^4+1),x, algorithm="maxima")`

[Out] `integrate((x^4 + 1)/(x^8 + b*x^4 + 1), x)`

**Fricas [B]** time = 1.5601, size = 3723, normalized size = 9.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^4+1)/(x^8+b*x^4+1),x, algorithm="fricas")`

[Out] `sqrt(sqrt(1/2)*sqrt(((b^2 + 4*b + 4)*sqrt((b - 2)/(b^3 + 6*b^2 + 12*b + 8)) - b)/(b^2 + 4*b + 4)))*arctan(1/2*sqrt(1/2)*(b^2 + (b^3 + 6*b^2 + 12*b + 8)*sqrt((b - 2)/(b^3 + 6*b^2 + 12*b + 8)) + 4*b + 4)*sqrt(x^2 + 1/2*sqrt(1/2)*(b^2 + (b^3 + 6*b^2 + 12*b + 8)*sqrt((b - 2)/(b^3 + 6*b^2 + 12*b + 8)) + 2*b)*sqrt(((b^2 + 4*b + 4)*sqrt((b - 2)/(b^3 + 6*b^2 + 12*b + 8)) - b)/(b^2 + 4*b + 4)))*sqrt(sqrt(1/2)*sqrt(((b^2 + 4*b + 4)*sqrt((b - 2)/(b^3 + 6*b^2 + 12*b + 8)) - b)/(b^2 + 4*b + 4)))*sqrt(((b^2 + 4*b + 4)*sqrt((b - 2)/(b^3 + 6*b^2 + 12*b + 8)) - b)/(b^2 + 4*b + 4)) - 1/2*sqrt(1/2)*((b^3 + 6*b^2 + 12*b + 8)*x*sqrt((b - 2)/(b^3 + 6*b^2 + 12*b + 8)) + (b^2 + 4*b + 4)*x)*sqrt(sqrt(1/2)*sqrt(((b^2 + 4*b + 4)*sqrt((b - 2)/(b^3 + 6*b^2 + 12*b + 8)) - b)/(b^2 + 4*b + 4)))*sqrt(((b^2 + 4*b + 4)*sqrt((b - 2)/(b^3 + 6*b^2 + 12*b + 8)) - b)/(b^2 + 4*b + 4)) - sqrt(sqrt(1/2)*sqrt(-((b^2 + 4*b + 4)*sqrt((b - 2)/(b^3 + 6*b^2 + 12*b + 8)) + b)/(b^2 + 4*b + 4)))*arctan(-1/2*(sqrt(1/2)*(b^2 - (b^3 + 6*b^2 + 12*b + 8)*sqrt((b - 2)/(b^3 + 6*b^2 + 12*b + 8)) + 4*b + 4)*sqrt(x^2 + 1/2*sqrt(1/2)*(b^2 - (b^3 + 6*b^2 + 12*b + 8)*sqrt((b - 2)/(b^3 + 6*b^2 + 12*b + 8)) + 2*b)*sqrt(-((b^2 + 4*b + 4)*sqrt((b -`

$$\begin{aligned}
& 2)/(b^3 + 6b^2 + 12b + 8)) + b)/(b^2 + 4b + 4)) * \sqrt{-((b^2 + 4b + 4) \\
& * \sqrt{(b - 2)/(b^3 + 6b^2 + 12b + 8)) + b)/(b^2 + 4b + 4)) + \sqrt{1/2} * ( \\
& (b^3 + 6b^2 + 12b + 8) * x * \sqrt{(b - 2)/(b^3 + 6b^2 + 12b + 8)) - (b^2 + \\
& 4b + 4) * x) * \sqrt{-((b^2 + 4b + 4) * \sqrt{(b - 2)/(b^3 + 6b^2 + 12b + 8)) + \\
& b)/(b^2 + 4b + 4)) * \sqrt{\sqrt{1/2} * \sqrt{-((b^2 + 4b + 4) * \sqrt{(b - 2)/(b \\
& ^3 + 6b^2 + 12b + 8)) + b)/(b^2 + 4b + 4))}} - 1/4 * \sqrt{\sqrt{1/2} * \sqrt{- \\
& ((b^2 + 4b + 4) * \sqrt{(b - 2)/(b^3 + 6b^2 + 12b + 8)) + b)/(b^2 + 4b + 4 \\
& ))} * \log(1/2 * ((b^2 + 4b + 4) * \sqrt{(b - 2)/(b^3 + 6b^2 + 12b + 8)) - b - 2 \\
& ) * \sqrt{\sqrt{1/2} * \sqrt{-((b^2 + 4b + 4) * \sqrt{(b - 2)/(b^3 + 6b^2 + 12b + \\
& 8)) + b)/(b^2 + 4b + 4))}} + x) + 1/4 * \sqrt{\sqrt{1/2} * \sqrt{-((b^2 + 4b + 4) \\
& * \sqrt{(b - 2)/(b^3 + 6b^2 + 12b + 8)) + b)/(b^2 + 4b + 4))} * \log(-1/2 * ((b \\
& ^2 + 4b + 4) * \sqrt{(b - 2)/(b^3 + 6b^2 + 12b + 8)) - b - 2) * \sqrt{\sqrt{1/2} \\
& } * \sqrt{-((b^2 + 4b + 4) * \sqrt{(b - 2)/(b^3 + 6b^2 + 12b + 8)) + b)/(b^2 + \\
& 4b + 4))}} + x) + 1/4 * \sqrt{\sqrt{1/2} * \sqrt{((b^2 + 4b + 4) * \sqrt{(b - 2)/(b \\
& ^3 + 6b^2 + 12b + 8)) - b)/(b^2 + 4b + 4))} * \log(1/2 * ((b^2 + 4b + 4) * \sqrt{ \\
& (b - 2)/(b^3 + 6b^2 + 12b + 8)) + b + 2) * \sqrt{\sqrt{1/2} * \sqrt{((b^2 + 4b \\
& + 4) * \sqrt{(b - 2)/(b^3 + 6b^2 + 12b + 8)) - b)/(b^2 + 4b + 4))}} + x) - \\
& 1/4 * \sqrt{\sqrt{1/2} * \sqrt{((b^2 + 4b + 4) * \sqrt{(b - 2)/(b^3 + 6b^2 + 12b \\
& + 8)) - b)/(b^2 + 4b + 4))} * \log(-1/2 * ((b^2 + 4b + 4) * \sqrt{(b - 2)/(b^3 + \\
& 6b^2 + 12b + 8)) + b + 2) * \sqrt{\sqrt{1/2} * \sqrt{((b^2 + 4b + 4) * \sqrt{(b - \\
& 2)/(b^3 + 6b^2 + 12b + 8)) - b)/(b^2 + 4b + 4))}} + x)
\end{aligned}$$

**Sympy [A]** time = 2.24891, size = 75, normalized size = 0.18

RootSum( $t^8(65536b^4 + 524288b^3 + 1572864b^2 + 2097152b + 1048576) + t^4(256b^3 + 1024b^2 + 1024b) + 1, (t \mapsto t \log$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x\*\*4+1)/(x\*\*8+b\*x\*\*4+1), x)

[Out] RootSum(\_t\*\*8\*(65536\*b\*\*4 + 524288\*b\*\*3 + 1572864\*b\*\*2 + 2097152\*b + 1048576) + \_t\*\*4\*(256\*b\*\*3 + 1024\*b\*\*2 + 1024\*b) + 1, Lambda(\_t, \_t\*log(1024\*\_t\*\*5\*b\*\*2 + 4096\*\_t\*\*5\*b + 4096\*\_t\*\*5 + 4\*\_t\*b + 4\*\_t + x)))

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4 + 1}{x^8 + bx^4 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)/(x^8+b*x^4+1),x, algorithm="giac")
```

```
[Out] integrate((x^4 + 1)/(x^8 + b*x^4 + 1), x)
```

### 3.10 $\int \frac{1+x^4}{1+3x^4+x^8} dx$

**Optimal.** Leaf size=451

$$\frac{\sqrt[4]{3+\sqrt{5}} \log\left(2x^2 - 2\sqrt[4]{2(3-\sqrt{5})}x + \sqrt{2(3-\sqrt{5})}\right)}{4 \cdot 2^{3/4}\sqrt{5}} + \frac{\sqrt[4]{3+\sqrt{5}} \log\left(2x^2 + 2\sqrt[4]{2(3-\sqrt{5})}x + \sqrt{2(3-\sqrt{5})}\right)}{4 \cdot 2^{3/4}\sqrt{5}} - \frac{\sqrt[4]{3-\sqrt{5}} \log\left(2x^2 - 2\sqrt[4]{2(3+\sqrt{5})}x + \sqrt{2(3+\sqrt{5})}\right)}{4 \cdot 2^{3/4}\sqrt{5}} + \frac{\sqrt[4]{3-\sqrt{5}} \log\left(2x^2 + 2\sqrt[4]{2(3+\sqrt{5})}x + \sqrt{2(3+\sqrt{5})}\right)}{4 \cdot 2^{3/4}\sqrt{5}}$$

[Out]  $-\left(\left(3 + \sqrt{5}\right)^{1/4} \operatorname{ArcTan}\left[\frac{1 - \left(2^{3/4}\right)x}{\left(3 - \sqrt{5}\right)^{1/4}}\right]\right) / \left(2 \cdot 2^{3/4} \sqrt{5}\right) + \left(\left(3 + \sqrt{5}\right)^{1/4} \operatorname{ArcTan}\left[\frac{1 + \left(2^{3/4}\right)x}{\left(3 - \sqrt{5}\right)^{1/4}}\right]\right) / \left(2 \cdot 2^{3/4} \sqrt{5}\right) - \left(\left(3 - \sqrt{5}\right)^{1/4} \operatorname{ArcTan}\left[\frac{1 - \left(2^{3/4}\right)x}{\left(3 + \sqrt{5}\right)^{1/4}}\right]\right) / \left(2 \cdot 2^{3/4} \sqrt{5}\right) + \left(\left(3 - \sqrt{5}\right)^{1/4} \operatorname{ArcTan}\left[\frac{1 + \left(2^{3/4}\right)x}{\left(3 + \sqrt{5}\right)^{1/4}}\right]\right) / \left(2 \cdot 2^{3/4} \sqrt{5}\right) - \left(\left(3 + \sqrt{5}\right)^{1/4} \operatorname{Log}\left[\sqrt{2 \cdot \left(3 - \sqrt{5}\right)}\right] - 2 \cdot \left(2 \cdot \left(3 - \sqrt{5}\right)\right)^{1/4} x + 2x^2\right) / \left(4 \cdot 2^{3/4} \sqrt{5}\right) + \left(\left(3 + \sqrt{5}\right)^{1/4} \operatorname{Log}\left[\sqrt{2 \cdot \left(3 - \sqrt{5}\right)}\right] + 2 \cdot \left(2 \cdot \left(3 - \sqrt{5}\right)\right)^{1/4} x + 2x^2\right) / \left(4 \cdot 2^{3/4} \sqrt{5}\right) - \left(\left(3 - \sqrt{5}\right)^{1/4} \operatorname{Log}\left[\sqrt{2 \cdot \left(3 + \sqrt{5}\right)}\right] - 2 \cdot \left(2 \cdot \left(3 + \sqrt{5}\right)\right)^{1/4} x + 2x^2\right) / \left(4 \cdot 2^{3/4} \sqrt{5}\right) + \left(\left(3 - \sqrt{5}\right)^{1/4} \operatorname{Log}\left[\sqrt{2 \cdot \left(3 + \sqrt{5}\right)}\right] + 2 \cdot \left(2 \cdot \left(3 + \sqrt{5}\right)\right)^{1/4} x + 2x^2\right) / \left(4 \cdot 2^{3/4} \sqrt{5}\right)$

**Rubi [A]** time = 0.406547, antiderivative size = 451, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 7, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$ , Rules used = {1420, 211, 1165, 628, 1162, 617, 204}

$$\frac{\sqrt[4]{3+\sqrt{5}} \log\left(2x^2 - 2\sqrt[4]{2(3-\sqrt{5})}x + \sqrt{2(3-\sqrt{5})}\right)}{4 \cdot 2^{3/4}\sqrt{5}} + \frac{\sqrt[4]{3+\sqrt{5}} \log\left(2x^2 + 2\sqrt[4]{2(3-\sqrt{5})}x + \sqrt{2(3-\sqrt{5})}\right)}{4 \cdot 2^{3/4}\sqrt{5}} - \frac{\sqrt[4]{3-\sqrt{5}} \log\left(2x^2 - 2\sqrt[4]{2(3+\sqrt{5})}x + \sqrt{2(3+\sqrt{5})}\right)}{4 \cdot 2^{3/4}\sqrt{5}} + \frac{\sqrt[4]{3-\sqrt{5}} \log\left(2x^2 + 2\sqrt[4]{2(3+\sqrt{5})}x + \sqrt{2(3+\sqrt{5})}\right)}{4 \cdot 2^{3/4}\sqrt{5}}$$

Antiderivative was successfully verified.

[In] Int[(1 + x^4)/(1 + 3\*x^4 + x^8), x]

[Out]  $-\left(\left(3 + \sqrt{5}\right)^{1/4} \operatorname{ArcTan}\left[\frac{1 - \left(2^{3/4}\right)x}{\left(3 - \sqrt{5}\right)^{1/4}}\right]\right) / \left(2 \cdot 2^{3/4} \sqrt{5}\right) + \left(\left(3 + \sqrt{5}\right)^{1/4} \operatorname{ArcTan}\left[\frac{1 + \left(2^{3/4}\right)x}{\left(3 - \sqrt{5}\right)^{1/4}}\right]\right) / \left(2 \cdot 2^{3/4} \sqrt{5}\right) - \left(\left(3 - \sqrt{5}\right)^{1/4} \operatorname{ArcTan}\left[\frac{1 - \left(2^{3/4}\right)x}{\left(3 + \sqrt{5}\right)^{1/4}}\right]\right) / \left(2 \cdot 2^{3/4} \sqrt{5}\right) + \left(\left(3 - \sqrt{5}\right)^{1/4} \operatorname{ArcTan}\left[\frac{1 + \left(2^{3/4}\right)x}{\left(3 + \sqrt{5}\right)^{1/4}}\right]\right) / \left(2 \cdot 2^{3/4} \sqrt{5}\right) - \left(\left(3 + \sqrt{5}\right)^{1/4} \operatorname{Log}\left[\sqrt{2 \cdot \left(3 - \sqrt{5}\right)}\right] - 2 \cdot \left(2 \cdot \left(3 - \sqrt{5}\right)\right)^{1/4} x + 2x^2\right) / \left(4 \cdot 2^{3/4} \sqrt{5}\right) + \left(\left(3 + \sqrt{5}\right)^{1/4} \operatorname{Log}\left[\sqrt{2 \cdot \left(3 - \sqrt{5}\right)}\right] + 2 \cdot \left(2 \cdot \left(3 - \sqrt{5}\right)\right)^{1/4} x + 2x^2\right) / \left(4 \cdot 2^{3/4} \sqrt{5}\right) - \left(\left(3 - \sqrt{5}\right)^{1/4} \operatorname{Log}\left[\sqrt{2 \cdot \left(3 + \sqrt{5}\right)}\right] - 2 \cdot \left(2 \cdot \left(3 + \sqrt{5}\right)\right)^{1/4} x + 2x^2\right) / \left(4 \cdot 2^{3/4} \sqrt{5}\right) + \left(\left(3 - \sqrt{5}\right)^{1/4} \operatorname{Log}\left[\sqrt{2 \cdot \left(3 + \sqrt{5}\right)}\right] + 2 \cdot \left(2 \cdot \left(3 + \sqrt{5}\right)\right)^{1/4} x + 2x^2\right) / \left(4 \cdot 2^{3/4} \sqrt{5}\right)$



$$\frac{2(3 + \sqrt{5}) - 2(2(3 + \sqrt{5}))^{1/4}x + 2x^2}{(4 \cdot 2^{3/4} \sqrt{5})} + \frac{((3 - \sqrt{5})^{1/4} \log[\sqrt{2(3 + \sqrt{5})}] + 2(2(3 + \sqrt{5}))^{1/4}x + 2x^2)}{(4 \cdot 2^{3/4} \sqrt{5})}$$

### Rule 1420

```
Int[((d_) + (e_)*(x_)^(n_))/((a_) + (b_)*(x_)^(n_) + (c_)*(x_)^(n2_)), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && IGtQ[n/2, 0] && GtQ[b^2 - 4*a*c, 0]
```

### Rule 211

```
Int[((a_) + (b_)*(x_)^4)^(-1), x_Symbol] := With[{r = Numerator[Rt[a/b, 2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4), x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] && AtomQ[SplitProduct[SumBaseQ, b]]))
```

### Rule 1165

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

### Rule 628

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

### Rule 1162

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

### Rule 617

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b]
```

], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4\*a\*c]) /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

### Rubi steps

$$\begin{aligned} \int \frac{1+x^4}{1+3x^4+x^8} dx &= \frac{1}{10} (5-\sqrt{5}) \int \frac{1}{\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4} dx + \frac{1}{10} (5+\sqrt{5}) \int \frac{1}{\frac{3}{2}+\frac{\sqrt{5}}{2}+x^4} dx \\ &= \frac{\int \frac{\sqrt{3-\sqrt{5}-\sqrt{2}x^2}}{\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4} dx}{2\sqrt{10}} + \frac{\int \frac{\sqrt{3-\sqrt{5}+\sqrt{2}x^2}}{\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4} dx}{2\sqrt{10}} + \frac{\int \frac{\sqrt{3+\sqrt{5}-\sqrt{2}x^2}}{\frac{3}{2}+\frac{\sqrt{5}}{2}+x^4} dx}{2\sqrt{10}} + \frac{\int \frac{\sqrt{3+\sqrt{5}+\sqrt{2}x^2}}{\frac{3}{2}+\frac{\sqrt{5}}{2}+x^4} dx}{2\sqrt{10}} \\ &= \frac{\int \frac{1}{\sqrt{\frac{1}{2}(3-\sqrt{5})}-\sqrt{2(3-\sqrt{5})x+x^2}} dx}{4\sqrt{5}} + \frac{\int \frac{1}{\sqrt{\frac{1}{2}(3-\sqrt{5})+\sqrt{2(3-\sqrt{5})x+x^2}} dx}{4\sqrt{5}} + \frac{\int \frac{1}{\sqrt{\frac{1}{2}(3+\sqrt{5})}-\sqrt{2(3+\sqrt{5})x+x^2}} dx}{4\sqrt{5}} + \frac{\int \frac{1}{\sqrt{\frac{1}{2}(3+\sqrt{5})+\sqrt{2(3+\sqrt{5})x+x^2}} dx}{4\sqrt{5}} \\ &= -\frac{\sqrt[4]{3+\sqrt{5}} \log\left(\sqrt{2(3-\sqrt{5})}-2\sqrt[4]{2(3-\sqrt{5})}x+2x^2\right)}{4 \cdot 2^{3/4}\sqrt{5}} + \frac{\sqrt[4]{3+\sqrt{5}} \log\left(\sqrt{2(3-\sqrt{5})}+2\sqrt[4]{2(3-\sqrt{5})}x+2x^2\right)}{4 \cdot 2^{3/4}\sqrt{5}} \\ &= -\frac{\tan^{-1}\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2\sqrt{5}\sqrt[4]{2(3-\sqrt{5})}} + \frac{\tan^{-1}\left(1+\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2\sqrt{5}\sqrt[4]{2(3-\sqrt{5})}} - \frac{\tan^{-1}\left(1-\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{2\sqrt{5}\sqrt[4]{2(3+\sqrt{5})}} + \frac{\tan^{-1}\left(1+\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{2\sqrt{5}\sqrt[4]{2(3+\sqrt{5})}} - \frac{\sqrt[4]{3+\sqrt{5}}}{\sqrt{5}} \end{aligned}$$

**Mathematica [C]** time = 0.0141974, size = 55, normalized size = 0.12

$$\frac{1}{4} \text{RootSum}\left[\#1^8 + 3\#1^4 + 1\&, \frac{\#1^4 \log(x - \#1) + \log(x - \#1)}{2\#1^7 + 3\#1^3} \&\right]$$

Antiderivative was successfully verified.

[In] Integrate[(1 + x^4)/(1 + 3\*x^4 + x^8), x]

[Out] RootSum[1 + 3\*#1^4 + #1^8 & , (Log[x - #1] + Log[x - #1]\*#1^4)/(3\*#1^3 + 2\*#1^7) & ]/4

**Maple [C]** time = 0.007, size = 42, normalized size = 0.1

$$\frac{1}{4} \sum_{_R=\text{RootOf}(_Z^8+3_Z^4+1)} \frac{(_R^4+1) \ln(x-_R)}{2_R^7+3_R^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^4+1)/(x^8+3\*x^4+1),x)

[Out] 1/4\*sum((\_R^4+1)/(2\*\_R^7+3\*\_R^3)\*ln(x-\_R),\_R=RootOf(\_Z^8+3\*\_Z^4+1))

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4+1}{x^8+3x^4+1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4+1)/(x^8+3\*x^4+1),x, algorithm="maxima")

[Out] integrate((x^4 + 1)/(x^8 + 3\*x^4 + 1), x)

**Fricas [B]** time = 1.62415, size = 3098, normalized size = 6.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4+1)/(x^8+3\*x^4+1),x, algorithm="fricas")

[Out] 1/80\*sqrt(10)\*(2\*sqrt(5) + 6)^(3/4)\*sqrt(sqrt(5) + 3)\*(sqrt(5) - 3)\*arctan(1/80\*sqrt(10)\*sqrt(20\*x^2 + sqrt(10)\*(sqrt(5)\*sqrt(2)\*x - 5\*sqrt(2)\*x))\*(2\*sqrt(5) + 6)^(1/4) - 5\*sqrt(2\*sqrt(5) + 6)\*(sqrt(5) - 3))\*(2\*sqrt(5) + 6)^(5

```

/4)*sqrt(sqrt(5) + 3)*(sqrt(5) - 2) + 1/40*sqrt(10)*(2*sqrt(5)*x - 5*x)*(2*
sqrt(5) + 6)^(5/4)*sqrt(sqrt(5) + 3) - 1/8*(sqrt(5)*sqrt(2) - 3*sqrt(2))*sq
rt(2*sqrt(5) + 6)*sqrt(sqrt(5) + 3)) + 1/80*sqrt(10)*(2*sqrt(5) + 6)^(3/4)*
sqrt(sqrt(5) + 3)*(sqrt(5) - 3)*arctan(1/80*sqrt(10)*sqrt(20*x^2 - sqrt(10)
*(sqrt(5)*sqrt(2)*x - 5*sqrt(2)*x)*(2*sqrt(5) + 6)^(1/4) - 5*sqrt(2*sqrt(5)
+ 6)*(sqrt(5) - 3))*(2*sqrt(5) + 6)^(5/4)*sqrt(sqrt(5) + 3)*(sqrt(5) - 2)
+ 1/40*sqrt(10)*(2*sqrt(5)*x - 5*x)*(2*sqrt(5) + 6)^(5/4)*sqrt(sqrt(5) + 3)
+ 1/8*(sqrt(5)*sqrt(2) - 3*sqrt(2))*sqrt(2*sqrt(5) + 6)*sqrt(sqrt(5) + 3))
- 1/80*sqrt(10)*(sqrt(5) + 3)*sqrt(-sqrt(5) + 3)*(-2*sqrt(5) + 6)^(3/4)*ar
ctan(1/80*sqrt(10)*sqrt(20*x^2 + sqrt(10)*(sqrt(5)*sqrt(2)*x + 5*sqrt(2)*x)
*(-2*sqrt(5) + 6)^(1/4) + 5*(sqrt(5) + 3)*sqrt(-2*sqrt(5) + 6))*(sqrt(5) +
2)*sqrt(-sqrt(5) + 3)*(-2*sqrt(5) + 6)^(5/4) - 1/40*(sqrt(10)*(2*sqrt(5)*x
+ 5*x)*(-2*sqrt(5) + 6)^(5/4) + 5*(sqrt(5)*sqrt(2) + 3*sqrt(2))*sqrt(-2*sq
rt(5) + 6))*sqrt(-sqrt(5) + 3)) - 1/80*sqrt(10)*(sqrt(5) + 3)*sqrt(-sqrt(5)
+ 3)*(-2*sqrt(5) + 6)^(3/4)*arctan(1/80*sqrt(10)*sqrt(20*x^2 - sqrt(10)*(sq
rt(5)*sqrt(2)*x + 5*sqrt(2)*x)*(-2*sqrt(5) + 6)^(1/4) + 5*(sqrt(5) + 3)*sq
rt(-2*sqrt(5) + 6))*(sqrt(5) + 2)*sqrt(-sqrt(5) + 3)*(-2*sqrt(5) + 6)^(5/4)
- 1/40*(sqrt(10)*(2*sqrt(5)*x + 5*x)*(-2*sqrt(5) + 6)^(5/4) - 5*(sqrt(5)*sq
rt(2) + 3*sqrt(2))*sqrt(-2*sqrt(5) + 6))*sqrt(-sqrt(5) + 3)) - 1/80*sqrt(10)
)*sqrt(2)*(2*sqrt(5) + 6)^(1/4)*log(20*x^2 + sqrt(10)*(sqrt(5)*sqrt(2)*x -
5*sqrt(2)*x)*(2*sqrt(5) + 6)^(1/4) - 5*sqrt(2*sqrt(5) + 6)*(sqrt(5) - 3)) +
1/80*sqrt(10)*sqrt(2)*(2*sqrt(5) + 6)^(1/4)*log(20*x^2 - sqrt(10)*(sqrt(5)
*sqrt(2)*x - 5*sqrt(2)*x)*(2*sqrt(5) + 6)^(1/4) - 5*sqrt(2*sqrt(5) + 6)*(sq
rt(5) - 3)) + 1/80*sqrt(10)*sqrt(2)*(-2*sqrt(5) + 6)^(1/4)*log(20*x^2 + sq
rt(10)*(sqrt(5)*sqrt(2)*x + 5*sqrt(2)*x)*(-2*sqrt(5) + 6)^(1/4) + 5*(sqrt(5)
+ 3)*sqrt(-2*sqrt(5) + 6)) - 1/80*sqrt(10)*sqrt(2)*(-2*sqrt(5) + 6)^(1/4)*
log(20*x^2 - sqrt(10)*(sqrt(5)*sqrt(2)*x + 5*sqrt(2)*x)*(-2*sqrt(5) + 6)^(
1/4) + 5*(sqrt(5) + 3)*sqrt(-2*sqrt(5) + 6))

```

**Sympy [A]** time = 1.06069, size = 24, normalized size = 0.05

$$\text{RootSum}\left(40960000t^8 + 19200t^4 + 1, \left(t \mapsto t \log(25600t^5 + 16t + x)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x\*\*4+1)/(x\*\*8+3\*x\*\*4+1), x)

[Out] RootSum(40960000\*\_t\*\*8 + 19200\*\_t\*\*4 + 1, Lambda(\_t, \_t\*log(25600\*\_t\*\*5 + 16\*\_t + x)))

**Giac [A]** time = 1.35736, size = 342, normalized size = 0.76

$$\frac{1}{40} (i+1)\sqrt{5\sqrt{5}-5} \log\left(130(i+1)x + 130i\sqrt{\sqrt{5}+1}\right) - \frac{1}{40} (i+1)\sqrt{5\sqrt{5}-5} \log\left(130(i+1)x - 130i\sqrt{\sqrt{5}+1}\right) - \frac{1}{40} (i-1)\sqrt{5\sqrt{5}+5} \log\left(130(i-1)x + 130i\sqrt{\sqrt{5}-1}\right) + \frac{1}{40} (i-1)\sqrt{5\sqrt{5}+5} \log\left(130(i-1)x - 130i\sqrt{\sqrt{5}-1}\right) + \frac{1}{40} (i+1)\sqrt{5\sqrt{5}-5} \log\left(50(i+1)x + 50i\sqrt{\sqrt{5}+1}\right) - \frac{1}{40} (i+1)\sqrt{5\sqrt{5}-5} \log\left(50(i+1)x - 50i\sqrt{\sqrt{5}+1}\right) + \frac{1}{40} (i-1)\sqrt{5\sqrt{5}+5} \log\left(50(i-1)x + 50i\sqrt{\sqrt{5}-1}\right) - \frac{1}{40} (i-1)\sqrt{5\sqrt{5}+5} \log\left(50(i-1)x - 50i\sqrt{\sqrt{5}-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4+1)/(x^8+3\*x^4+1),x, algorithm="giac")

[Out] 1/40\*(i + 1)\*sqrt(5\*sqrt(5) - 5)\*log(130\*(i + 1)\*x + 130\*i\*sqrt(sqrt(5) + 1)) - 1/40\*(i + 1)\*sqrt(5\*sqrt(5) - 5)\*log(130\*(i + 1)\*x - 130\*i\*sqrt(sqrt(5) + 1)) - 1/40\*(i - 1)\*sqrt(5\*sqrt(5) - 5)\*log(130\*(i + 1)\*x + 130\*sqrt(sqrt(5) + 1)) + 1/40\*(i - 1)\*sqrt(5\*sqrt(5) - 5)\*log(130\*(i + 1)\*x - 130\*sqrt(sqrt(5) + 1)) + 1/40\*(i + 1)\*sqrt(5\*sqrt(5) + 5)\*log(50\*(i + 1)\*x + 50\*i\*sqrt(sqrt(5) - 1)) - 1/40\*(i + 1)\*sqrt(5\*sqrt(5) + 5)\*log(50\*(i + 1)\*x - 50\*i\*sqrt(sqrt(5) - 1)) - 1/40\*(i - 1)\*sqrt(5\*sqrt(5) + 5)\*log(50\*(i + 1)\*x + 50\*sqrt(sqrt(5) - 1)) + 1/40\*(i - 1)\*sqrt(5\*sqrt(5) + 5)\*log(50\*(i + 1)\*x - 50\*sqrt(sqrt(5) - 1))

$$3.11 \quad \int \frac{1+x^4}{1+2x^4+x^8} dx$$

**Optimal.** Leaf size=85

$$-\frac{\log(x^2 - \sqrt{2}x + 1)}{4\sqrt{2}} + \frac{\log(x^2 + \sqrt{2}x + 1)}{4\sqrt{2}} - \frac{\tan^{-1}(1 - \sqrt{2}x)}{2\sqrt{2}} + \frac{\tan^{-1}(\sqrt{2}x + 1)}{2\sqrt{2}}$$

[Out] -ArcTan[1 - Sqrt[2]\*x]/(2\*Sqrt[2]) + ArcTan[1 + Sqrt[2]\*x]/(2\*Sqrt[2]) - Log[1 - Sqrt[2]\*x + x^2]/(4\*Sqrt[2]) + Log[1 + Sqrt[2]\*x + x^2]/(4\*Sqrt[2])

**Rubi [A]** time = 0.0454182, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$ , Rules used = {28, 211, 1165, 628, 1162, 617, 204}

$$-\frac{\log(x^2 - \sqrt{2}x + 1)}{4\sqrt{2}} + \frac{\log(x^2 + \sqrt{2}x + 1)}{4\sqrt{2}} - \frac{\tan^{-1}(1 - \sqrt{2}x)}{2\sqrt{2}} + \frac{\tan^{-1}(\sqrt{2}x + 1)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[(1 + x^4)/(1 + 2\*x^4 + x^8), x]

[Out] -ArcTan[1 - Sqrt[2]\*x]/(2\*Sqrt[2]) + ArcTan[1 + Sqrt[2]\*x]/(2\*Sqrt[2]) - Log[1 - Sqrt[2]\*x + x^2]/(4\*Sqrt[2]) + Log[1 + Sqrt[2]\*x + x^2]/(4\*Sqrt[2])

### Rule 28

Int[(u\_.)\*((a\_.) + (c\_.)\*(x\_)^(n2\_.) + (b\_.)\*(x\_)^(n\_.))^(p\_.), x\_Symbol] :> Dist[1/c^p, Int[u\*(b/2 + c\*x^n)^(2\*p), x], x] /; FreeQ[{a, b, c, n}, x] && EqQ[n2, 2\*n] && EqQ[b^2 - 4\*a\*c, 0] && IntegerQ[p]

### Rule 211

Int[((a\_.) + (b\_.)\*(x\_)^4)^(-1), x\_Symbol] :> With[{r = Numerator[Rt[a/b, 2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2\*r), Int[(r - s\*x^2)/(a + b\*x^4), x], x] + Dist[1/(2\*r), Int[(r + s\*x^2)/(a + b\*x^4), x], x]] /; FreeQ[{a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] && AtomQ[SplitProduct[SumBaseQ, b]]))

### Rule 1165

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Free
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

### Rule 628

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

### Rule 1162

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

### Rule 617

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c]) /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

### Rule 204

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

### Rubi steps

$$\begin{aligned}
\int \frac{1+x^4}{1+2x^4+x^8} dx &= \int \frac{1}{1+x^4} dx \\
&= \frac{1}{2} \int \frac{1-x^2}{1+x^4} dx + \frac{1}{2} \int \frac{1+x^2}{1+x^4} dx \\
&= \frac{1}{4} \int \frac{1}{1-\sqrt{2}x+x^2} dx + \frac{1}{4} \int \frac{1}{1+\sqrt{2}x+x^2} dx - \frac{\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2} dx}{4\sqrt{2}} - \frac{\int \frac{\sqrt{2}-2x}{-1+\sqrt{2}x-x^2} dx}{4\sqrt{2}} \\
&= -\frac{\log(1-\sqrt{2}x+x^2)}{4\sqrt{2}} + \frac{\log(1+\sqrt{2}x+x^2)}{4\sqrt{2}} + \frac{\text{Subst}\left(\int \frac{1}{-1-x^2} dx, x, 1-\sqrt{2}x\right)}{2\sqrt{2}} - \frac{\text{Subst}\left(\int \frac{1}{-1-x^2} dx, x, 1+\sqrt{2}x\right)}{2\sqrt{2}} \\
&= -\frac{\tan^{-1}(1-\sqrt{2}x)}{2\sqrt{2}} + \frac{\tan^{-1}(1+\sqrt{2}x)}{2\sqrt{2}} - \frac{\log(1-\sqrt{2}x+x^2)}{4\sqrt{2}} + \frac{\log(1+\sqrt{2}x+x^2)}{4\sqrt{2}}
\end{aligned}$$

**Mathematica [A]** time = 0.0199455, size = 64, normalized size = 0.75

$$\frac{-\log(x^2 - \sqrt{2}x + 1) + \log(x^2 + \sqrt{2}x + 1) - 2 \tan^{-1}(1 - \sqrt{2}x) + 2 \tan^{-1}(\sqrt{2}x + 1)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Integrate[(1 + x^4)/(1 + 2\*x^4 + x^8), x]

[Out] (-2\*ArcTan[1 - Sqrt[2]\*x] + 2\*ArcTan[1 + Sqrt[2]\*x] - Log[1 - Sqrt[2]\*x + x^2] + Log[1 + Sqrt[2]\*x + x^2])/(4\*Sqrt[2])

**Maple [A]** time = 0.001, size = 58, normalized size = 0.7

$$\frac{\arctan(1+x\sqrt{2})\sqrt{2}}{4} + \frac{\arctan(-1+x\sqrt{2})\sqrt{2}}{4} + \frac{\sqrt{2}}{8} \ln\left(\frac{1+x^2+x\sqrt{2}}{1+x^2-x\sqrt{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^4+1)/(x^8+2\*x^4+1), x)

[Out] 1/4\*arctan(1+x\*2^(1/2))\*2^(1/2)+1/4\*arctan(-1+x\*2^(1/2))\*2^(1/2)+1/8\*2^(1/2)\*ln((1+x^2+x\*2^(1/2))/(1+x^2-x\*2^(1/2)))



---

**Maxima [A]** time = 1.50123, size = 97, normalized size = 1.14

$$\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(2x+\sqrt{2})\right)+\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(2x-\sqrt{2})\right)+\frac{1}{8}\sqrt{2}\log(x^2+\sqrt{2}x+1)-\frac{1}{8}\sqrt{2}\log(x^2-\sqrt{2}x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4+1)/(x^8+2\*x^4+1),x, algorithm="maxima")

[Out] 1/4\*sqrt(2)\*arctan(1/2\*sqrt(2)\*(2\*x + sqrt(2))) + 1/4\*sqrt(2)\*arctan(1/2\*sqrt(2)\*(2\*x - sqrt(2))) + 1/8\*sqrt(2)\*log(x^2 + sqrt(2)\*x + 1) - 1/8\*sqrt(2)\*log(x^2 - sqrt(2)\*x + 1)

---

**Fricas [A]** time = 1.35686, size = 302, normalized size = 3.55

$$-\frac{1}{2}\sqrt{2}\arctan\left(-\sqrt{2}x+\sqrt{2}\sqrt{x^2+\sqrt{2}x+1}-1\right)-\frac{1}{2}\sqrt{2}\arctan\left(-\sqrt{2}x+\sqrt{2}\sqrt{x^2-\sqrt{2}x+1}+1\right)+\frac{1}{8}\sqrt{2}\log(x^2+\sqrt{2}x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4+1)/(x^8+2\*x^4+1),x, algorithm="fricas")

[Out] -1/2\*sqrt(2)\*arctan(-sqrt(2)\*x + sqrt(2)\*sqrt(x^2 + sqrt(2)\*x + 1) - 1) - 1/2\*sqrt(2)\*arctan(-sqrt(2)\*x + sqrt(2)\*sqrt(x^2 - sqrt(2)\*x + 1) + 1) + 1/8\*sqrt(2)\*log(x^2 + sqrt(2)\*x + 1) - 1/8\*sqrt(2)\*log(x^2 - sqrt(2)\*x + 1)

---

**Sympy [A]** time = 0.146687, size = 73, normalized size = 0.86

$$-\frac{\sqrt{2}\log(x^2-\sqrt{2}x+1)}{8}+\frac{\sqrt{2}\log(x^2+\sqrt{2}x+1)}{8}+\frac{\sqrt{2}\operatorname{atan}(\sqrt{2}x-1)}{4}+\frac{\sqrt{2}\operatorname{atan}(\sqrt{2}x+1)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x\*\*4+1)/(x\*\*8+2\*x\*\*4+1),x)

[Out] -sqrt(2)\*log(x\*\*2 - sqrt(2)\*x + 1)/8 + sqrt(2)\*log(x\*\*2 + sqrt(2)\*x + 1)/8 + sqrt(2)\*atan(sqrt(2)\*x - 1)/4 + sqrt(2)\*atan(sqrt(2)\*x + 1)/4

---

**Giac [A]** time = 1.12382, size = 97, normalized size = 1.14

$$\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(2x+\sqrt{2})\right)+\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(2x-\sqrt{2})\right)+\frac{1}{8}\sqrt{2}\log(x^2+\sqrt{2}x+1)-\frac{1}{8}\sqrt{2}\log(x^2-\sqrt{2}x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4+1)/(x^8+2\*x^4+1),x, algorithm="giac")

[Out] 1/4\*sqrt(2)\*arctan(1/2\*sqrt(2)\*(2\*x + sqrt(2))) + 1/4\*sqrt(2)\*arctan(1/2\*sqrt(2)\*(2\*x - sqrt(2))) + 1/8\*sqrt(2)\*log(x^2 + sqrt(2)\*x + 1) - 1/8\*sqrt(2)\*log(x^2 - sqrt(2)\*x + 1)

$$3.12 \quad \int \frac{1+x^4}{1+x^4+x^8} dx$$

**Optimal.** Leaf size=140

$$-\frac{1}{8} \log(x^2 - x + 1) + \frac{1}{8} \log(x^2 + x + 1) - \frac{\log(x^2 - \sqrt{3}x + 1)}{8\sqrt{3}} + \frac{\log(x^2 + \sqrt{3}x + 1)}{8\sqrt{3}} - \frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{4\sqrt{3}} - \frac{1}{4} \tan^{-1}(\sqrt{3} - 2x)$$

[Out] -ArcTan[(1 - 2\*x)/Sqrt[3]]/(4\*Sqrt[3]) - ArcTan[Sqrt[3] - 2\*x]/4 + ArcTan[(1 + 2\*x)/Sqrt[3]]/(4\*Sqrt[3]) + ArcTan[Sqrt[3] + 2\*x]/4 - Log[1 - x + x^2]/8 + Log[1 + x + x^2]/8 - Log[1 - Sqrt[3]\*x + x^2]/(8\*Sqrt[3]) + Log[1 + Sqrt[3]\*x + x^2]/(8\*Sqrt[3])

**Rubi [A]** time = 0.0954812, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 6, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$ , Rules used = {1419, 1094, 634, 618, 204, 628}

$$-\frac{1}{8} \log(x^2 - x + 1) + \frac{1}{8} \log(x^2 + x + 1) - \frac{\log(x^2 - \sqrt{3}x + 1)}{8\sqrt{3}} + \frac{\log(x^2 + \sqrt{3}x + 1)}{8\sqrt{3}} - \frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{4\sqrt{3}} - \frac{1}{4} \tan^{-1}(\sqrt{3} - 2x)$$

Antiderivative was successfully verified.

[In] Int[(1 + x^4)/(1 + x^4 + x^8), x]

[Out] -ArcTan[(1 - 2\*x)/Sqrt[3]]/(4\*Sqrt[3]) - ArcTan[Sqrt[3] - 2\*x]/4 + ArcTan[(1 + 2\*x)/Sqrt[3]]/(4\*Sqrt[3]) + ArcTan[Sqrt[3] + 2\*x]/4 - Log[1 - x + x^2]/8 + Log[1 + x + x^2]/8 - Log[1 - Sqrt[3]\*x + x^2]/(8\*Sqrt[3]) + Log[1 + Sqrt[3]\*x + x^2]/(8\*Sqrt[3])

#### Rule 1419

Int[((d\_) + (e\_.)\*(x\_)^(n\_))/((a\_) + (b\_.)\*(x\_)^(n\_) + (c\_.)\*(x\_)^(n2\_)), x\_Symbol] :> With[{q = Rt[(2\*d)/e - b/c, 2]}, Dist[e/(2\*c), Int[1/Simp[d/e + q\*x^(n/2) + x^n, x], x], x] + Dist[e/(2\*c), Int[1/Simp[d/e - q\*x^(n/2) + x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[n2, 2\*n] && NeQ[b^2 - 4\*a\*c, 0] && EqQ[c\*d^2 - a\*e^2, 0] && IGtQ[n/2, 0] && (GtQ[(2\*d)/e - b/c, 0] || (!LtQ[(2\*d)/e - b/c, 0] && EqQ[d, e\*Rt[a/c, 2]]))

#### Rule 1094

```
Int[((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(-1), x_Symbol] := With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(r - x)/(q - r*x
+ x^2), x], x] + Dist[1/(2*c*q*r), Int[(r + x)/(q + r*x + x^2), x], x]]] /
; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

### Rule 634

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

### Rule 618

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[In
t[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

### Rule 204

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

### Rule 628

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

### Rubi steps

$$\begin{aligned}
\int \frac{1+x^4}{1+x^4+x^8} dx &= \frac{1}{2} \int \frac{1}{1-x^2+x^4} dx + \frac{1}{2} \int \frac{1}{1+x^2+x^4} dx \\
&= \frac{1}{4} \int \frac{1-x}{1-x+x^2} dx + \frac{1}{4} \int \frac{1+x}{1+x+x^2} dx + \frac{\int \frac{\sqrt{3}-x}{1-\sqrt{3}x+x^2} dx}{4\sqrt{3}} + \frac{\int \frac{\sqrt{3}+x}{1+\sqrt{3}x+x^2} dx}{4\sqrt{3}} \\
&= \frac{1}{8} \int \frac{1}{1-x+x^2} dx - \frac{1}{8} \int \frac{-1+2x}{1-x+x^2} dx + \frac{1}{8} \int \frac{1}{1+x+x^2} dx + \frac{1}{8} \int \frac{1+2x}{1+x+x^2} dx + \frac{1}{8} \int \frac{1}{1-\sqrt{3}x+x^2} dx \\
&= -\frac{1}{8} \log(1-x+x^2) + \frac{1}{8} \log(1+x+x^2) - \frac{\log(1-\sqrt{3}x+x^2)}{8\sqrt{3}} + \frac{\log(1+\sqrt{3}x+x^2)}{8\sqrt{3}} - \frac{1}{4} \text{Subst}\left(\int \frac{1}{1-\sqrt{3}x+x^2} dx\right) \\
&= -\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{4\sqrt{3}} - \frac{1}{4} \tan^{-1}(\sqrt{3}-2x) + \frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{4\sqrt{3}} + \frac{1}{4} \tan^{-1}(\sqrt{3}+2x) - \frac{1}{8} \log(1-x+x^2) + \frac{1}{8} \log(1+x+x^2)
\end{aligned}$$

**Mathematica [C]** time = 0.184966, size = 135, normalized size = 0.96

$$\frac{1}{48} \left( -6 \log(x^2 - x + 1) + 6 \log(x^2 + x + 1) + 4i\sqrt{-6 - 6i\sqrt{3}} \tan^{-1}\left(\frac{1}{2}(1 - i\sqrt{3})x\right) - 4i\sqrt{-6 + 6i\sqrt{3}} \tan^{-1}\left(\frac{1}{2}(1 + i\sqrt{3})x\right) \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[(1 + x^4)/(1 + x^4 + x^8), x]

[Out] ((4\*I)\*Sqrt[-6 - (6\*I)\*Sqrt[3]]\*ArcTan[((1 - I\*Sqrt[3])\*x)/2] - (4\*I)\*Sqrt[-6 + (6\*I)\*Sqrt[3]]\*ArcTan[((1 + I\*Sqrt[3])\*x)/2] + 4\*Sqrt[3]\*ArcTan[(-1 + 2\*x)/Sqrt[3]] + 4\*Sqrt[3]\*ArcTan[(1 + 2\*x)/Sqrt[3]] - 6\*Log[1 - x + x^2] + 6\*Log[1 + x + x^2])/48

**Maple [A]** time = 0.016, size = 109, normalized size = 0.8

$$\frac{\ln(x^2 + x + 1)}{8} + \frac{\sqrt{3}}{12} \arctan\left(\frac{(1 + 2x)\sqrt{3}}{3}\right) - \frac{\ln(x^2 - x + 1)}{8} + \frac{\sqrt{3}}{12} \arctan\left(\frac{(2x - 1)\sqrt{3}}{3}\right) + \frac{\ln(1 + x^2 + x\sqrt{3})\sqrt{3}}{24} + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^4+1)/(x^8+x^4+1), x)

```
[Out] 1/8*ln(x^2+x+1)+1/12*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)-1/8*ln(x^2-x+1)+1/
12*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))+1/24*ln(1+x^2+x*3^(1/2))*3^(1/2)+1/4
*arctan(2*x+3^(1/2))-1/24*ln(1+x^2-x*3^(1/2))*3^(1/2)+1/4*arctan(2*x-3^(1/2
))
```

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{1}{12} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3}(2x+1)\right) + \frac{1}{12} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3}(2x-1)\right) + \frac{1}{2} \int \frac{1}{x^4 - x^2 + 1} dx + \frac{1}{8} \log(x^2 + x + 1) - \frac{1}{8} \log(x^2 - x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)/(x^8+x^4+1),x, algorithm="maxima")
```

```
[Out] 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/12*sqrt(3)*arctan(1/3*sqrt(3)
)*(2*x - 1)) + 1/2*integrate(1/(x^4 - x^2 + 1), x) + 1/8*log(x^2 + x + 1) -
1/8*log(x^2 - x + 1)
```

**Fricas [A]** time = 1.43924, size = 716, normalized size = 5.11

$$-\frac{1}{12} \sqrt{6}\sqrt{3}\sqrt{2} \arctan\left(-\frac{1}{3} \sqrt{6}\sqrt{3}\sqrt{2}x + \frac{1}{3} \sqrt{6}\sqrt{3}\sqrt{\sqrt{6}\sqrt{2}x + 2x^2 + 2} - \sqrt{3}\right) - \frac{1}{12} \sqrt{6}\sqrt{3}\sqrt{2} \arctan\left(-\frac{1}{3} \sqrt{6}\sqrt{3}\sqrt{2}x + \frac{1}{3} \sqrt{6}\sqrt{3}\sqrt{\sqrt{6}\sqrt{2}x + 2x^2 + 2} + \sqrt{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)/(x^8+x^4+1),x, algorithm="fricas")
```

```
[Out] -1/12*sqrt(6)*sqrt(3)*sqrt(2)*arctan(-1/3*sqrt(6)*sqrt(3)*sqrt(2)*x + 1/3*s
qrt(6)*sqrt(3)*sqrt(sqrt(6)*sqrt(2)*x + 2*x^2 + 2) - sqrt(3)) - 1/12*sqrt(6
)*sqrt(3)*sqrt(2)*arctan(-1/3*sqrt(6)*sqrt(3)*sqrt(2)*x + 1/3*sqrt(6)*sqrt(
3)*sqrt(-sqrt(6)*sqrt(2)*x + 2*x^2 + 2) + sqrt(3)) + 1/48*sqrt(6)*sqrt(2)*l
og(sqrt(6)*sqrt(2)*x + 2*x^2 + 2) - 1/48*sqrt(6)*sqrt(2)*log(-sqrt(6)*sqrt(
2)*x + 2*x^2 + 2) + 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/12*sqrt(
3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/8*log(x^2 + x + 1) - 1/8*log(x^2 - x +
1)
```

**Sympy [C]** time = 0.692687, size = 190, normalized size = 1.36

$$\left(-\frac{1}{8} - \frac{\sqrt{3}i}{24}\right) \log\left(x - 1 - \frac{\sqrt{3}i}{3} + 9216\left(-\frac{1}{8} - \frac{\sqrt{3}i}{24}\right)^5\right) + \left(-\frac{1}{8} + \frac{\sqrt{3}i}{24}\right) \log\left(x - 1 + 9216\left(-\frac{1}{8} + \frac{\sqrt{3}i}{24}\right)^5 + \frac{\sqrt{3}i}{3}\right) + \left(\frac{1}{8} - \frac{\sqrt{3}i}{24}\right) \log\left(x + 1 - \frac{\sqrt{3}i}{3} + 9216\left(\frac{1}{8} - \frac{\sqrt{3}i}{24}\right)^5\right) + \left(\frac{1}{8} + \frac{\sqrt{3}i}{24}\right) \log\left(x + 1 + 9216\left(\frac{1}{8} + \frac{\sqrt{3}i}{24}\right)^5 + \frac{\sqrt{3}i}{3}\right) + \text{RootSum}\left(2304*_t^{*4} + 48*_t^{*2} + 1, \text{Lambda}(_t, _t \log(9216*_t^{*5} + 8*_t + x))\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x\*\*4+1)/(x\*\*8+x\*\*4+1),x)

[Out] (-1/8 - sqrt(3)\*I/24)\*log(x - 1 - sqrt(3)\*I/3 + 9216\*(-1/8 - sqrt(3)\*I/24)\*  
\*5) + (-1/8 + sqrt(3)\*I/24)\*log(x - 1 + 9216\*(-1/8 + sqrt(3)\*I/24)\*\*5 + sqrt  
t(3)\*I/3) + (1/8 - sqrt(3)\*I/24)\*log(x + 1 - sqrt(3)\*I/3 + 9216\*(1/8 - sqrt  
(3)\*I/24)\*\*5) + (1/8 + sqrt(3)\*I/24)\*log(x + 1 + 9216\*(1/8 + sqrt(3)\*I/24)\*  
\*5 + sqrt(3)\*I/3) + RootSum(2304\*\_t\*\*4 + 48\*\_t\*\*2 + 1, Lambda(\_t, \_t\*log(92  
16\*\_t\*\*5 + 8\*\_t + x)))

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4 + 1}{x^8 + x^4 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4+1)/(x^8+x^4+1),x, algorithm="giac")

[Out] integrate((x^4 + 1)/(x^8 + x^4 + 1), x)

### 3.13 $\int \frac{1+x^4}{1+x^8} dx$

**Optimal.** Leaf size=347

$$-\frac{\log\left(x^2 - \sqrt{2 - \sqrt{2}}x + 1\right)}{8\sqrt{2 - \sqrt{2}}} + \frac{\log\left(x^2 + \sqrt{2 - \sqrt{2}}x + 1\right)}{8\sqrt{2 - \sqrt{2}}} - \frac{\log\left(x^2 - \sqrt{2 + \sqrt{2}}x + 1\right)}{8\sqrt{2 + \sqrt{2}}} + \frac{\log\left(x^2 + \sqrt{2 + \sqrt{2}}x + 1\right)}{8\sqrt{2 + \sqrt{2}}} - \frac{1}{4}\sqrt{\frac{1}{2}}$$

```
[Out] -(Sqrt[(2 - Sqrt[2])/2]*ArcTan[(Sqrt[2 - Sqrt[2]] - 2*x)/Sqrt[2 + Sqrt[2]]])
)/4 - (Sqrt[(2 + Sqrt[2])/2]*ArcTan[(Sqrt[2 + Sqrt[2]] - 2*x)/Sqrt[2 - Sqrt
[2]]])/4 + (Sqrt[(2 - Sqrt[2])/2]*ArcTan[(Sqrt[2 - Sqrt[2]] + 2*x)/Sqrt[2 +
Sqrt[2]]])/4 + (Sqrt[(2 + Sqrt[2])/2]*ArcTan[(Sqrt[2 + Sqrt[2]] + 2*x)/Sqr
t[2 - Sqrt[2]]])/4 - Log[1 - Sqrt[2 - Sqrt[2]]*x + x^2]/(8*Sqrt[2 - Sqrt[2]
]) + Log[1 + Sqrt[2 - Sqrt[2]]*x + x^2]/(8*Sqrt[2 - Sqrt[2]]) - Log[1 - Sqr
t[2 + Sqrt[2]]*x + x^2]/(8*Sqrt[2 + Sqrt[2]]) + Log[1 + Sqrt[2 + Sqrt[2]]*x
+ x^2]/(8*Sqrt[2 + Sqrt[2]])
```

---

**Rubi [A]** time = 0.246808, antiderivative size = 347, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 6, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$ , Rules used = {1413, 1094, 634, 618, 204, 628}

$$-\frac{\log\left(x^2 - \sqrt{2 - \sqrt{2}}x + 1\right)}{8\sqrt{2 - \sqrt{2}}} + \frac{\log\left(x^2 + \sqrt{2 - \sqrt{2}}x + 1\right)}{8\sqrt{2 - \sqrt{2}}} - \frac{\log\left(x^2 - \sqrt{2 + \sqrt{2}}x + 1\right)}{8\sqrt{2 + \sqrt{2}}} + \frac{\log\left(x^2 + \sqrt{2 + \sqrt{2}}x + 1\right)}{8\sqrt{2 + \sqrt{2}}} - \frac{1}{4}\sqrt{\frac{1}{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^4)/(1 + x^8),x]
```

```
[Out] -(Sqrt[(2 - Sqrt[2])/2]*ArcTan[(Sqrt[2 - Sqrt[2]] - 2*x)/Sqrt[2 + Sqrt[2]]])
)/4 - (Sqrt[(2 + Sqrt[2])/2]*ArcTan[(Sqrt[2 + Sqrt[2]] - 2*x)/Sqrt[2 - Sqrt
[2]]])/4 + (Sqrt[(2 - Sqrt[2])/2]*ArcTan[(Sqrt[2 - Sqrt[2]] + 2*x)/Sqrt[2 +
Sqrt[2]]])/4 + (Sqrt[(2 + Sqrt[2])/2]*ArcTan[(Sqrt[2 + Sqrt[2]] + 2*x)/Sqr
t[2 - Sqrt[2]]])/4 - Log[1 - Sqrt[2 - Sqrt[2]]*x + x^2]/(8*Sqrt[2 - Sqrt[2]
]) + Log[1 + Sqrt[2 - Sqrt[2]]*x + x^2]/(8*Sqrt[2 - Sqrt[2]]) - Log[1 - Sqr
t[2 + Sqrt[2]]*x + x^2]/(8*Sqrt[2 + Sqrt[2]]) + Log[1 + Sqrt[2 + Sqrt[2]]*x
+ x^2]/(8*Sqrt[2 + Sqrt[2]])
```

Rule 1413



```
Int[((d_) + (e_)*(x_)^(n_))/((a_) + (c_)*(x_)^(n2_)), x_Symbol] := With[{
q = Rt[2*d*e, 2]}, Dist[e^2/(2*c), Int[1/(d + q*x^(n/2) + e*x^n), x], x] +
Dist[e^2/(2*c), Int[1/(d - q*x^(n/2) + e*x^n), x], x]] /; FreeQ[{a, c, d, e
}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 - a*e^2, 0] && IGtQ[n/2, 0] && PosQ[d*e]
```

### Rule 1094

```
Int[((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(-1), x_Symbol] := With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(r - x)/(q - r*x
+ x^2), x], x] + Dist[1/(2*c*q*r), Int[(r + x)/(q + r*x + x^2), x], x]]] /
; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

### Rule 634

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

### Rule 618

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

### Rule 204

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

### Rule 628

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

### Rubi steps

$$\begin{aligned}
\int \frac{1+x^4}{1+x^8} dx &= \frac{1}{2} \int \frac{1}{1-\sqrt{2}x^2+x^4} dx + \frac{1}{2} \int \frac{1}{1+\sqrt{2}x^2+x^4} dx \\
&= \frac{\int \frac{\sqrt{2-\sqrt{2}-x}}{1-\sqrt{2-\sqrt{2}x+x^2}} dx}{4\sqrt{2-\sqrt{2}}} + \frac{\int \frac{\sqrt{2-\sqrt{2}+x}}{1+\sqrt{2-\sqrt{2}x+x^2}} dx}{4\sqrt{2-\sqrt{2}}} + \frac{\int \frac{\sqrt{2+\sqrt{2}-x}}{1-\sqrt{2+\sqrt{2}x+x^2}} dx}{4\sqrt{2+\sqrt{2}}} + \frac{\int \frac{\sqrt{2+\sqrt{2}+x}}{1+\sqrt{2+\sqrt{2}x+x^2}} dx}{4\sqrt{2+\sqrt{2}}} \\
&= \frac{1}{8} \int \frac{1}{1-\sqrt{2-\sqrt{2}x+x^2}} dx + \frac{1}{8} \int \frac{1}{1+\sqrt{2-\sqrt{2}x+x^2}} dx + \frac{1}{8} \int \frac{1}{1-\sqrt{2+\sqrt{2}x+x^2}} dx + \frac{1}{8} \int \frac{1}{1+\sqrt{2+\sqrt{2}x+x^2}} dx \\
&= -\frac{\log\left(1-\sqrt{2-\sqrt{2}x+x^2}\right)}{8\sqrt{2-\sqrt{2}}} + \frac{\log\left(1+\sqrt{2-\sqrt{2}x+x^2}\right)}{8\sqrt{2-\sqrt{2}}} - \frac{\log\left(1-\sqrt{2+\sqrt{2}x+x^2}\right)}{8\sqrt{2+\sqrt{2}}} + \frac{\log\left(1+\sqrt{2+\sqrt{2}x+x^2}\right)}{8\sqrt{2+\sqrt{2}}} \\
&= -\frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{2}-2x}}{\sqrt{2+\sqrt{2}}}\right)}{4\sqrt{2+\sqrt{2}}} - \frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{2}-2x}}{\sqrt{2-\sqrt{2}}}\right)}{4\sqrt{2-\sqrt{2}}} + \frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{2}+2x}}{\sqrt{2+\sqrt{2}}}\right)}{4\sqrt{2+\sqrt{2}}} + \frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{2}+2x}}{\sqrt{2-\sqrt{2}}}\right)}{4\sqrt{2-\sqrt{2}}} - \frac{\log\left(1-\sqrt{2-\sqrt{2}x}\right)}{8\sqrt{2-\sqrt{2}}}
\end{aligned}$$

**Mathematica [A]** time = 0.181807, size = 258, normalized size = 0.74

$$\frac{1}{8} \left( -\left( \sin\left(\frac{\pi}{8}\right) + \cos\left(\frac{\pi}{8}\right) \right) \log\left(x^2 - 2x \sin\left(\frac{\pi}{8}\right) + 1\right) + \left( \sin\left(\frac{\pi}{8}\right) + \cos\left(\frac{\pi}{8}\right) \right) \log\left(x^2 + 2x \sin\left(\frac{\pi}{8}\right) + 1\right) + \left( \sin\left(\frac{\pi}{8}\right) - \cos\left(\frac{\pi}{8}\right) \right) \log\left(x^2 - 2x \cos\left(\frac{\pi}{8}\right) + 1\right) + \left( \sin\left(\frac{\pi}{8}\right) - \cos\left(\frac{\pi}{8}\right) \right) \log\left(x^2 + 2x \cos\left(\frac{\pi}{8}\right) + 1\right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(1 + x^4)/(1 + x^8), x]

[Out] (2\*ArcTan[Sec[Pi/8]\*(x + Sin[Pi/8])]\*(Cos[Pi/8] - Sin[Pi/8]) + 2\*ArcTan[x\*Sec[Pi/8] - Tan[Pi/8]]\*(Cos[Pi/8] - Sin[Pi/8]) + Log[1 + x^2 + 2\*x\*Cos[Pi/8]]\*(Cos[Pi/8] - Sin[Pi/8]) + Log[1 + x^2 - 2\*x\*Cos[Pi/8]]\*(-Cos[Pi/8] + Sin[Pi/8]) + 2\*ArcTan[(x - Cos[Pi/8])\*Csc[Pi/8]]\*(Cos[Pi/8] + Sin[Pi/8]) + 2\*ArcTan[(x + Cos[Pi/8])\*Csc[Pi/8]]\*(Cos[Pi/8] + Sin[Pi/8]) - Log[1 + x^2 - 2\*x\*Sin[Pi/8]]\*(Cos[Pi/8] + Sin[Pi/8]) + Log[1 + x^2 + 2\*x\*Sin[Pi/8]]\*(Cos[Pi/8] + Sin[Pi/8]))/8

**Maple [C]** time = 0.006, size = 27, normalized size = 0.1

$$\frac{1}{8} \sum_{_R=\text{RootOf}(_Z^8+1)} \frac{(_R^4+1) \ln(x-_R)}{_R^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^4+1)/(x^8+1),x)

[Out] 1/8\*sum((\_R^4+1)/\_R^7\*ln(x-\_R),\_R=RootOf(\_Z^8+1))

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4+1}{x^8+1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4+1)/(x^8+1),x, algorithm="maxima")

[Out] integrate((x^4 + 1)/(x^8 + 1), x)

**Fricas [B]** time = 1.50932, size = 3092, normalized size = 8.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4+1)/(x^8+1),x, algorithm="fricas")

[Out] -1/8\*(sqrt(sqrt(2) + 2) - sqrt(-sqrt(2) + 2))\*arctan(-(2\*x - 2\*sqrt(x^2 + x\*sqrt(-sqrt(2) + 2) + 1) + sqrt(-sqrt(2) + 2))/sqrt(sqrt(2) + 2)) - 1/8\*(sqrt(sqrt(2) + 2) - sqrt(-sqrt(2) + 2))\*arctan(-(2\*x - 2\*sqrt(x^2 - x\*sqrt(-sqrt(2) + 2) + 1) - sqrt(-sqrt(2) + 2))/sqrt(sqrt(2) + 2)) - 1/8\*(sqrt(sqrt(2) + 2) + sqrt(-sqrt(2) + 2))\*arctan(-(2\*x - 2\*sqrt(x^2 + x\*sqrt(sqrt(2) + 2) + 1) + sqrt(sqrt(2) + 2))/sqrt(-sqrt(2) + 2)) - 1/8\*(sqrt(sqrt(2) + 2) + sqrt(-sqrt(2) + 2))\*arctan(-(2\*x - 2\*sqrt(x^2 - x\*sqrt(sqrt(2) + 2) + 1) - sqrt(sqrt(2) + 2))/sqrt(-sqrt(2) + 2)) - 1/8\*sqrt(2)\*sqrt(-sqrt(2) + 2)\*ar

```

ctan(-(2*sqrt(2)*x - 2*sqrt(2)*sqrt(x^2 + 1/2*sqrt(2)*x*sqrt(sqrt(2) + 2) -
1/2*sqrt(2)*x*sqrt(-sqrt(2) + 2) + 1) + sqrt(sqrt(2) + 2) - sqrt(-sqrt(2)
+ 2))/(sqrt(sqrt(2) + 2) + sqrt(-sqrt(2) + 2))) - 1/8*sqrt(2)*sqrt(-sqrt(2)
+ 2)*arctan(-(2*sqrt(2)*x - 2*sqrt(2)*sqrt(x^2 - 1/2*sqrt(2)*x*sqrt(sqrt(2)
) + 2) + 1/2*sqrt(2)*x*sqrt(-sqrt(2) + 2) + 1) - sqrt(sqrt(2) + 2) + sqrt(-
sqrt(2) + 2))/(sqrt(sqrt(2) + 2) + sqrt(-sqrt(2) + 2))) + 1/8*sqrt(2)*sqrt(
sqrt(2) + 2)*arctan((2*sqrt(2)*x - 2*sqrt(2)*sqrt(x^2 + 1/2*sqrt(2)*x*sqrt(
sqrt(2) + 2) + 1/2*sqrt(2)*x*sqrt(-sqrt(2) + 2) + 1) + sqrt(sqrt(2) + 2) +
sqrt(-sqrt(2) + 2))/(sqrt(sqrt(2) + 2) - sqrt(-sqrt(2) + 2))) + 1/8*sqrt(2)
*sqrt(sqrt(2) + 2)*arctan((2*sqrt(2)*x - 2*sqrt(2)*sqrt(x^2 - 1/2*sqrt(2)*x
*sqrt(sqrt(2) + 2) - 1/2*sqrt(2)*x*sqrt(-sqrt(2) + 2) + 1) - sqrt(sqrt(2) +
2) - sqrt(-sqrt(2) + 2))/(sqrt(sqrt(2) + 2) - sqrt(-sqrt(2) + 2))) + 1/32*
sqrt(2)*sqrt(-sqrt(2) + 2)*log(x^2 + 1/2*sqrt(2)*x*sqrt(sqrt(2) + 2) + 1/2*
sqrt(2)*x*sqrt(-sqrt(2) + 2) + 1) + 1/32*sqrt(2)*sqrt(sqrt(2) + 2)*log(x^2
+ 1/2*sqrt(2)*x*sqrt(sqrt(2) + 2) - 1/2*sqrt(2)*x*sqrt(-sqrt(2) + 2) + 1) -
1/32*sqrt(2)*sqrt(sqrt(2) + 2)*log(x^2 - 1/2*sqrt(2)*x*sqrt(sqrt(2) + 2) +
1/2*sqrt(2)*x*sqrt(-sqrt(2) + 2) + 1) - 1/32*sqrt(2)*sqrt(-sqrt(2) + 2)*lo
g(x^2 - 1/2*sqrt(2)*x*sqrt(sqrt(2) + 2) - 1/2*sqrt(2)*x*sqrt(-sqrt(2) + 2)
+ 1) + 1/32*(sqrt(sqrt(2) + 2) - sqrt(-sqrt(2) + 2))*log(x^2 + x*sqrt(sqrt(
2) + 2) + 1) - 1/32*(sqrt(sqrt(2) + 2) - sqrt(-sqrt(2) + 2))*log(x^2 - x*sq
rt(sqrt(2) + 2) + 1) + 1/32*(sqrt(sqrt(2) + 2) + sqrt(-sqrt(2) + 2))*log(x^
2 + x*sqrt(-sqrt(2) + 2) + 1) - 1/32*(sqrt(sqrt(2) + 2) + sqrt(-sqrt(2) + 2
))*log(x^2 - x*sqrt(-sqrt(2) + 2) + 1)

```

**Sympy [A]** time = 1.12383, size = 19, normalized size = 0.05

$$\text{RootSum}\left(1048576t^8 + 1, \left(t \mapsto t \log(4096t^5 + 4t + x)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x\*\*4+1)/(x\*\*8+1), x)

[Out] RootSum(1048576\*\_t\*\*8 + 1, Lambda(\_t, \_t\*log(4096\*\_t\*\*5 + 4\*\_t + x)))

**Giac [A]** time = 1.1791, size = 333, normalized size = 0.96

$$\frac{1}{8} \sqrt{-2\sqrt{2} + 4} \arctan\left(\frac{2x + \sqrt{-\sqrt{2} + 2}}{\sqrt{\sqrt{2} + 2}}\right) + \frac{1}{8} \sqrt{-2\sqrt{2} + 4} \arctan\left(\frac{2x - \sqrt{-\sqrt{2} + 2}}{\sqrt{\sqrt{2} + 2}}\right) + \frac{1}{8} \sqrt{2\sqrt{2} + 4} \arctan\left(\frac{2x + \sqrt{\sqrt{2} + 2}}{\sqrt{-\sqrt{2} + 2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)/(x^8+1),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(-2*sqrt(2) + 4)*arctan((2*x + sqrt(-sqrt(2) + 2))/sqrt(sqrt(2) + 2)) + 1/8*sqrt(-2*sqrt(2) + 4)*arctan((2*x - sqrt(-sqrt(2) + 2))/sqrt(sqrt(2) + 2)) + 1/8*sqrt(2*sqrt(2) + 4)*arctan((2*x + sqrt(sqrt(2) + 2))/sqrt(-sqrt(2) + 2)) + 1/8*sqrt(2*sqrt(2) + 4)*arctan((2*x - sqrt(sqrt(2) + 2))/sqrt(-sqrt(2) + 2)) + 1/16*sqrt(-2*sqrt(2) + 4)*log(x^2 + x*sqrt(sqrt(2) + 2) + 1) - 1/16*sqrt(-2*sqrt(2) + 4)*log(x^2 - x*sqrt(sqrt(2) + 2) + 1) + 1/16*sqrt(2*sqrt(2) + 4)*log(x^2 + x*sqrt(-sqrt(2) + 2) + 1) - 1/16*sqrt(2*sqrt(2) + 4)*log(x^2 - x*sqrt(-sqrt(2) + 2) + 1)
```

### 3.14 $\int \frac{1+x^4}{1-x^4+x^8} dx$

**Optimal.** Leaf size=331

$$-\frac{\log\left(x^2 - \sqrt{2 - \sqrt{3}}x + 1\right)}{8\sqrt{2 - \sqrt{3}}} + \frac{\log\left(x^2 + \sqrt{2 - \sqrt{3}}x + 1\right)}{8\sqrt{2 - \sqrt{3}}} - \frac{\log\left(x^2 - \sqrt{2 + \sqrt{3}}x + 1\right)}{8\sqrt{2 + \sqrt{3}}} + \frac{\log\left(x^2 + \sqrt{2 + \sqrt{3}}x + 1\right)}{8\sqrt{2 + \sqrt{3}}} - \frac{1}{4}\sqrt{2}$$

```
[Out] -(Sqrt[2 - Sqrt[3]]*ArcTan[(Sqrt[2 - Sqrt[3]] - 2*x)/Sqrt[2 + Sqrt[3]]])/4
- (Sqrt[2 + Sqrt[3]]*ArcTan[(Sqrt[2 + Sqrt[3]] - 2*x)/Sqrt[2 - Sqrt[3]]])/4
+ (Sqrt[2 - Sqrt[3]]*ArcTan[(Sqrt[2 - Sqrt[3]] + 2*x)/Sqrt[2 + Sqrt[3]]])/
4 + (Sqrt[2 + Sqrt[3]]*ArcTan[(Sqrt[2 + Sqrt[3]] + 2*x)/Sqrt[2 - Sqrt[3]]])/
4 - Log[1 - Sqrt[2 - Sqrt[3]]*x + x^2]/(8*Sqrt[2 - Sqrt[3]]) + Log[1 + Sqr
t[2 - Sqrt[3]]*x + x^2]/(8*Sqrt[2 - Sqrt[3]]) - Log[1 - Sqrt[2 + Sqrt[3]]*x
+ x^2]/(8*Sqrt[2 + Sqrt[3]]) + Log[1 + Sqrt[2 + Sqrt[3]]*x + x^2]/(8*Sqrt[
2 + Sqrt[3]])
```

---

**Rubi [A]** time = 0.234564, antiderivative size = 331, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 6, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {1419, 1094, 634, 618, 204, 628}

$$-\frac{\log\left(x^2 - \sqrt{2 - \sqrt{3}}x + 1\right)}{8\sqrt{2 - \sqrt{3}}} + \frac{\log\left(x^2 + \sqrt{2 - \sqrt{3}}x + 1\right)}{8\sqrt{2 - \sqrt{3}}} - \frac{\log\left(x^2 - \sqrt{2 + \sqrt{3}}x + 1\right)}{8\sqrt{2 + \sqrt{3}}} + \frac{\log\left(x^2 + \sqrt{2 + \sqrt{3}}x + 1\right)}{8\sqrt{2 + \sqrt{3}}} - \frac{1}{4}\sqrt{2}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^4)/(1 - x^4 + x^8), x]
```

```
[Out] -(Sqrt[2 - Sqrt[3]]*ArcTan[(Sqrt[2 - Sqrt[3]] - 2*x)/Sqrt[2 + Sqrt[3]]])/4
- (Sqrt[2 + Sqrt[3]]*ArcTan[(Sqrt[2 + Sqrt[3]] - 2*x)/Sqrt[2 - Sqrt[3]]])/4
+ (Sqrt[2 - Sqrt[3]]*ArcTan[(Sqrt[2 - Sqrt[3]] + 2*x)/Sqrt[2 + Sqrt[3]]])/
4 + (Sqrt[2 + Sqrt[3]]*ArcTan[(Sqrt[2 + Sqrt[3]] + 2*x)/Sqrt[2 - Sqrt[3]]])/
4 - Log[1 - Sqrt[2 - Sqrt[3]]*x + x^2]/(8*Sqrt[2 - Sqrt[3]]) + Log[1 + Sqr
t[2 - Sqrt[3]]*x + x^2]/(8*Sqrt[2 - Sqrt[3]]) - Log[1 - Sqrt[2 + Sqrt[3]]*x
+ x^2]/(8*Sqrt[2 + Sqrt[3]]) + Log[1 + Sqrt[2 + Sqrt[3]]*x + x^2]/(8*Sqrt[
2 + Sqrt[3]])
```

Rule 1419

```
Int[((d_) + (e_)*(x_)^(n_))/((a_) + (b_)*(x_)^(n_) + (c_)*(x_)^(n2_)), x
_Symbol] := With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e +
q*x^(n/2) + x^n, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x^(n/2) + x
^n, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4
*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && IGtQ[n/2, 0] && (GtQ[(2*d)/e - b/c, 0]
|| (!LtQ[(2*d)/e - b/c, 0] && EqQ[d, e*Rt[a/c, 2]]))
```

### Rule 1094

```
Int[((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(-1), x_Symbol] := With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(r - x)/(q - r*x
+ x^2), x], x] + Dist[1/(2*c*q*r), Int[(r + x)/(q + r*x + x^2), x], x]]] /
; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

### Rule 634

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

### Rule 618

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

### Rule 204

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

### Rule 628

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

### Rubi steps

$$\begin{aligned}
\int \frac{1+x^4}{1-x^4+x^8} dx &= \frac{1}{2} \int \frac{1}{1-\sqrt{3}x^2+x^4} dx + \frac{1}{2} \int \frac{1}{1+\sqrt{3}x^2+x^4} dx \\
&= \frac{\int \frac{\sqrt{2-\sqrt{3}}-x}{1-\sqrt{2-\sqrt{3}}x+x^2} dx}{4\sqrt{2-\sqrt{3}}} + \frac{\int \frac{\sqrt{2-\sqrt{3}}+x}{1+\sqrt{2-\sqrt{3}}x+x^2} dx}{4\sqrt{2-\sqrt{3}}} + \frac{\int \frac{\sqrt{2+\sqrt{3}}-x}{1-\sqrt{2+\sqrt{3}}x+x^2} dx}{4\sqrt{2+\sqrt{3}}} + \frac{\int \frac{\sqrt{2+\sqrt{3}}+x}{1+\sqrt{2+\sqrt{3}}x+x^2} dx}{4\sqrt{2+\sqrt{3}}} \\
&= \frac{1}{8} \int \frac{1}{1-\sqrt{2-\sqrt{3}}x+x^2} dx + \frac{1}{8} \int \frac{1}{1+\sqrt{2-\sqrt{3}}x+x^2} dx + \frac{1}{8} \int \frac{1}{1-\sqrt{2+\sqrt{3}}x+x^2} dx + \frac{1}{8} \int \frac{1}{1+\sqrt{2+\sqrt{3}}x+x^2} dx \\
&= -\frac{\log\left(1-\sqrt{2-\sqrt{3}}x+x^2\right)}{8\sqrt{2-\sqrt{3}}} + \frac{\log\left(1+\sqrt{2-\sqrt{3}}x+x^2\right)}{8\sqrt{2-\sqrt{3}}} - \frac{\log\left(1-\sqrt{2+\sqrt{3}}x+x^2\right)}{8\sqrt{2+\sqrt{3}}} + \frac{\log\left(1+\sqrt{2+\sqrt{3}}x+x^2\right)}{8\sqrt{2+\sqrt{3}}} \\
&= -\frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)}{4\sqrt{2+\sqrt{3}}} - \frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)}{4\sqrt{2-\sqrt{3}}} + \frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}+2x}{\sqrt{2+\sqrt{3}}}\right)}{4\sqrt{2+\sqrt{3}}} + \frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}+2x}{\sqrt{2-\sqrt{3}}}\right)}{4\sqrt{2-\sqrt{3}}} - \frac{\log\left(1-\sqrt{2-\sqrt{3}}x+x^2\right)}{8\sqrt{2-\sqrt{3}}}
\end{aligned}$$

**Mathematica [C]** time = 0.0155387, size = 55, normalized size = 0.17

$$\frac{1}{4} \text{RootSum}\left[\#1^8 - \#1^4 + 1 \&, \frac{\#1^4 \log(x - \#1) + \log(x - \#1)}{2\#1^7 - \#1^3} \&\right]$$

Antiderivative was successfully verified.

[In] Integrate[(1 + x^4)/(1 - x^4 + x^8), x]

[Out] RootSum[1 - #1^4 + #1^8 &, (Log[x - #1] + Log[x - #1]\*#1^4)/(-#1^3 + 2\*#1^7) & ]/4

**Maple [C]** time = 0.006, size = 42, normalized size = 0.1

$$\frac{1}{4} \sum_{_R=\text{RootOf}(-Z^8-Z^4+1)} \frac{(_R^4+1)\ln(x-_R)}{2\_R^7-\_R^3}$$

Verification of antiderivative is not currently implemented for this CAS.



[In] int((x^4+1)/(x^8-x^4+1),x)

[Out] 1/4\*sum((\_R^4+1)/(2\*\_R^7-\_R^3)\*ln(x-\_R),\_R=RootOf(\_Z^8-\_Z^4+1))

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4 + 1}{x^8 - x^4 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4+1)/(x^8-x^4+1),x, algorithm="maxima")

[Out] integrate((x^4 + 1)/(x^8 - x^4 + 1), x)

**Fricas [A]** time = 1.46206, size = 1196, normalized size = 3.61

$$-\frac{1}{8} \sqrt{\sqrt{3} + 2} (\sqrt{3} - 2) \log\left(2x^2 + 2x\sqrt{\sqrt{3} + 2} + 2\right) + \frac{1}{8} \sqrt{\sqrt{3} + 2} (\sqrt{3} - 2) \log\left(2x^2 - 2x\sqrt{\sqrt{3} + 2} + 2\right) + \frac{1}{16} (\sqrt{3} + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4+1)/(x^8-x^4+1),x, algorithm="fricas")

[Out] -1/8\*sqrt(sqrt(3) + 2)\*(sqrt(3) - 2)\*log(2\*x^2 + 2\*x\*sqrt(sqrt(3) + 2) + 2) + 1/8\*sqrt(sqrt(3) + 2)\*(sqrt(3) - 2)\*log(2\*x^2 - 2\*x\*sqrt(sqrt(3) + 2) + 2) + 1/16\*(sqrt(3) + 2)\*sqrt(-4\*sqrt(3) + 8)\*log(2\*x^2 + x\*sqrt(-4\*sqrt(3) + 8) + 2) - 1/16\*(sqrt(3) + 2)\*sqrt(-4\*sqrt(3) + 8)\*log(2\*x^2 - x\*sqrt(-4\*sqrt(3) + 8) + 2) - 1/2\*sqrt(sqrt(3) + 2)\*arctan(sqrt(2)\*sqrt(2\*x^2 + 2\*x\*sqrt(sqrt(3) + 2) + 2)\*sqrt(sqrt(3) + 2) - 2\*x\*sqrt(sqrt(3) + 2) - sqrt(3) - 2) - 1/2\*sqrt(sqrt(3) + 2)\*arctan(sqrt(2)\*sqrt(2\*x^2 - 2\*x\*sqrt(sqrt(3) + 2) + 2)\*sqrt(sqrt(3) + 2) - 2\*x\*sqrt(sqrt(3) + 2) + sqrt(3) + 2) - 1/4\*sqrt(-4\*sqrt(3) + 8)\*arctan(1/2\*sqrt(2)\*sqrt(2\*x^2 + x\*sqrt(-4\*sqrt(3) + 8) + 2)\*sqrt(-4\*sqrt(3) + 8) - x\*sqrt(-4\*sqrt(3) + 8) + sqrt(3) - 2) - 1/4\*sqrt(-4\*sqrt(3) + 8)\*arctan(1/2\*sqrt(2)\*sqrt(2\*x^2 - x\*sqrt(-4\*sqrt(3) + 8) + 2)\*sqrt(-4\*sqrt(3) + 8) - x\*sqrt(-4\*sqrt(3) + 8) - sqrt(3) + 2)

**Sympy [A]** time = 1.2298, size = 20, normalized size = 0.06

$$\text{RootSum}\left(65536t^8 - 256t^4 + 1, (t \mapsto t \log(1024t^5 + x))\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x\*\*4+1)/(x\*\*8-x\*\*4+1),x)

[Out] RootSum(65536\*\_t\*\*8 - 256\*\_t\*\*4 + 1, Lambda(\_t, \_t\*log(1024\*\_t\*\*5 + x)))

**Giac [A]** time = 1.1562, size = 331, normalized size = 1.

$$\frac{1}{8}(\sqrt{6} - \sqrt{2}) \arctan\left(\frac{4x + \sqrt{6} - \sqrt{2}}{\sqrt{6} + \sqrt{2}}\right) + \frac{1}{8}(\sqrt{6} - \sqrt{2}) \arctan\left(\frac{4x - \sqrt{6} + \sqrt{2}}{\sqrt{6} + \sqrt{2}}\right) + \frac{1}{8}(\sqrt{6} + \sqrt{2}) \arctan\left(\frac{4x + \sqrt{6} + \sqrt{2}}{\sqrt{6} - \sqrt{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4+1)/(x^8-x^4+1),x, algorithm="giac")

[Out] 1/8\*(sqrt(6) - sqrt(2))\*arctan((4\*x + sqrt(6) - sqrt(2))/(sqrt(6) + sqrt(2))) + 1/8\*(sqrt(6) - sqrt(2))\*arctan((4\*x - sqrt(6) + sqrt(2))/(sqrt(6) + sqrt(2))) + 1/8\*(sqrt(6) + sqrt(2))\*arctan((4\*x + sqrt(6) + sqrt(2))/(sqrt(6) - sqrt(2))) + 1/8\*(sqrt(6) + sqrt(2))\*arctan((4\*x - sqrt(6) - sqrt(2))/(sqrt(6) - sqrt(2))) + 1/16\*(sqrt(6) - sqrt(2))\*log(x^2 + 1/2\*x\*(sqrt(6) + sqrt(2)) + 1) - 1/16\*(sqrt(6) - sqrt(2))\*log(x^2 - 1/2\*x\*(sqrt(6) + sqrt(2)) + 1) + 1/16\*(sqrt(6) + sqrt(2))\*log(x^2 + 1/2\*x\*(sqrt(6) - sqrt(2)) + 1) - 1/16\*(sqrt(6) + sqrt(2))\*log(x^2 - 1/2\*x\*(sqrt(6) - sqrt(2)) + 1)

$$3.15 \quad \int \frac{1+x^4}{1-2x^4+x^8} dx$$

**Optimal.** Leaf size=27

$$\frac{x}{2(1-x^4)} + \frac{1}{4} \tan^{-1}(x) + \frac{1}{4} \tanh^{-1}(x)$$

[Out]  $x/(2*(1 - x^4)) + \text{ArcTan}[x]/4 + \text{ArcTanh}[x]/4$

**Rubi [A]** time = 0.0077244, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$ , Rules used = {28, 385, 212, 206, 203}

$$\frac{x}{2(1-x^4)} + \frac{1}{4} \tan^{-1}(x) + \frac{1}{4} \tanh^{-1}(x)$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(1 + x^4)/(1 - 2*x^4 + x^8), x]$

[Out]  $x/(2*(1 - x^4)) + \text{ArcTan}[x]/4 + \text{ArcTanh}[x]/4$

### Rule 28

$\text{Int}[(u_.)*((a_.) + (c_.)*(x_)^{(n2_.)} + (b_.)*(x_)^{(n_)})^{(p_.)}, x\_Symbol] \rightarrow \text{Dist}[1/c^p, \text{Int}[u*(b/2 + c*x^n)^{(2*p)}, x], x] /; \text{FreeQ}\{a, b, c, n\}, x] \&\& \text{EqQ}[n2, 2*n] \&\& \text{EqQ}[b^2 - 4*a*c, 0] \&\& \text{IntegerQ}[p]$

### Rule 385

$\text{Int}[(a_.) + (b_.)*(x_)^{(n_)}]^{(p_)}*((c_.) + (d_.)*(x_)^{(n_)}), x\_Symbol] \rightarrow -\text{Simp}[(b*c - a*d)*x*(a + b*x^n)^{(p+1)}/(a*b*n*(p+1)), x] - \text{Dist}[(a*d - b*c*(n*(p+1) + 1))/(a*b*n*(p+1)), \text{Int}[(a + b*x^n)^{(p+1)}, x], x] /; \text{FreeQ}\{a, b, c, d, n, p\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& (\text{LtQ}[p, -1] \parallel \text{ILtQ}[1/n + p, 0])$

### Rule 212

$\text{Int}[(a_.) + (b_.)*(x_)^4]^{(-1)}, x\_Symbol] \rightarrow \text{With}\{r = \text{Numerator}[\text{Rt}[-(a/b), 2]], s = \text{Denominator}[\text{Rt}[-(a/b), 2]]\}, \text{Dist}[r/(2*a), \text{Int}[1/(r - s*x^2), x], x] + \text{Dist}[r/(2*a), \text{Int}[1/(r + s*x^2), x], x] /; \text{FreeQ}\{a, b\}, x] \&\& !\text{GtQ}$

[a/b, 0]

### Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[
Q[a, 0] || LtQ[b, 0])
```

### Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

### Rubi steps

$$\begin{aligned} \int \frac{1+x^4}{1-2x^4+x^8} dx &= \int \frac{1+x^4}{(-1+x^4)^2} dx \\ &= \frac{x}{2(1-x^4)} - \frac{1}{2} \int \frac{1}{-1+x^4} dx \\ &= \frac{x}{2(1-x^4)} + \frac{1}{4} \int \frac{1}{1-x^2} dx + \frac{1}{4} \int \frac{1}{1+x^2} dx \\ &= \frac{x}{2(1-x^4)} + \frac{1}{4} \tan^{-1}(x) + \frac{1}{4} \tanh^{-1}(x) \end{aligned}$$

**Mathematica [A]** time = 0.0135165, size = 31, normalized size = 1.15

$$\frac{1}{8} \left( -\frac{4x}{x^4-1} - \log(1-x) + \log(x+1) + 2 \tan^{-1}(x) \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^4)/(1 - 2*x^4 + x^8), x]
```

```
[Out] ((-4*x)/(-1 + x^4) + 2*ArcTan[x] - Log[1 - x] + Log[1 + x])/8
```

---

**Maple [A]** time = 0.011, size = 42, normalized size = 1.6

$$\frac{x}{4x^2+4} + \frac{\arctan(x)}{4} - \frac{1}{8+8x} + \frac{\ln(1+x)}{8} - \frac{1}{8x-8} - \frac{\ln(x-1)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^4+1)/(x^8-2\*x^4+1),x)

[Out] 1/4\*x/(x^2+1)+1/4\*arctan(x)-1/8/(1+x)+1/8\*ln(1+x)-1/8/(x-1)-1/8\*ln(x-1)

**Maxima [A]** time = 1.49966, size = 36, normalized size = 1.33

$$-\frac{x}{2(x^4-1)} + \frac{1}{4} \arctan(x) + \frac{1}{8} \log(x+1) - \frac{1}{8} \log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4+1)/(x^8-2\*x^4+1),x, algorithm="maxima")

[Out] -1/2\*x/(x^4 - 1) + 1/4\*arctan(x) + 1/8\*log(x + 1) - 1/8\*log(x - 1)

**Fricas [B]** time = 1.23094, size = 123, normalized size = 4.56

$$\frac{2(x^4-1)\arctan(x) + (x^4-1)\log(x+1) - (x^4-1)\log(x-1) - 4x}{8(x^4-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4+1)/(x^8-2\*x^4+1),x, algorithm="fricas")

[Out] 1/8\*(2\*(x^4 - 1)\*arctan(x) + (x^4 - 1)\*log(x + 1) - (x^4 - 1)\*log(x - 1) - 4\*x)/(x^4 - 1)

**Sympy [A]** time = 0.142127, size = 26, normalized size = 0.96

$$-\frac{x}{2x^4-2} - \frac{\log(x-1)}{8} + \frac{\log(x+1)}{8} + \frac{\operatorname{atan}(x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x\*\*4+1)/(x\*\*8-2\*x\*\*4+1),x)

[Out] -x/(2\*x\*\*4 - 2) - log(x - 1)/8 + log(x + 1)/8 + atan(x)/4

**Giac [A]** time = 1.09585, size = 39, normalized size = 1.44

$$-\frac{x}{2(x^4-1)} + \frac{1}{4} \arctan(x) + \frac{1}{8} \log(|x+1|) - \frac{1}{8} \log(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4+1)/(x^8-2\*x^4+1),x, algorithm="giac")

[Out] -1/2\*x/(x^4 - 1) + 1/4\*arctan(x) + 1/8\*log(abs(x + 1)) - 1/8\*log(abs(x - 1))

$$3.16 \quad \int \frac{1+x^4}{1-3x^4+x^8} dx$$

**Optimal.** Leaf size=131

$$\frac{\tan^{-1}\left(\sqrt{\frac{2}{\sqrt{5}-1}}x\right)}{\sqrt{2}(\sqrt{5}-1)} - \frac{\tan^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right)}{\sqrt{2}(1+\sqrt{5})} + \frac{\tanh^{-1}\left(\sqrt{\frac{2}{\sqrt{5}-1}}x\right)}{\sqrt{2}(\sqrt{5}-1)} - \frac{\tanh^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right)}{\sqrt{2}(1+\sqrt{5})}$$

[Out] ArcTan[Sqrt[2/(-1 + Sqrt[5]])\*x]/Sqrt[2\*(-1 + Sqrt[5])] - ArcTan[Sqrt[2/(1 + Sqrt[5]])\*x]/Sqrt[2\*(1 + Sqrt[5])] + ArcTanh[Sqrt[2/(-1 + Sqrt[5]])\*x]/Sqrt[2\*(-1 + Sqrt[5])] - ArcTanh[Sqrt[2/(1 + Sqrt[5]])\*x]/Sqrt[2\*(1 + Sqrt[5])]

**Rubi [A]** time = 0.0862422, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$ , Rules used = {1419, 1093, 203, 207}

$$\frac{\tan^{-1}\left(\sqrt{\frac{2}{\sqrt{5}-1}}x\right)}{\sqrt{2}(\sqrt{5}-1)} - \frac{\tan^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right)}{\sqrt{2}(1+\sqrt{5})} + \frac{\tanh^{-1}\left(\sqrt{\frac{2}{\sqrt{5}-1}}x\right)}{\sqrt{2}(\sqrt{5}-1)} - \frac{\tanh^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right)}{\sqrt{2}(1+\sqrt{5})}$$

Antiderivative was successfully verified.

[In] Int[(1 + x^4)/(1 - 3\*x^4 + x^8),x]

[Out] ArcTan[Sqrt[2/(-1 + Sqrt[5]])\*x]/Sqrt[2\*(-1 + Sqrt[5])] - ArcTan[Sqrt[2/(1 + Sqrt[5]])\*x]/Sqrt[2\*(1 + Sqrt[5])] + ArcTanh[Sqrt[2/(-1 + Sqrt[5]])\*x]/Sqrt[2\*(-1 + Sqrt[5])] - ArcTanh[Sqrt[2/(1 + Sqrt[5]])\*x]/Sqrt[2\*(1 + Sqrt[5])]

### Rule 1419

Int[((d\_) + (e\_)\*(x\_)^(n\_))/((a\_) + (b\_)\*(x\_)^(n\_) + (c\_)\*(x\_)^(2n\_)), x  
\_Symbol] := With[{q = Rt[(2\*d)/e - b/c, 2]}, Dist[e/(2\*c), Int[1/Simp[d/e + q\*x^(n/2) + x^n, x], x], x] + Dist[e/(2\*c), Int[1/Simp[d/e - q\*x^(n/2) + x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[n2, 2\*n] && NeQ[b^2 - 4\*a\*c, 0] && EqQ[c\*d^2 - a\*e^2, 0] && IGtQ[n/2, 0] && (GtQ[(2\*d)/e - b/c, 0] || (!LtQ[(2\*d)/e - b/c, 0] && EqQ[d, e\*Rt[a/c, 2]]))

Rule 1093

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{1+x^4}{1-3x^4+x^8} dx &= \frac{1}{2} \int \frac{1}{1-\sqrt{5}x^2+x^4} dx + \frac{1}{2} \int \frac{1}{1+\sqrt{5}x^2+x^4} dx \\ &= \frac{1}{2} \int \frac{1}{-\frac{1}{2}-\frac{\sqrt{5}}{2}+x^2} dx - \frac{1}{2} \int \frac{1}{\frac{1}{2}-\frac{\sqrt{5}}{2}+x^2} dx + \frac{1}{2} \int \frac{1}{-\frac{1}{2}+\frac{\sqrt{5}}{2}+x^2} dx - \frac{1}{2} \int \frac{1}{\frac{1}{2}+\frac{\sqrt{5}}{2}+x^2} dx \\ &= \frac{\tan^{-1}\left(\sqrt{\frac{2}{-1+\sqrt{5}}}x\right)}{\sqrt{2(-1+\sqrt{5})}} - \frac{\tan^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right)}{\sqrt{2(1+\sqrt{5})}} + \frac{\tanh^{-1}\left(\sqrt{\frac{2}{-1+\sqrt{5}}}x\right)}{\sqrt{2(-1+\sqrt{5})}} - \frac{\tanh^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right)}{\sqrt{2(1+\sqrt{5})}} \end{aligned}$$

**Mathematica [A]** time = 0.0798201, size = 131, normalized size = 1.

$$\frac{\tan^{-1}\left(\sqrt{\frac{2}{\sqrt{5}-1}}x\right)}{\sqrt{2(\sqrt{5}-1)}} - \frac{\tan^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right)}{\sqrt{2(1+\sqrt{5})}} + \frac{\tanh^{-1}\left(\sqrt{\frac{2}{\sqrt{5}-1}}x\right)}{\sqrt{2(\sqrt{5}-1)}} - \frac{\tanh^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right)}{\sqrt{2(1+\sqrt{5})}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^4)/(1 - 3*x^4 + x^8), x]
```



[Out] ArcTan[Sqrt[2/(-1 + Sqrt[5])]x]/Sqrt[2\*(-1 + Sqrt[5])] - ArcTan[Sqrt[2/(1 + Sqrt[5])]x]/Sqrt[2\*(1 + Sqrt[5])] + ArcTanh[Sqrt[2/(-1 + Sqrt[5])]x]/Sqrt[2\*(-1 + Sqrt[5])] - ArcTanh[Sqrt[2/(1 + Sqrt[5])]x]/Sqrt[2\*(1 + Sqrt[5])]

**Maple [A]** time = 0.029, size = 96, normalized size = 0.7

$$-\frac{1}{\sqrt{2+2\sqrt{5}}}\operatorname{Artanh}\left(2\frac{x}{\sqrt{2+2\sqrt{5}}}\right)+\frac{1}{\sqrt{-2+2\sqrt{5}}}\arctan\left(2\frac{x}{\sqrt{-2+2\sqrt{5}}}\right)+\frac{1}{\sqrt{-2+2\sqrt{5}}}\operatorname{Artanh}\left(2\frac{x}{\sqrt{-2+2\sqrt{5}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^4+1)/(x^8-3\*x^4+1),x)

[Out] -1/(2+2\*5^(1/2))^(1/2)\*arctanh(2\*x/(2+2\*5^(1/2))^(1/2))+1/(-2+2\*5^(1/2))^(1/2)\*arctan(2\*x/(-2+2\*5^(1/2))^(1/2))+1/(-2+2\*5^(1/2))^(1/2)\*arctanh(2\*x/(-2+2\*5^(1/2))^(1/2))-1/(2+2\*5^(1/2))^(1/2)\*arctan(2\*x/(2+2\*5^(1/2))^(1/2))

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4 + 1}{x^8 - 3x^4 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4+1)/(x^8-3\*x^4+1),x, algorithm="maxima")

[Out] integrate((x^4 + 1)/(x^8 - 3\*x^4 + 1), x)

**Fricas [B]** time = 1.37286, size = 815, normalized size = 6.22

$$-\frac{1}{2}\sqrt{2}\sqrt{\sqrt{5}+1}\arctan\left(-\frac{1}{2}\sqrt{2x}\sqrt{\sqrt{5}+1}+\frac{1}{2}\sqrt{2x^2+\sqrt{5}-1}\sqrt{\sqrt{5}+1}\right)+\frac{1}{2}\sqrt{2}\sqrt{\sqrt{5}-1}\arctan\left(-\frac{1}{2}\sqrt{2x}\sqrt{\sqrt{5}-1}+\frac{1}{2}\sqrt{2x^2+\sqrt{5}-1}\sqrt{\sqrt{5}-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4+1)/(x^8-3\*x^4+1),x, algorithm="fricas")

[Out]  $-1/2\sqrt{2}\sqrt{\sqrt{5} + 1}\arctan(-1/2\sqrt{2}x\sqrt{\sqrt{5} + 1}) + 1/2\sqrt{2}\sqrt{2x^2 + \sqrt{5} - 1}\sqrt{\sqrt{5} + 1}) + 1/2\sqrt{2}\sqrt{2x^2 + \sqrt{5} - 1}\arctan(-1/2\sqrt{2}x\sqrt{\sqrt{5} - 1}) + 1/2\sqrt{2}\sqrt{2x^2 + \sqrt{5} + 1}\sqrt{\sqrt{5} - 1}) + 1/8\sqrt{2}\sqrt{\sqrt{5} + 1}\log((\sqrt{5}\sqrt{2} - \sqrt{2})\sqrt{\sqrt{5} + 1} + 4x) - 1/8\sqrt{2}\sqrt{\sqrt{5} + 1}\log(-(\sqrt{5}\sqrt{2} - \sqrt{2})\sqrt{\sqrt{5} + 1} + 4x) - 1/8\sqrt{2}\sqrt{\sqrt{5} - 1}\log((\sqrt{5}\sqrt{2} + \sqrt{2})\sqrt{\sqrt{5} - 1} + 4x) + 1/8\sqrt{2}\sqrt{\sqrt{5} - 1}\log(-(\sqrt{5}\sqrt{2} + \sqrt{2})\sqrt{\sqrt{5} - 1} + 4x)$

**Sympy [A]** time = 0.882531, size = 49, normalized size = 0.37

$\text{RootSum}(256t^4 - 16t^2 - 1, (t \mapsto t \log(1024t^5 - 8t + x))) + \text{RootSum}(256t^4 + 16t^2 - 1, (t \mapsto t \log(1024t^5 - 8t + x)))$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x\*\*4+1)/(x\*\*8-3\*x\*\*4+1),x)

[Out]  $\text{RootSum}(256*_t**4 - 16*_t**2 - 1, \text{Lambda}(_t, _t*\log(1024*_t**5 - 8*_t + x))) + \text{RootSum}(256*_t**4 + 16*_t**2 - 1, \text{Lambda}(_t, _t*\log(1024*_t**5 - 8*_t + x)))$

**Giac [A]** time = 1.23239, size = 198, normalized size = 1.51

$-\frac{1}{4}\sqrt{2\sqrt{5}-2}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}}\right) + \frac{1}{4}\sqrt{2\sqrt{5}+2}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}}\right) - \frac{1}{8}\sqrt{2\sqrt{5}-2}\log\left(\left|x + \sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}\right|\right) + \frac{1}{8}\sqrt{2\sqrt{5}+2}\log\left(\left|x + \sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}\right|\right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4+1)/(x^8-3\*x^4+1),x, algorithm="giac")

[Out]  $-1/4\sqrt{2}\sqrt{\sqrt{5} - 2}\arctan(x/\sqrt{1/2\sqrt{5} + 1/2}) + 1/4\sqrt{2}\sqrt{2\sqrt{5} + 2}\arctan(x/\sqrt{1/2\sqrt{5} - 1/2}) - 1/8\sqrt{2}\sqrt{\sqrt{5} - 2}\log(\text{abs}(x + \sqrt{1/2\sqrt{5} + 1/2})) + 1/8\sqrt{2}\sqrt{\sqrt{5} - 2}\log(\text{abs}(x - \sqrt{1/2\sqrt{5} + 1/2})) + 1/8\sqrt{2}\sqrt{2\sqrt{5} + 2}\log(\text{abs}(x + \sqrt{1/2\sqrt{5} - 1/2})) - 1/8\sqrt{2}\sqrt{2\sqrt{5} + 2}\log(\text{abs}(x - \sqrt{1/2\sqrt{5} - 1/2}))$

$$3.17 \quad \int \frac{1+x^4}{1-4x^4+x^8} dx$$

**Optimal.** Leaf size=157

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt{\sqrt{3}-1}}\right)}{2\sqrt[4]{2}\sqrt{\sqrt{3}-1}} - \frac{\tan^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt{1+\sqrt{3}}}\right)}{2\sqrt[4]{2}\sqrt{1+\sqrt{3}}} + \frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt{\sqrt{3}-1}}\right)}{2\sqrt[4]{2}\sqrt{\sqrt{3}-1}} - \frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt{1+\sqrt{3}}}\right)}{2\sqrt[4]{2}\sqrt{1+\sqrt{3}}}$$

[Out] ArcTan[(2^(1/4)\*x)/Sqrt[-1 + Sqrt[3]]]/(2\*2^(1/4)\*Sqrt[-1 + Sqrt[3]]) - ArcTan[(2^(1/4)\*x)/Sqrt[1 + Sqrt[3]]]/(2\*2^(1/4)\*Sqrt[1 + Sqrt[3]]) + ArcTanh[(2^(1/4)\*x)/Sqrt[-1 + Sqrt[3]]]/(2\*2^(1/4)\*Sqrt[-1 + Sqrt[3]]) - ArcTanh[(2^(1/4)\*x)/Sqrt[1 + Sqrt[3]]]/(2\*2^(1/4)\*Sqrt[1 + Sqrt[3]])

**Rubi [A]** time = 0.0865303, antiderivative size = 157, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$ , Rules used = {1419, 1093, 203, 207}

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt{\sqrt{3}-1}}\right)}{2\sqrt[4]{2}\sqrt{\sqrt{3}-1}} - \frac{\tan^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt{1+\sqrt{3}}}\right)}{2\sqrt[4]{2}\sqrt{1+\sqrt{3}}} + \frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt{\sqrt{3}-1}}\right)}{2\sqrt[4]{2}\sqrt{\sqrt{3}-1}} - \frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt{1+\sqrt{3}}}\right)}{2\sqrt[4]{2}\sqrt{1+\sqrt{3}}}$$

Antiderivative was successfully verified.

[In] Int[(1 + x^4)/(1 - 4\*x^4 + x^8),x]

[Out] ArcTan[(2^(1/4)\*x)/Sqrt[-1 + Sqrt[3]]]/(2\*2^(1/4)\*Sqrt[-1 + Sqrt[3]]) - ArcTan[(2^(1/4)\*x)/Sqrt[1 + Sqrt[3]]]/(2\*2^(1/4)\*Sqrt[1 + Sqrt[3]]) + ArcTanh[(2^(1/4)\*x)/Sqrt[-1 + Sqrt[3]]]/(2\*2^(1/4)\*Sqrt[-1 + Sqrt[3]]) - ArcTanh[(2^(1/4)\*x)/Sqrt[1 + Sqrt[3]]]/(2\*2^(1/4)\*Sqrt[1 + Sqrt[3]])

### Rule 1419

Int[((d\_) + (e\_)\*(x\_)^(n\_))/((a\_) + (b\_)\*(x\_)^(n\_) + (c\_)\*(x\_)^(n2\_)), x\_Symbol] := With[{q = Rt[(2\*d)/e - b/c, 2]}, Dist[e/(2\*c), Int[1/Simp[d/e + q\*x^(n/2) + x^n, x], x], x] + Dist[e/(2\*c), Int[1/Simp[d/e - q\*x^(n/2) + x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[n2, 2\*n] && NeQ[b^2 - 4\*a\*c, 0] && EqQ[c\*d^2 - a\*e^2, 0] && IGtQ[n/2, 0] && (GtQ[(2\*d)/e - b/c, 0] || (!LtQ[(2\*d)/e - b/c, 0] && EqQ[d, e\*Rt[a/c, 2]]))

Rule 1093

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{1+x^4}{1-4x^4+x^8} dx &= \frac{1}{2} \int \frac{1}{1-\sqrt{6}x^2+x^4} dx + \frac{1}{2} \int \frac{1}{1+\sqrt{6}x^2+x^4} dx \\ &= \frac{\int \frac{1}{-\sqrt{\frac{3}{2}-\frac{1}{\sqrt{2}}+x^2}} dx}{2\sqrt{2}} + \frac{\int \frac{1}{\sqrt{\frac{3}{2}-\frac{1}{\sqrt{2}}+x^2}} dx}{2\sqrt{2}} - \frac{\int \frac{1}{-\sqrt{\frac{3}{2}+\frac{1}{\sqrt{2}}+x^2}} dx}{2\sqrt{2}} - \frac{\int \frac{1}{\sqrt{\frac{3}{2}+\frac{1}{\sqrt{2}}+x^2}} dx}{2\sqrt{2}} \\ &= \frac{\tan^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt{-1+\sqrt{3}}}\right)}{2\sqrt[4]{2}\sqrt{-1+\sqrt{3}}} - \frac{\tan^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt{1+\sqrt{3}}}\right)}{2\sqrt[4]{2}\sqrt{1+\sqrt{3}}} + \frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt{-1+\sqrt{3}}}\right)}{2\sqrt[4]{2}\sqrt{-1+\sqrt{3}}} - \frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt{1+\sqrt{3}}}\right)}{2\sqrt[4]{2}\sqrt{1+\sqrt{3}}} \end{aligned}$$

**Mathematica [C]** time = 0.0129564, size = 53, normalized size = 0.34

$$\frac{1}{8} \text{RootSum} \left[ \#1^8 - 4\#1^4 + 1 \&, \frac{\#1^4 \log(x - \#1) + \log(x - \#1)}{\#1^7 - 2\#1^3} \& \right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^4)/(1 - 4*x^4 + x^8), x]
```

[Out] RootSum[1 - 4\*#1^4 + #1^8 & , (Log[x - #1] + Log[x - #1]\*#1^4)/(-2\*#1^3 + #1^7) & ]/8

**Maple [C]** time = 0.008, size = 40, normalized size = 0.3

$$\frac{1}{8} \sum_{_R=\text{RootOf}(_Z^8-4_Z^4+1)} \frac{(_R^4+1) \ln(x-_R)}{-_R^7-2_R^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^4+1)/(x^8-4\*x^4+1),x)

[Out] 1/8\*sum((\_R^4+1)/(\_R^7-2\*\_R^3)\*ln(x-\_R),\_R=RootOf(\_Z^8-4\*\_Z^4+1))

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4 + 1}{x^8 - 4x^4 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4+1)/(x^8-4\*x^4+1),x, algorithm="maxima")

[Out] integrate((x^4 + 1)/(x^8 - 4\*x^4 + 1), x)

**Fricas [B]** time = 1.35917, size = 1062, normalized size = 6.76

$$\frac{1}{2} \sqrt{2}(-\sqrt{3}+2)^{\frac{1}{4}} \arctan\left(\frac{1}{2} \sqrt{x^2 + (\sqrt{3}+2)\sqrt{-\sqrt{3}+2}}(\sqrt{3}\sqrt{2} + \sqrt{2})(-\sqrt{3}+2)^{\frac{3}{4}} - \frac{1}{2}(\sqrt{3}\sqrt{2}x + \sqrt{2}x)(-\sqrt{3}+2)^{\frac{3}{4}}\right) -$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4+1)/(x^8-4\*x^4+1),x, algorithm="fricas")

[Out] 1/2\*sqrt(2)\*(-sqrt(3) + 2)^(1/4)\*arctan(1/2\*sqrt(x^2 + (sqrt(3) + 2)\*sqrt(-sqrt(3) + 2))\*(sqrt(3)\*sqrt(2) + sqrt(2))\*(-sqrt(3) + 2)^(3/4) - 1/2\*(sqrt(

$$3)\sqrt{2}x + \sqrt{2}x(-\sqrt{3} + 2)^{3/4} - 1/2\sqrt{2}(\sqrt{3} + 2)^{1/4}\arctan(1/2(\sqrt{x^2 - \sqrt{3} + 2})(\sqrt{3} - 2)(\sqrt{3}\sqrt{2} - \sqrt{2}))\sqrt{\sqrt{3} + 2} - (\sqrt{3}\sqrt{2}x - \sqrt{2}x)\sqrt{\sqrt{3} + 2})(\sqrt{3} + 2)^{1/4} + 1/8\sqrt{2}(\sqrt{3} + 2)^{1/4}\log((\sqrt{3}\sqrt{2} - \sqrt{2})(\sqrt{3} + 2)^{1/4} + 2x) - 1/8\sqrt{2}(\sqrt{3} + 2)^{1/4}\log(-(\sqrt{3}\sqrt{2} - \sqrt{2})(\sqrt{3} + 2)^{1/4} + 2x) - 1/8\sqrt{2}(-\sqrt{3} + 2)^{1/4}\log((\sqrt{3}\sqrt{2} + \sqrt{2})(-\sqrt{3} + 2)^{1/4} + 2x) + 1/8\sqrt{2}(-\sqrt{3} + 2)^{1/4}\log(-(\sqrt{3}\sqrt{2} + \sqrt{2})(-\sqrt{3} + 2)^{1/4} + 2x)$$

**Sympy [A]** time = 0.179358, size = 24, normalized size = 0.15

$$\text{RootSum}\left(1048576t^8 - 4096t^4 + 1, \left(t \mapsto t \log(4096t^5 - 12t + x)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x\*\*4+1)/(x\*\*8-4\*x\*\*4+1),x)

[Out] RootSum(1048576\*\_t\*\*8 - 4096\*\_t\*\*4 + 1, Lambda(\_t, \_t\*log(4096\*\_t\*\*5 - 12\*\_t + x)))

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4 + 1}{x^8 - 4x^4 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4+1)/(x^8-4\*x^4+1),x, algorithm="giac")

[Out] integrate((x^4 + 1)/(x^8 - 4\*x^4 + 1), x)

$$3.18 \quad \int \frac{1+x^4}{1-5x^4+x^8} dx$$

**Optimal.** Leaf size=171

$$\frac{\tan^{-1}\left(\sqrt{\frac{2}{\sqrt{7}-\sqrt{3}}}x\right)}{\sqrt{6(\sqrt{7}-\sqrt{3})}} - \frac{\tan^{-1}\left(\sqrt{\frac{2}{\sqrt{3}+\sqrt{7}}}x\right)}{\sqrt{6(\sqrt{3}+\sqrt{7})}} + \frac{\tanh^{-1}\left(\sqrt{\frac{2}{\sqrt{7}-\sqrt{3}}}x\right)}{\sqrt{6(\sqrt{7}-\sqrt{3})}} - \frac{\tanh^{-1}\left(\sqrt{\frac{2}{\sqrt{3}+\sqrt{7}}}x\right)}{\sqrt{6(\sqrt{3}+\sqrt{7})}}$$

[Out] ArcTan[Sqrt[2/(-Sqrt[3] + Sqrt[7])]\*x]/Sqrt[6\*(-Sqrt[3] + Sqrt[7])] - ArcTan[Sqrt[2/(Sqrt[3] + Sqrt[7])]\*x]/Sqrt[6\*(Sqrt[3] + Sqrt[7])] + ArcTanh[Sqrt[2/(-Sqrt[3] + Sqrt[7])]\*x]/Sqrt[6\*(-Sqrt[3] + Sqrt[7])] - ArcTanh[Sqrt[2/(Sqrt[3] + Sqrt[7])]\*x]/Sqrt[6\*(Sqrt[3] + Sqrt[7])]

**Rubi [A]** time = 0.151257, antiderivative size = 171, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$ , Rules used = {1419, 1093, 203, 207}

$$\frac{\tan^{-1}\left(\sqrt{\frac{2}{\sqrt{7}-\sqrt{3}}}x\right)}{\sqrt{6(\sqrt{7}-\sqrt{3})}} - \frac{\tan^{-1}\left(\sqrt{\frac{2}{\sqrt{3}+\sqrt{7}}}x\right)}{\sqrt{6(\sqrt{3}+\sqrt{7})}} + \frac{\tanh^{-1}\left(\sqrt{\frac{2}{\sqrt{7}-\sqrt{3}}}x\right)}{\sqrt{6(\sqrt{7}-\sqrt{3})}} - \frac{\tanh^{-1}\left(\sqrt{\frac{2}{\sqrt{3}+\sqrt{7}}}x\right)}{\sqrt{6(\sqrt{3}+\sqrt{7})}}$$

Antiderivative was successfully verified.

[In] Int[(1 + x^4)/(1 - 5\*x^4 + x^8),x]

[Out] ArcTan[Sqrt[2/(-Sqrt[3] + Sqrt[7])]\*x]/Sqrt[6\*(-Sqrt[3] + Sqrt[7])] - ArcTan[Sqrt[2/(Sqrt[3] + Sqrt[7])]\*x]/Sqrt[6\*(Sqrt[3] + Sqrt[7])] + ArcTanh[Sqrt[2/(-Sqrt[3] + Sqrt[7])]\*x]/Sqrt[6\*(-Sqrt[3] + Sqrt[7])] - ArcTanh[Sqrt[2/(Sqrt[3] + Sqrt[7])]\*x]/Sqrt[6\*(Sqrt[3] + Sqrt[7])]

### Rule 1419

Int[((d\_) + (e\_.)\*(x\_)^(n\_))/((a\_) + (b\_.)\*(x\_)^(n\_) + (c\_.)\*(x\_)^(2n\_)), x\_Symbol] := With[{q = Rt[(2\*d)/e - b/c, 2]}, Dist[e/(2\*c), Int[1/Simp[d/e + q\*x^(n/2) + x^n, x], x], x] + Dist[e/(2\*c), Int[1/Simp[d/e - q\*x^(n/2) + x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[n2, 2\*n] && NeQ[b^2 - 4\*a\*c, 0] && EqQ[c\*d^2 - a\*e^2, 0] && IGtQ[n/2, 0] && (GtQ[(2\*d)/e - b/c, 0] || (!LtQ[(2\*d)/e - b/c, 0] && EqQ[d, e\*Rt[a/c, 2]]))

Rule 1093

```
Int[((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(-1), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]
```

Rule 203

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])
```

Rule 207

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{1+x^4}{1-5x^4+x^8} dx &= \frac{1}{2} \int \frac{1}{1-\sqrt{7}x^2+x^4} dx + \frac{1}{2} \int \frac{1}{1+\sqrt{7}x^2+x^4} dx \\ &= \frac{\int \frac{1}{-\frac{\sqrt{3}-\sqrt{7}}{2}+x^2} dx}{2\sqrt{3}} - \frac{\int \frac{1}{\frac{\sqrt{3}-\sqrt{7}}{2}+x^2} dx}{2\sqrt{3}} + \frac{\int \frac{1}{-\frac{\sqrt{3}+\sqrt{7}}{2}+x^2} dx}{2\sqrt{3}} - \frac{\int \frac{1}{\frac{\sqrt{3}+\sqrt{7}}{2}+x^2} dx}{2\sqrt{3}} \\ &= \frac{\tan^{-1}\left(\sqrt{\frac{2}{-\sqrt{3}+\sqrt{7}}}x\right)}{\sqrt{6(-\sqrt{3}+\sqrt{7})}} - \frac{\tan^{-1}\left(\sqrt{\frac{2}{\sqrt{3}+\sqrt{7}}}x\right)}{\sqrt{6(\sqrt{3}+\sqrt{7})}} + \frac{\tanh^{-1}\left(\sqrt{\frac{2}{-\sqrt{3}+\sqrt{7}}}x\right)}{\sqrt{6(-\sqrt{3}+\sqrt{7})}} - \frac{\tanh^{-1}\left(\sqrt{\frac{2}{\sqrt{3}+\sqrt{7}}}x\right)}{\sqrt{6(\sqrt{3}+\sqrt{7})}} \end{aligned}$$

**Mathematica [C]** time = 0.0132405, size = 55, normalized size = 0.32

$$\frac{1}{4} \text{RootSum}\left[\#1^8 - 5\#1^4 + 1 \&, \frac{\#1^4 \log(x - \#1) + \log(x - \#1)}{2\#1^7 - 5\#1^3} \&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^4)/(1 - 5*x^4 + x^8), x]
```



[Out] RootSum[1 - 5\*#1^4 + #1^8 & , (Log[x - #1] + Log[x - #1]\*#1^4)/(-5\*#1^3 + 2\*#1^7) & ]/4

**Maple [C]** time = 0.008, size = 42, normalized size = 0.3

$$\frac{1}{4} \sum_{_R=\text{RootOf}(_Z^8-5_Z^4+1)} \frac{(_R^4+1) \ln(x-_R)}{2_R^7-5_R^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^4+1)/(x^8-5\*x^4+1),x)

[Out] 1/4\*sum((\_R^4+1)/(2\*\_R^7-5\*\_R^3)\*ln(x-\_R),\_R=RootOf(\_Z^8-5\*\_Z^4+1))

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4+1}{x^8-5x^4+1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4+1)/(x^8-5\*x^4+1),x, algorithm="maxima")

[Out] integrate((x^4 + 1)/(x^8 - 5\*x^4 + 1), x)

**Fricas [B]** time = 1.45293, size = 1812, normalized size = 10.6

$$\frac{1}{6} \sqrt{6} \sqrt{\sqrt{2} \sqrt{-\sqrt{7} \sqrt{3} + 5}} \arctan \left( \frac{1}{48} \left( \sqrt{7} \sqrt{6} \sqrt{3} \sqrt{2} + 3 \sqrt{6} \sqrt{2} \right) \sqrt{4x^2 + \left( \sqrt{7} \sqrt{3} \sqrt{2} + 5 \sqrt{2} \right) \sqrt{-\sqrt{7} \sqrt{3} + 5}} \sqrt{-\sqrt{7} \sqrt{3} + 5} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4+1)/(x^8-5\*x^4+1),x, algorithm="fricas")

[Out] 1/6\*sqrt(6)\*sqrt(sqrt(2)\*sqrt(-sqrt(7)\*sqrt(3) + 5))\*arctan(1/48\*(sqrt(7)\*sqrt(6)\*sqrt(3)\*sqrt(2) + 3\*sqrt(6)\*sqrt(2))\*sqrt(4\*x^2 + (sqrt(7)\*sqrt(3)\*sqrt(2) + 5\*sqrt(2))\*sqrt(-sqrt(7)\*sqrt(3) + 5)\*sqrt(-sqrt(7)\*sqrt(3) + 5))

```

qrt(2) + 5*sqrt(2))*sqrt(-sqrt(7)*sqrt(3) + 5))*sqrt(-sqrt(7)*sqrt(3) + 5)*
sqrt(sqrt(2)*sqrt(-sqrt(7)*sqrt(3) + 5)) - 1/24*(sqrt(7)*sqrt(6)*sqrt(3)*sq
rt(2)*x + 3*sqrt(6)*sqrt(2)*x)*sqrt(-sqrt(7)*sqrt(3) + 5)*sqrt(sqrt(2)*sqrt
(-sqrt(7)*sqrt(3) + 5))) - 1/6*sqrt(6)*sqrt(sqrt(2)*sqrt(sqrt(7)*sqrt(3) +
5))*arctan(1/48*((sqrt(7)*sqrt(6)*sqrt(3)*sqrt(2) - 3*sqrt(6)*sqrt(2))*sqrt
(4*x^2 - (sqrt(7)*sqrt(3)*sqrt(2) - 5*sqrt(2))*sqrt(sqrt(7)*sqrt(3) + 5))*s
qrt(sqrt(7)*sqrt(3) + 5) - 2*(sqrt(7)*sqrt(6)*sqrt(3)*sqrt(2)*x - 3*sqrt(6)
*sqrt(2)*x)*sqrt(sqrt(7)*sqrt(3) + 5))*sqrt(sqrt(2)*sqrt(sqrt(7)*sqrt(3) +
5))) + 1/24*sqrt(6)*sqrt(sqrt(2)*sqrt(sqrt(7)*sqrt(3) + 5))*log((sqrt(7)*sq
rt(6)*sqrt(3) - 3*sqrt(6))*sqrt(sqrt(2)*sqrt(sqrt(7)*sqrt(3) + 5)) + 12*x)
- 1/24*sqrt(6)*sqrt(sqrt(2)*sqrt(sqrt(7)*sqrt(3) + 5))*log(-(sqrt(7)*sqrt(6)
)*sqrt(3) - 3*sqrt(6))*sqrt(sqrt(2)*sqrt(sqrt(7)*sqrt(3) + 5)) + 12*x) - 1/
24*sqrt(6)*sqrt(sqrt(2)*sqrt(-sqrt(7)*sqrt(3) + 5))*log((sqrt(7)*sqrt(6)*sq
rt(3) + 3*sqrt(6))*sqrt(sqrt(2)*sqrt(-sqrt(7)*sqrt(3) + 5)) + 12*x) + 1/24*
sqrt(6)*sqrt(sqrt(2)*sqrt(-sqrt(7)*sqrt(3) + 5))*log(-(sqrt(7)*sqrt(6)*sqrt
(3) + 3*sqrt(6))*sqrt(sqrt(2)*sqrt(-sqrt(7)*sqrt(3) + 5)) + 12*x)

```

**Sympy [A]** time = 0.190294, size = 24, normalized size = 0.14

$$\text{RootSum}\left(5308416t^8 - 11520t^4 + 1, (t \mapsto t \log(9216t^5 - 16t + x))\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+1)/(x**8-5*x**4+1), x)
```

```
[Out] RootSum(5308416*_t**8 - 11520*_t**4 + 1, Lambda(_t, _t*log(9216*_t**5 - 16*_t + x)))
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4 + 1}{x^8 - 5x^4 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)/(x^8-5*x^4+1), x, algorithm="giac")
```

```
[Out] integrate((x^4 + 1)/(x^8 - 5*x^4 + 1), x)
```

$$3.19 \quad \int \frac{1+x^4}{1-6x^4+x^8} dx$$

**Optimal.** Leaf size=117

$$\frac{\tan^{-1}\left(\frac{x}{\sqrt{\sqrt{2}-1}}\right)}{4\sqrt{\sqrt{2}-1}} - \frac{\tan^{-1}\left(\frac{x}{\sqrt{1+\sqrt{2}}}\right)}{4\sqrt{1+\sqrt{2}}} + \frac{\tanh^{-1}\left(\frac{x}{\sqrt{\sqrt{2}-1}}\right)}{4\sqrt{\sqrt{2}-1}} - \frac{\tanh^{-1}\left(\frac{x}{\sqrt{1+\sqrt{2}}}\right)}{4\sqrt{1+\sqrt{2}}}$$

[Out] ArcTan[x/Sqrt[-1 + Sqrt[2]]]/(4\*Sqrt[-1 + Sqrt[2]]) - ArcTan[x/Sqrt[1 + Sqrt[2]]]/(4\*Sqrt[1 + Sqrt[2]]) + ArcTanh[x/Sqrt[-1 + Sqrt[2]]]/(4\*Sqrt[-1 + Sqrt[2]]) - ArcTanh[x/Sqrt[1 + Sqrt[2]]]/(4\*Sqrt[1 + Sqrt[2]])

**Rubi [A]** time = 0.0574979, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$ , Rules used = {1419, 1093, 203, 207}

$$\frac{\tan^{-1}\left(\frac{x}{\sqrt{\sqrt{2}-1}}\right)}{4\sqrt{\sqrt{2}-1}} - \frac{\tan^{-1}\left(\frac{x}{\sqrt{1+\sqrt{2}}}\right)}{4\sqrt{1+\sqrt{2}}} + \frac{\tanh^{-1}\left(\frac{x}{\sqrt{\sqrt{2}-1}}\right)}{4\sqrt{\sqrt{2}-1}} - \frac{\tanh^{-1}\left(\frac{x}{\sqrt{1+\sqrt{2}}}\right)}{4\sqrt{1+\sqrt{2}}}$$

Antiderivative was successfully verified.

[In] Int[(1 + x^4)/(1 - 6\*x^4 + x^8),x]

[Out] ArcTan[x/Sqrt[-1 + Sqrt[2]]]/(4\*Sqrt[-1 + Sqrt[2]]) - ArcTan[x/Sqrt[1 + Sqrt[2]]]/(4\*Sqrt[1 + Sqrt[2]]) + ArcTanh[x/Sqrt[-1 + Sqrt[2]]]/(4\*Sqrt[-1 + Sqrt[2]]) - ArcTanh[x/Sqrt[1 + Sqrt[2]]]/(4\*Sqrt[1 + Sqrt[2]])

### Rule 1419

Int[((d\_) + (e\_)\*(x\_)^(n\_))/((a\_) + (b\_)\*(x\_)^(n\_) + (c\_)\*(x\_)^(n2\_)), x\_Symbol] :> With[{q = Rt[(2\*d)/e - b/c, 2]}, Dist[e/(2\*c), Int[1/Simp[d/e + q\*x^(n/2) + x^n, x], x], x] + Dist[e/(2\*c), Int[1/Simp[d/e - q\*x^(n/2) + x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[n2, 2\*n] && NeQ[b^2 - 4\*a\*c, 0] && EqQ[c\*d^2 - a\*e^2, 0] && IGtQ[n/2, 0] && (GtQ[(2\*d)/e - b/c, 0] || (!LtQ[(2\*d)/e - b/c, 0] && EqQ[d, e\*Rt[a/c, 2]]))

### Rule 1093

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]
```

### Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])
```

### Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])
```

### Rubi steps

$$\begin{aligned} \int \frac{1+x^4}{1-6x^4+x^8} dx &= \frac{1}{2} \int \frac{1}{1-2\sqrt{2}x^2+x^4} dx + \frac{1}{2} \int \frac{1}{1+2\sqrt{2}x^2+x^4} dx \\ &= \frac{1}{4} \int \frac{1}{-1-\sqrt{2}+x^2} dx - \frac{1}{4} \int \frac{1}{1-\sqrt{2}+x^2} dx + \frac{1}{4} \int \frac{1}{-1+\sqrt{2}+x^2} dx - \frac{1}{4} \int \frac{1}{1+\sqrt{2}+x^2} dx \\ &= \frac{\tan^{-1}\left(\frac{x}{\sqrt{-1+\sqrt{2}}}\right)}{4\sqrt{-1+\sqrt{2}}} - \frac{\tan^{-1}\left(\frac{x}{\sqrt{1+\sqrt{2}}}\right)}{4\sqrt{1+\sqrt{2}}} + \frac{\tanh^{-1}\left(\frac{x}{\sqrt{-1+\sqrt{2}}}\right)}{4\sqrt{-1+\sqrt{2}}} - \frac{\tanh^{-1}\left(\frac{x}{\sqrt{1+\sqrt{2}}}\right)}{4\sqrt{1+\sqrt{2}}} \end{aligned}$$

**Mathematica [A]** time = 0.0452306, size = 111, normalized size = 0.95

$$\frac{1}{4} \left( \sqrt{1+\sqrt{2}} \tan^{-1}\left(\frac{x}{\sqrt{\sqrt{2}-1}}\right) - \sqrt{\sqrt{2}-1} \tan^{-1}\left(\frac{x}{\sqrt{1+\sqrt{2}}}\right) + \sqrt{1+\sqrt{2}} \tanh^{-1}\left(\frac{x}{\sqrt{\sqrt{2}-1}}\right) - \sqrt{\sqrt{2}-1} \tanh^{-1}\left(\frac{x}{\sqrt{1+\sqrt{2}}}\right) \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^4)/(1 - 6*x^4 + x^8), x]
```

```
[Out] (Sqrt[1 + Sqrt[2]]*ArcTan[x/Sqrt[-1 + Sqrt[2]]] - Sqrt[-1 + Sqrt[2]]*ArcTan[x/Sqrt[1 + Sqrt[2]]] + Sqrt[1 + Sqrt[2]]*ArcTanh[x/Sqrt[-1 + Sqrt[2]]] - S
```

`qrt[-1 + Sqrt[2]]*ArcTanh[x/Sqrt[1 + Sqrt[2]]])/4`

**Maple [A]** time = 0.051, size = 78, normalized size = 0.7

$$\frac{1}{4\sqrt{\sqrt{2}-1}} \arctan\left(\frac{x}{\sqrt{\sqrt{2}-1}}\right) + \frac{1}{4\sqrt{\sqrt{2}-1}} \operatorname{Artanh}\left(\frac{x}{\sqrt{\sqrt{2}-1}}\right) - \frac{1}{4\sqrt{1+\sqrt{2}}} \arctan\left(\frac{x}{\sqrt{1+\sqrt{2}}}\right) - \frac{1}{4\sqrt{1+\sqrt{2}}} \operatorname{Artanh}\left(\frac{x}{\sqrt{1+\sqrt{2}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^4+1)/(x^8-6*x^4+1),x)`

[Out] `1/4*arctan(x/(2^(1/2)-1)^(1/2))/(2^(1/2)-1)^(1/2)+1/4*arctanh(x/(2^(1/2)-1)^(1/2))/(2^(1/2)-1)^(1/2)-1/4*arctan(x/(1+2^(1/2))^(1/2))/(1+2^(1/2))^(1/2)-1/4*arctanh(x/(1+2^(1/2))^(1/2))/(1+2^(1/2))^(1/2)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4 + 1}{x^8 - 6x^4 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^4+1)/(x^8-6*x^4+1),x, algorithm="maxima")`

[Out] `integrate((x^4 + 1)/(x^8 - 6*x^4 + 1), x)`

**Fricas [B]** time = 1.27806, size = 616, normalized size = 5.26

$$-\frac{1}{2}\sqrt{\sqrt{2}+1} \arctan\left(-x\sqrt{\sqrt{2}+1} + \sqrt{x^2 + \sqrt{2}-1}\sqrt{\sqrt{2}+1}\right) + \frac{1}{2}\sqrt{\sqrt{2}-1} \arctan\left(-x\sqrt{\sqrt{2}-1} + \sqrt{x^2 + \sqrt{2}+1}\sqrt{\sqrt{2}-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^4+1)/(x^8-6*x^4+1),x, algorithm="fricas")`

[Out] `-1/2*sqrt(sqrt(2) + 1)*arctan(-x*sqrt(sqrt(2) + 1) + sqrt(x^2 + sqrt(2) - 1)*sqrt(sqrt(2) + 1)) + 1/2*sqrt(sqrt(2) - 1)*arctan(-x*sqrt(sqrt(2) - 1) +`

$$\begin{aligned} & \sqrt{x^2 + \sqrt{2} + 1} \sqrt{\sqrt{2} - 1} - \frac{1}{8} \sqrt{\sqrt{2} - 1} \log((\sqrt{2} + 1) \sqrt{\sqrt{2} - 1} + x) + \frac{1}{8} \sqrt{\sqrt{2} - 1} \log(-(\sqrt{2} + 1) \\ & \sqrt{\sqrt{2} - 1} + x) + \frac{1}{8} \sqrt{\sqrt{2} + 1} \log(\sqrt{\sqrt{2} + 1} (\sqrt{2} - 1) + x) - \frac{1}{8} \sqrt{\sqrt{2} + 1} \log(-\sqrt{\sqrt{2} + 1} (\sqrt{2} - 1) \\ & + x) \end{aligned}$$

**Sympy [A]** time = 0.860665, size = 49, normalized size = 0.42

$$\text{RootSum}(4096t^4 - 128t^2 - 1, (t \mapsto t \log(16384t^5 - 20t + x))) + \text{RootSum}(4096t^4 + 128t^2 - 1, (t \mapsto t \log(16384t^5 -$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x\*\*4+1)/(x\*\*8-6\*x\*\*4+1),x)

[Out] RootSum(4096\*\_t\*\*4 - 128\*\_t\*\*2 - 1, Lambda(\_t, \_t\*log(16384\*\_t\*\*5 - 20\*\_t + x))) + RootSum(4096\*\_t\*\*4 + 128\*\_t\*\*2 - 1, Lambda(\_t, \_t\*log(16384\*\_t\*\*5 - 20\*\_t + x)))

**Giac [A]** time = 1.21443, size = 166, normalized size = 1.42

$$-\frac{1}{4} \sqrt{\sqrt{2}-1} \arctan\left(\frac{x}{\sqrt{\sqrt{2}+1}}\right) + \frac{1}{4} \sqrt{\sqrt{2}+1} \arctan\left(\frac{x}{\sqrt{\sqrt{2}-1}}\right) - \frac{1}{8} \sqrt{\sqrt{2}-1} \log\left(\left|x + \sqrt{\sqrt{2}+1}\right|\right) + \frac{1}{8} \sqrt{\sqrt{2}-1} \log\left(\left|x - \sqrt{\sqrt{2}+1}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4+1)/(x^8-6\*x^4+1),x, algorithm="giac")

[Out] -1/4\*sqrt(sqrt(2) - 1)\*arctan(x/sqrt(sqrt(2) + 1)) + 1/4\*sqrt(sqrt(2) + 1)\*arctan(x/sqrt(sqrt(2) - 1)) - 1/8\*sqrt(sqrt(2) - 1)\*log(abs(x + sqrt(sqrt(2) + 1))) + 1/8\*sqrt(sqrt(2) - 1)\*log(abs(x - sqrt(sqrt(2) + 1))) + 1/8\*sqrt(sqrt(2) + 1)\*log(abs(x + sqrt(sqrt(2) - 1))) - 1/8\*sqrt(sqrt(2) + 1)\*log(abs(x - sqrt(sqrt(2) - 1)))

$$3.20 \quad \int \frac{1-x^4}{1+bx^4+x^8} dx$$

**Optimal.** Leaf size=511

$$\frac{\sqrt{2-\sqrt{2-b}} \log\left(-\sqrt{2-\sqrt{2-b}x} + x^2 + 1\right)}{8\sqrt{2-b}} - \frac{\sqrt{2-\sqrt{2-b}} \log\left(\sqrt{2-\sqrt{2-b}x} + x^2 + 1\right)}{8\sqrt{2-b}} - \frac{\sqrt{\sqrt{2-b}+2} \log\left(-\sqrt{\sqrt{2-b}+2x} + x^2 + 1\right)}{8\sqrt{2-b}}$$

```
[Out] -(Sqrt[2 + b]*ArcTan[(Sqrt[2 - Sqrt[2 - b]] - 2*x)/Sqrt[2 + Sqrt[2 - b]])/(
(4*Sqrt[2 - Sqrt[2 - b]]*Sqrt[2 - b]) + (Sqrt[2 + b]*ArcTan[(Sqrt[2 + Sqrt[
2 - b]] - 2*x)/Sqrt[2 - Sqrt[2 - b]])/(4*Sqrt[2 + Sqrt[2 - b]]*Sqrt[2 - b]
) + (Sqrt[2 + b]*ArcTan[(Sqrt[2 - Sqrt[2 - b]] + 2*x)/Sqrt[2 + Sqrt[2 - b]]
])/ (4*Sqrt[2 - Sqrt[2 - b]]*Sqrt[2 - b]) - (Sqrt[2 + b]*ArcTan[(Sqrt[2 + Sq
rt[2 - b]] + 2*x)/Sqrt[2 - Sqrt[2 - b]])/(4*Sqrt[2 + Sqrt[2 - b]]*Sqrt[2 -
b]) + (Sqrt[2 - Sqrt[2 - b]]*Log[1 - Sqrt[2 - Sqrt[2 - b]]*x + x^2))/(8*Sq
rt[2 - b]) - (Sqrt[2 - Sqrt[2 - b]]*Log[1 + Sqrt[2 - Sqrt[2 - b]]*x + x^2))
/(8*Sqrt[2 - b]) - (Sqrt[2 + Sqrt[2 - b]]*Log[1 - Sqrt[2 + Sqrt[2 - b]]*x +
x^2))/(8*Sqrt[2 - b]) + (Sqrt[2 + Sqrt[2 - b]]*Log[1 + Sqrt[2 + Sqrt[2 - b]
]]*x + x^2))/(8*Sqrt[2 - b])
```

---

**Rubi [A]** time = 0.358509, antiderivative size = 511, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 6, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.3$ , Rules used = {1421, 1169, 634, 618, 204, 628}

$$\frac{\sqrt{2-\sqrt{2-b}} \log\left(-\sqrt{2-\sqrt{2-b}x} + x^2 + 1\right)}{8\sqrt{2-b}} - \frac{\sqrt{2-\sqrt{2-b}} \log\left(\sqrt{2-\sqrt{2-b}x} + x^2 + 1\right)}{8\sqrt{2-b}} - \frac{\sqrt{\sqrt{2-b}+2} \log\left(-\sqrt{\sqrt{2-b}+2x} + x^2 + 1\right)}{8\sqrt{2-b}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^4)/(1 + b*x^4 + x^8), x]
```

```
[Out] -(Sqrt[2 + b]*ArcTan[(Sqrt[2 - Sqrt[2 - b]] - 2*x)/Sqrt[2 + Sqrt[2 - b]])/(
(4*Sqrt[2 - Sqrt[2 - b]]*Sqrt[2 - b]) + (Sqrt[2 + b]*ArcTan[(Sqrt[2 + Sqrt[
2 - b]] - 2*x)/Sqrt[2 - Sqrt[2 - b]])/(4*Sqrt[2 + Sqrt[2 - b]]*Sqrt[2 - b]
) + (Sqrt[2 + b]*ArcTan[(Sqrt[2 - Sqrt[2 - b]] + 2*x)/Sqrt[2 + Sqrt[2 - b]]
])/ (4*Sqrt[2 - Sqrt[2 - b]]*Sqrt[2 - b]) - (Sqrt[2 + b]*ArcTan[(Sqrt[2 + Sq
rt[2 - b]] + 2*x)/Sqrt[2 - Sqrt[2 - b]])/(4*Sqrt[2 + Sqrt[2 - b]]*Sqrt[2 -
b]) + (Sqrt[2 - Sqrt[2 - b]]*Log[1 - Sqrt[2 - Sqrt[2 - b]]*x + x^2))/(8*Sq
```

```
rt[2 - b]) - (Sqrt[2 - Sqrt[2 - b]]*Log[1 + Sqrt[2 - Sqrt[2 - b]]*x + x^2])
/(8*Sqrt[2 - b]) - (Sqrt[2 + Sqrt[2 - b]]*Log[1 - Sqrt[2 + Sqrt[2 - b]]*x +
x^2])/(8*Sqrt[2 - b]) + (Sqrt[2 + Sqrt[2 - b]]*Log[1 + Sqrt[2 + Sqrt[2 - b]
]]*x + x^2))/(8*Sqrt[2 - b])
```

### Rule 1421

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[(-2*d)/e - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x^
(n/2))/Simp[d/e + q*x^(n/2) - x^n, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*
x^(n/2))/Simp[d/e - q*x^(n/2) - x^n, x], x], x]] /; FreeQ[{a, b, c, d, e},
x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && IGtQ[
n/2, 0] && !GtQ[b^2 - 4*a*c, 0]
```

### Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
(d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

### Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

### Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

### Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

### Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d},
```



e}, x] && EqQ[2\*c\*d - b\*e, 0]

### Rubi steps

$$\begin{aligned}
 \int \frac{1-x^4}{1+bx^4+x^8} dx &= -\frac{\int \frac{\sqrt{2-b}+2x^2}{-1-\sqrt{2-b}x^2-x^4} dx}{2\sqrt{2-b}} - \frac{\int \frac{\sqrt{2-b}-2x^2}{-1+\sqrt{2-b}x^2-x^4} dx}{2\sqrt{2-b}} \\
 &= \frac{\int \frac{\sqrt{2-\sqrt{2-b}}\sqrt{2-b}-(-2+\sqrt{2-b})x}{1-\sqrt{2-\sqrt{2-b}}x+x^2} dx}{4\sqrt{2-\sqrt{2-b}}\sqrt{2-b}} + \frac{\int \frac{\sqrt{2-\sqrt{2-b}}\sqrt{2-b}+(-2+\sqrt{2-b})x}{1+\sqrt{2-\sqrt{2-b}}x+x^2} dx}{4\sqrt{2-\sqrt{2-b}}\sqrt{2-b}} + \frac{\int \frac{\sqrt{2+\sqrt{2-b}}\sqrt{2-b}-(2+\sqrt{2-b})x}{1-\sqrt{2+\sqrt{2-b}}x+x^2} dx}{4\sqrt{2+\sqrt{2-b}}\sqrt{2-b}} + \frac{\int \frac{\sqrt{2+\sqrt{2-b}}\sqrt{2-b}+(2+\sqrt{2-b})x}{1+\sqrt{2+\sqrt{2-b}}x+x^2} dx}{4\sqrt{2+\sqrt{2-b}}\sqrt{2-b}} \\
 &= -\left(\frac{1}{8}\left(-1+\frac{2}{\sqrt{2-b}}\right)\int \frac{1}{1-\sqrt{2+\sqrt{2-b}}x+x^2} dx\right) - \frac{1}{8}\left(-1+\frac{2}{\sqrt{2-b}}\right)\int \frac{1}{1+\sqrt{2+\sqrt{2-b}}x+x^2} dx \\
 &= \frac{\sqrt{2-\sqrt{2-b}}\log\left(1-\sqrt{2-\sqrt{2-b}}x+x^2\right)}{8\sqrt{2-b}} - \frac{\sqrt{2-\sqrt{2-b}}\log\left(1+\sqrt{2-\sqrt{2-b}}x+x^2\right)}{8\sqrt{2-b}} - \frac{\sqrt{2+\sqrt{2-b}}\log\left(1-\sqrt{2+\sqrt{2-b}}x+x^2\right)}{8\sqrt{2-b}} + \frac{\sqrt{2+\sqrt{2-b}}\log\left(1+\sqrt{2+\sqrt{2-b}}x+x^2\right)}{8\sqrt{2-b}} \\
 &= -\frac{\sqrt{2+\sqrt{2-b}}\tan^{-1}\left(\frac{\sqrt{2-\sqrt{2-b}}-2x}{\sqrt{2+\sqrt{2-b}}}\right)}{4\sqrt{2-b}} + \frac{\sqrt{2-\sqrt{2-b}}\tan^{-1}\left(\frac{\sqrt{2+\sqrt{2-b}}-2x}{\sqrt{2-\sqrt{2-b}}}\right)}{4\sqrt{2-b}} + \frac{\sqrt{2+\sqrt{2-b}}\tan^{-1}\left(\frac{\sqrt{2-\sqrt{2-b}}+2x}{\sqrt{2+\sqrt{2-b}}}\right)}{4\sqrt{2-b}} - \frac{\sqrt{2-\sqrt{2-b}}\tan^{-1}\left(\frac{\sqrt{2+\sqrt{2-b}}+2x}{\sqrt{2-\sqrt{2-b}}}\right)}{4\sqrt{2-b}}
 \end{aligned}$$

**Mathematica [C]** time = 0.0246006, size = 57, normalized size = 0.11

$$-\frac{1}{4}\text{RootSum}\left[\#1^4b + \#1^8 + 1\&, \frac{\#1^4\log(x - \#1) - \log(x - \#1)}{\#1^3b + 2\#1^7}\&\right]$$

Antiderivative was successfully verified.

[In] Integrate[(1 - x^4)/(1 + b\*x^4 + x^8), x]

[Out] -RootSum[1 + b\*#1^4 + #1^8 & , (-Log[x - #1] + Log[x - #1]\*#1^4)/(b\*#1^3 + 2\*#1^7) & ]/4

**Maple [C]** time = 0.003, size = 44, normalized size = 0.1

$$\frac{1}{4} \sum_{-R=\text{RootOf}(-Z^8+Z^4b+1)} \frac{(-R^4+1)\ln(x-R)}{2R^7+R^3b}$$



8)) - 4\*b + 4)\*sqrt(x^2 + 1/2\*sqrt(1/2)\*(b^2 - (b^3 - 6\*b^2 + 12\*b - 8)\*sqrt((b + 2)/(b^3 - 6\*b^2 + 12\*b - 8)) - 2\*b)\*sqrt(-((b^2 - 4\*b + 4)\*sqrt((b + 2)/(b^3 - 6\*b^2 + 12\*b - 8)) + b)/(b^2 - 4\*b + 4)))\*sqrt(-((b^2 - 4\*b + 4)\*sqrt((b + 2)/(b^3 - 6\*b^2 + 12\*b - 8)) + b)/(b^2 - 4\*b + 4)) + sqrt(1/2)\*((b^3 - 6\*b^2 + 12\*b - 8)\*x\*sqrt((b + 2)/(b^3 - 6\*b^2 + 12\*b - 8)) - (b^2 - 4\*b + 4)\*x)\*sqrt(-((b^2 - 4\*b + 4)\*sqrt((b + 2)/(b^3 - 6\*b^2 + 12\*b - 8)) + b)/(b^2 - 4\*b + 4)))\*sqrt(sqrt(1/2)\*sqrt(-((b^2 - 4\*b + 4)\*sqrt((b + 2)/(b^3 - 6\*b^2 + 12\*b - 8)) + b)/(b^2 - 4\*b + 4)))) + 1/4\*sqrt(sqrt(1/2)\*sqrt(-((b^2 - 4\*b + 4)\*sqrt((b + 2)/(b^3 - 6\*b^2 + 12\*b - 8)) + b)/(b^2 - 4\*b + 4)))\*log(1/2\*((b^2 - 4\*b + 4)\*sqrt((b + 2)/(b^3 - 6\*b^2 + 12\*b - 8)) - b + 2)\*sqrt(sqrt(1/2)\*sqrt(-((b^2 - 4\*b + 4)\*sqrt((b + 2)/(b^3 - 6\*b^2 + 12\*b - 8)) + b)/(b^2 - 4\*b + 4)) + x) - 1/4\*sqrt(sqrt(1/2)\*sqrt(-((b^2 - 4\*b + 4)\*sqrt((b + 2)/(b^3 - 6\*b^2 + 12\*b - 8)) + b)/(b^2 - 4\*b + 4)))\*log(-1/2\*((b^2 - 4\*b + 4)\*sqrt((b + 2)/(b^3 - 6\*b^2 + 12\*b - 8)) - b + 2)\*sqrt(sqrt(1/2)\*sqrt(-((b^2 - 4\*b + 4)\*sqrt((b + 2)/(b^3 - 6\*b^2 + 12\*b - 8)) + b)/(b^2 - 4\*b + 4)) + x) - 1/4\*sqrt(sqrt(1/2)\*sqrt(((b^2 - 4\*b + 4)\*sqrt((b + 2)/(b^3 - 6\*b^2 + 12\*b - 8)) - b)/(b^2 - 4\*b + 4)))\*log(1/2\*((b^2 - 4\*b + 4)\*sqrt((b + 2)/(b^3 - 6\*b^2 + 12\*b - 8)) + b - 2)\*sqrt(sqrt(1/2)\*sqrt(((b^2 - 4\*b + 4)\*sqrt((b + 2)/(b^3 - 6\*b^2 + 12\*b - 8)) - b)/(b^2 - 4\*b + 4)) + x) + 1/4\*sqrt(sqrt(1/2)\*sqrt(((b^2 - 4\*b + 4)\*sqrt((b + 2)/(b^3 - 6\*b^2 + 12\*b - 8)) - b)/(b^2 - 4\*b + 4)))\*log(-1/2\*((b^2 - 4\*b + 4)\*sqrt((b + 2)/(b^3 - 6\*b^2 + 12\*b - 8)) + b - 2)\*sqrt(sqrt(1/2)\*sqrt(((b^2 - 4\*b + 4)\*sqrt((b + 2)/(b^3 - 6\*b^2 + 12\*b - 8)) - b)/(b^2 - 4\*b + 4)) + x)

**Sympy [A]** time = 2.26308, size = 76, normalized size = 0.15

-RootSum( $t^8(65536b^4 - 524288b^3 + 1572864b^2 - 2097152b + 1048576) + t^4(256b^3 - 1024b^2 + 1024b) + 1, (t \mapsto t$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x\*\*4+1)/(x\*\*8+b\*x\*\*4+1),x)

[Out] -RootSum(\_t\*\*8\*(65536\*b\*\*4 - 524288\*b\*\*3 + 1572864\*b\*\*2 - 2097152\*b + 1048576) + \_t\*\*4\*(256\*b\*\*3 - 1024\*b\*\*2 + 1024\*b) + 1, Lambda(\_t, \_t\*log(1024\*\_t\*\*5\*b\*\*2 - 4096\*\_t\*\*5\*b + 4096\*\_t\*\*5 + 4\*\_t\*b - 4\*\_t + x)))

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4 - 1}{x^8 + bx^4 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)/(x^8+b*x^4+1),x, algorithm="giac")
```

```
[Out] integrate(-(x^4 - 1)/(x^8 + b*x^4 + 1), x)
```

### 3.21 $\int \frac{1-x^4}{1+3x^4+x^8} dx$

**Optimal.** Leaf size=411

$$\frac{\sqrt[4]{3+\sqrt{5}} \log\left(2x^2 - 2\sqrt[4]{2(3-\sqrt{5})}x + \sqrt{2(3-\sqrt{5})}\right)}{4 \cdot 2^{3/4}} + \frac{\sqrt[4]{3+\sqrt{5}} \log\left(2x^2 + 2\sqrt[4]{2(3-\sqrt{5})}x + \sqrt{2(3-\sqrt{5})}\right)}{4 \cdot 2^{3/4}} + \frac{\sqrt[4]{3-\sqrt{5}} \log\left(2x^2 - 2\sqrt[4]{2(3+\sqrt{5})}x + \sqrt{2(3+\sqrt{5})}\right)}{4 \cdot 2^{3/4}} + \frac{\sqrt[4]{3-\sqrt{5}} \log\left(2x^2 + 2\sqrt[4]{2(3+\sqrt{5})}x + \sqrt{2(3+\sqrt{5})}\right)}{4 \cdot 2^{3/4}}$$

```
[Out] -((3 + Sqrt[5])^(1/4)*ArcTan[1 - (2^(3/4)*x)/(3 - Sqrt[5])^(1/4)]/(2*2^(3/4)) + ((3 + Sqrt[5])^(1/4)*ArcTan[1 + (2^(3/4)*x)/(3 - Sqrt[5])^(1/4)]/(2*2^(3/4)) + ((3 - Sqrt[5])^(1/4)*ArcTan[1 - (2^(3/4)*x)/(3 + Sqrt[5])^(1/4)]/(2*2^(3/4)) - ((3 - Sqrt[5])^(1/4)*ArcTan[1 + (2^(3/4)*x)/(3 + Sqrt[5])^(1/4)]/(2*2^(3/4)) - ((3 + Sqrt[5])^(1/4)*Log[Sqrt[2*(3 - Sqrt[5])]] - 2*(2*(3 - Sqrt[5]))^(1/4)*x + 2*x^2))/(4*2^(3/4)) + ((3 + Sqrt[5])^(1/4)*Log[Sqrt[2*(3 - Sqrt[5])]] + 2*(2*(3 - Sqrt[5]))^(1/4)*x + 2*x^2))/(4*2^(3/4)) + ((3 - Sqrt[5])^(1/4)*Log[Sqrt[2*(3 + Sqrt[5])]] - 2*(2*(3 + Sqrt[5]))^(1/4)*x + 2*x^2))/(4*2^(3/4)) - ((3 - Sqrt[5])^(1/4)*Log[Sqrt[2*(3 + Sqrt[5])]] + 2*(2*(3 + Sqrt[5]))^(1/4)*x + 2*x^2))/(4*2^(3/4))
```

**Rubi [A]** time = 0.321287, antiderivative size = 411, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 7, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.35$ , Rules used = {1420, 211, 1165, 628, 1162, 617, 204}

$$\frac{\sqrt[4]{3+\sqrt{5}} \log\left(2x^2 - 2\sqrt[4]{2(3-\sqrt{5})}x + \sqrt{2(3-\sqrt{5})}\right)}{4 \cdot 2^{3/4}} + \frac{\sqrt[4]{3+\sqrt{5}} \log\left(2x^2 + 2\sqrt[4]{2(3-\sqrt{5})}x + \sqrt{2(3-\sqrt{5})}\right)}{4 \cdot 2^{3/4}} + \frac{\sqrt[4]{3-\sqrt{5}} \log\left(2x^2 - 2\sqrt[4]{2(3+\sqrt{5})}x + \sqrt{2(3+\sqrt{5})}\right)}{4 \cdot 2^{3/4}} + \frac{\sqrt[4]{3-\sqrt{5}} \log\left(2x^2 + 2\sqrt[4]{2(3+\sqrt{5})}x + \sqrt{2(3+\sqrt{5})}\right)}{4 \cdot 2^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^4)/(1 + 3*x^4 + x^8), x]
```

```
[Out] -((3 + Sqrt[5])^(1/4)*ArcTan[1 - (2^(3/4)*x)/(3 - Sqrt[5])^(1/4)]/(2*2^(3/4)) + ((3 + Sqrt[5])^(1/4)*ArcTan[1 + (2^(3/4)*x)/(3 - Sqrt[5])^(1/4)]/(2*2^(3/4)) + ((3 - Sqrt[5])^(1/4)*ArcTan[1 - (2^(3/4)*x)/(3 + Sqrt[5])^(1/4)]/(2*2^(3/4)) - ((3 - Sqrt[5])^(1/4)*ArcTan[1 + (2^(3/4)*x)/(3 + Sqrt[5])^(1/4)]/(2*2^(3/4)) - ((3 + Sqrt[5])^(1/4)*Log[Sqrt[2*(3 - Sqrt[5])]] - 2*(2*(3 - Sqrt[5]))^(1/4)*x + 2*x^2))/(4*2^(3/4)) + ((3 + Sqrt[5])^(1/4)*Log[Sqrt[2*(3 - Sqrt[5])]] + 2*(2*(3 - Sqrt[5]))^(1/4)*x + 2*x^2))/(4*2^(3/4)) + ((3 - Sqrt[5])^(1/4)*Log[Sqrt[2*(3 + Sqrt[5])]] - 2*(2*(3 + Sqrt[5]))^(1/4)*x + 2*x^2))/(4*2^(3/4)) - ((3 - Sqrt[5])^(1/4)*Log[Sqrt[2*(3 + Sqrt[5])]] + 2*(2*(3 + Sqrt[5]))^(1/4)*x + 2*x^2))/(4*2^(3/4))
```

$(2*(3 + \text{Sqrt}[5]))^{(1/4)*x + 2*x^2} / (4*2^{(3/4)})$

### Rule 1420

Int[((d\_) + (e\_.)\*(x\_)^(n\_))/((a\_) + (b\_.)\*(x\_)^(n\_) + (c\_.)\*(x\_)^(n2\_)), x\_Symbol] := With[{q = Rt[b^2 - 4\*a\*c, 2]}, Dist[e/2 + (2\*c\*d - b\*e)/(2\*q), Int[1/(b/2 - q/2 + c\*x^n), x], x] + Dist[e/2 - (2\*c\*d - b\*e)/(2\*q), Int[1/(b/2 + q/2 + c\*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2\*n] && NeQ[b^2 - 4\*a\*c, 0] && EqQ[c\*d^2 - a\*e^2, 0] && IGtQ[n/2, 0] && GtQ[b^2 - 4\*a\*c, 0]

### Rule 211

Int[((a\_) + (b\_.)\*(x\_)^4)^(-1), x\_Symbol] := With[{r = Numerator[Rt[a/b, 2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2\*r), Int[(r - s\*x^2)/(a + b\*x^4), x], x] + Dist[1/(2\*r), Int[(r + s\*x^2)/(a + b\*x^4), x], x]] /; FreeQ[{a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] && AtomQ[SplitProduct[SumBaseQ, b]]))

### Rule 1165

Int[((d\_) + (e\_.)\*(x\_)^2)/((a\_) + (c\_.)\*(x\_)^4), x\_Symbol] := With[{q = Rt[(-2\*d)/e, 2]}, Dist[e/(2\*c\*q), Int[(q - 2\*x)/Simp[d/e + q\*x - x^2, x], x], x] + Dist[e/(2\*c\*q), Int[(q + 2\*x)/Simp[d/e - q\*x - x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && NegQ[d\*e]

### Rule 628

Int[((d\_) + (e\_.)\*(x\_))/((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Simp[(d\*Log[RemoveContent[a + b\*x + c\*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

### Rule 1162

Int[((d\_) + (e\_.)\*(x\_)^2)/((a\_) + (c\_.)\*(x\_)^4), x\_Symbol] := With[{q = Rt[(2\*d)/e, 2]}, Dist[e/(2\*c), Int[1/Simp[d/e + q\*x + x^2, x], x], x] + Dist[e/(2\*c), Int[1/Simp[d/e - q\*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && PosQ[d\*e]

### Rule 617

Int[((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := With[{q = 1 - 4\*Simplify[(a\*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2\*c\*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4\*a\*c])] /; Free

$Q[\{a, b, c\}, x] \ \&\& \ \text{NeQ}[b^2 - 4ac, 0]$

### Rule 204

$\text{Int}[\{(a\_)+(b\_)*(x\_)^2\}^{-1}, x\_Symbol] \rightarrow -\text{Simp}[\text{ArcTan}[\text{Rt}[-b, 2]*x]/\text{Rt}[-a, 2]]/(\text{Rt}[-a, 2]*\text{Rt}[-b, 2]), x] \ /; \ \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{PosQ}[a/b] \ \&\& \ (\text{LtQ}[a, 0] \ || \ \text{LtQ}[b, 0])$

### Rubi steps

$$\begin{aligned} \int \frac{1-x^4}{1+3x^4+x^8} dx &= \frac{1}{2}(-1-\sqrt{5}) \int \frac{1}{\frac{3}{2} + \frac{\sqrt{5}}{2} + x^4} dx + \frac{1}{2}(-1+\sqrt{5}) \int \frac{1}{\frac{3}{2} - \frac{\sqrt{5}}{2} + x^4} dx \\ &= \frac{\int \frac{\sqrt{3-\sqrt{5}-\sqrt{2}x^2}}{\frac{3}{2} + \frac{\sqrt{5}}{2} + x^4} dx}{2\sqrt{2}} + \frac{\int \frac{\sqrt{3-\sqrt{5}+\sqrt{2}x^2}}{\frac{3}{2} + \frac{\sqrt{5}}{2} + x^4} dx}{2\sqrt{2}} - \frac{\int \frac{\sqrt{3+\sqrt{5}-\sqrt{2}x^2}}{\frac{3}{2} + \frac{\sqrt{5}}{2} + x^4} dx}{2\sqrt{2}} - \frac{\int \frac{\sqrt{3+\sqrt{5}+\sqrt{2}x^2}}{\frac{3}{2} + \frac{\sqrt{5}}{2} + x^4} dx}{2\sqrt{2}} \\ &= \frac{1}{4} \int \frac{1}{\sqrt{\frac{1}{2}(3-\sqrt{5})} - \sqrt[4]{2(3-\sqrt{5})}x + x^2} dx + \frac{1}{4} \int \frac{1}{\sqrt{\frac{1}{2}(3-\sqrt{5})} + \sqrt[4]{2(3-\sqrt{5})}x + x^2} dx - \frac{1}{4} \int \frac{1}{\sqrt{\frac{1}{2}(3+\sqrt{5})} - \sqrt[4]{2(3+\sqrt{5})}x + x^2} dx - \frac{1}{4} \int \frac{1}{\sqrt{\frac{1}{2}(3+\sqrt{5})} + \sqrt[4]{2(3+\sqrt{5})}x + x^2} dx \\ &= -\frac{\sqrt[4]{3+\sqrt{5}} \log\left(\sqrt{2(3-\sqrt{5})} - 2\sqrt[4]{2(3-\sqrt{5})}x + 2x^2\right)}{4 \cdot 2^{3/4}} + \frac{\sqrt[4]{3+\sqrt{5}} \log\left(\sqrt{2(3-\sqrt{5})} + 2\sqrt[4]{2(3-\sqrt{5})}x + 2x^2\right)}{4 \cdot 2^{3/4}} \\ &= -\frac{\tan^{-1}\left(1 - \frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2\sqrt[4]{2(3-\sqrt{5})}} + \frac{\tan^{-1}\left(1 + \frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2\sqrt[4]{2(3-\sqrt{5})}} + \frac{\tan^{-1}\left(1 - \frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{2\sqrt[4]{2(3+\sqrt{5})}} - \frac{\tan^{-1}\left(1 + \frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{2\sqrt[4]{2(3+\sqrt{5})}} - \frac{\sqrt[4]{3+\sqrt{5}}}{4} \end{aligned}$$

**Mathematica [C]** time = 0.0140134, size = 57, normalized size = 0.14

$$-\frac{1}{4} \text{RootSum}\left[\#1^8 + 3\#1^4 + 1 \&, \frac{\#1^4 \log(x - \#1) - \log(x - \#1)}{2\#1^7 + 3\#1^3} \&\right]$$

Antiderivative was successfully verified.

[In] Integrate[(1 - x^4)/(1 + 3\*x^4 + x^8), x]

[Out] -RootSum[1 + 3\*#1^4 + #1^8 &, (-Log[x - #1] + Log[x - #1]\*#1^4)/(3\*#1^3 + 2\*#1^7) & ]/4

---

**Maple [C]** time = 0.006, size = 44, normalized size = 0.1

$$\frac{1}{4} \sum_{_R=\text{RootOf}(_Z^8+3_Z^4+1)} \frac{(-_R^4+1)\ln(x-_R)}{2_R^7+3_R^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-x^4+1)/(x^8+3\*x^4+1),x)

[Out] 1/4\*sum((-\_R^4+1)/(2\*\_R^7+3\*\_R^3)\*ln(x-\_R),\_R=RootOf(\_Z^8+3\*\_Z^4+1))

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^4-1}{x^8+3x^4+1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^4+1)/(x^8+3\*x^4+1),x, algorithm="maxima")

[Out] -integrate((x^4 - 1)/(x^8 + 3\*x^4 + 1), x)

---

**Fricas [B]** time = 1.58495, size = 2790, normalized size = 6.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^4+1)/(x^8+3\*x^4+1),x, algorithm="fricas")

[Out] 1/16\*(sqrt(5)\*sqrt(2) - 3\*sqrt(2))\*(2\*sqrt(5) + 6)^(3/4)\*sqrt(sqrt(5) + 3)\*  
arctan(1/16\*sqrt(4\*x^2 - sqrt(2\*sqrt(5) + 6))\*(sqrt(5) - 3) + 2\*(sqrt(5)\*x -  
x)\*(2\*sqrt(5) + 6)^(1/4))\*(sqrt(5)\*sqrt(2) - 2\*sqrt(2))\*(2\*sqrt(5) + 6)^(5  
/4)\*sqrt(sqrt(5) + 3) - 1/8\*(sqrt(5)\*sqrt(2)\*x - 2\*sqrt(2)\*x)\*(2\*sqrt(5) +  
6)^(5/4)\*sqrt(sqrt(5) + 3) + 1/8\*(sqrt(5)\*sqrt(2) - 3\*sqrt(2))\*sqrt(2\*sqrt(  
5) + 6)\*sqrt(sqrt(5) + 3) + 1/16\*(sqrt(5)\*sqrt(2) - 3\*sqrt(2))\*(2\*sqrt(5)  
+ 6)^(3/4)\*sqrt(sqrt(5) + 3)\*arctan(1/16\*sqrt(4\*x^2 - sqrt(2\*sqrt(5) + 6))\*



$$\begin{aligned} & \sqrt{5} - 3 - 2(\sqrt{5}x - x)(2\sqrt{5} + 6)^{1/4}(\sqrt{5}\sqrt{2} - 2\sqrt{2})(2\sqrt{5} + 6)^{5/4}\sqrt{\sqrt{5} + 3} - 1/8(\sqrt{5}\sqrt{2}x - 2\sqrt{2}x)(2\sqrt{5} + 6)^{5/4}\sqrt{\sqrt{5} + 3} - 1/8(\sqrt{5}\sqrt{2} - 3\sqrt{2})\sqrt{2\sqrt{5} + 6}\sqrt{\sqrt{5} + 3} + 1/16(\sqrt{5}\sqrt{2} + 3\sqrt{2})\sqrt{-\sqrt{5} + 3}(-2\sqrt{5} + 6)^{3/4}\arctan(1/16\sqrt{4x^2 + (\sqrt{5} + 3)\sqrt{-2\sqrt{5} + 6}} + 2(\sqrt{5}x + x)(-2\sqrt{5} + 6)^{1/4})(\sqrt{5}\sqrt{2} + 2\sqrt{2})\sqrt{-\sqrt{5} + 3}(-2\sqrt{5} + 6)^{5/4} - 1/8((\sqrt{5}\sqrt{2}x + 2\sqrt{2}x)(-2\sqrt{5} + 6)^{5/4} + (\sqrt{5}\sqrt{2} + 3\sqrt{2})\sqrt{-2\sqrt{5} + 6})\sqrt{-\sqrt{5} + 3} + 1/16(\sqrt{5}\sqrt{2} + 3\sqrt{2})\sqrt{-\sqrt{5} + 3}(-2\sqrt{5} + 6)^{3/4}\arctan(1/16\sqrt{4x^2 + (\sqrt{5} + 3)\sqrt{-2\sqrt{5} + 6}} - 2(\sqrt{5}x + x)(-2\sqrt{5} + 6)^{1/4})(\sqrt{5}\sqrt{2} + 2\sqrt{2})\sqrt{-\sqrt{5} + 3}(-2\sqrt{5} + 6)^{5/4} - 1/8((\sqrt{5}\sqrt{2}x + 2\sqrt{2}x)(-2\sqrt{5} + 6)^{5/4} - (\sqrt{5}\sqrt{2} + 3\sqrt{2})\sqrt{-2\sqrt{5} + 6})\sqrt{-\sqrt{5} + 3} + 1/8(2\sqrt{5} + 6)^{1/4}\log(4x^2 - \sqrt{2\sqrt{5} + 6})(\sqrt{5} - 3) + 2(\sqrt{5}x - x)(2\sqrt{5} + 6)^{1/4} - 1/8(2\sqrt{5} + 6)^{1/4}\log(4x^2 - \sqrt{2\sqrt{5} + 6})(\sqrt{5} - 3) - 2(\sqrt{5}x - x)(2\sqrt{5} + 6)^{1/4} - 1/8(-2\sqrt{5} + 6)^{1/4}\log(4x^2 + (\sqrt{5} + 3)\sqrt{-2\sqrt{5} + 6}) + 2(\sqrt{5}x + x)(-2\sqrt{5} + 6)^{1/4} + 1/8(-2\sqrt{5} + 6)^{1/4}\log(4x^2 + (\sqrt{5} + 3)\sqrt{-2\sqrt{5} + 6}) - 2(\sqrt{5}x + x)(-2\sqrt{5} + 6)^{1/4} \end{aligned}$$

**Sympy [A]** time = 1.05375, size = 26, normalized size = 0.06

$$-\text{RootSum}\left(65536t^8 + 768t^4 + 1, \left(t \mapsto t \log(1024t^5 + 8t + x)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x\*\*4+1)/(x\*\*8+3\*x\*\*4+1), x)

[Out] -RootSum(65536\*\_t\*\*8 + 768\*\_t\*\*4 + 1, Lambda(\_t, \_t\*log(1024\*\_t\*\*5 + 8\*\_t + x)))

**Giac [A]** time = 1.33808, size = 320, normalized size = 0.78

$$-\frac{1}{8}(i+1)\sqrt{\sqrt{5}-1}\log\left(50(i+1)x + 50i\sqrt{\sqrt{5}+1}\right) + \frac{1}{8}(i+1)\sqrt{\sqrt{5}-1}\log\left(50(i+1)x - 50i\sqrt{\sqrt{5}+1}\right) + \frac{1}{8}(i-1)\sqrt{\sqrt{5}+1}\log\left(50(i-1)x + 50(i-1)\sqrt{\sqrt{5}-1}\right) - \frac{1}{8}(i-1)\sqrt{\sqrt{5}+1}\log\left(50(i-1)x - 50(i-1)\sqrt{\sqrt{5}-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)/(x^8+3*x^4+1),x, algorithm="giac")
```

```
[Out] -1/8*(i + 1)*sqrt(sqrt(5) - 1)*log(50*(i + 1)*x + 50*i*sqrt(sqrt(5) + 1)) +
1/8*(i + 1)*sqrt(sqrt(5) - 1)*log(50*(i + 1)*x - 50*i*sqrt(sqrt(5) + 1)) +
1/8*(i - 1)*sqrt(sqrt(5) - 1)*log(50*(i + 1)*x + 50*sqrt(sqrt(5) + 1)) -
1/8*(i - 1)*sqrt(sqrt(5) - 1)*log(50*(i + 1)*x - 50*sqrt(sqrt(5) + 1)) + 1/8
*(i + 1)*sqrt(sqrt(5) + 1)*log(34*(i + 1)*x + 34*i*sqrt(sqrt(5) - 1)) - 1/8
*(i + 1)*sqrt(sqrt(5) + 1)*log(34*(i + 1)*x - 34*i*sqrt(sqrt(5) - 1)) - 1/8
*(i - 1)*sqrt(sqrt(5) + 1)*log(34*(i + 1)*x + 34*sqrt(sqrt(5) - 1)) + 1/8*(
i - 1)*sqrt(sqrt(5) + 1)*log(34*(i + 1)*x - 34*sqrt(sqrt(5) - 1))
```

$$3.22 \quad \int \frac{1-x^4}{1+2x^4+x^8} dx$$

**Optimal.** Leaf size=97

$$\frac{x}{2(x^4+1)} - \frac{\log(x^2 - \sqrt{2}x + 1)}{8\sqrt{2}} + \frac{\log(x^2 + \sqrt{2}x + 1)}{8\sqrt{2}} - \frac{\tan^{-1}(1 - \sqrt{2}x)}{4\sqrt{2}} + \frac{\tan^{-1}(\sqrt{2}x + 1)}{4\sqrt{2}}$$

[Out] x/(2\*(1 + x^4)) - ArcTan[1 - Sqrt[2]\*x]/(4\*Sqrt[2]) + ArcTan[1 + Sqrt[2]\*x]/(4\*Sqrt[2]) - Log[1 - Sqrt[2]\*x + x^2]/(8\*Sqrt[2]) + Log[1 + Sqrt[2]\*x + x^2]/(8\*Sqrt[2])

**Rubi [A]** time = 0.0523075, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.4$ , Rules used = {28, 385, 211, 1165, 628, 1162, 617, 204}

$$\frac{x}{2(x^4+1)} - \frac{\log(x^2 - \sqrt{2}x + 1)}{8\sqrt{2}} + \frac{\log(x^2 + \sqrt{2}x + 1)}{8\sqrt{2}} - \frac{\tan^{-1}(1 - \sqrt{2}x)}{4\sqrt{2}} + \frac{\tan^{-1}(\sqrt{2}x + 1)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[(1 - x^4)/(1 + 2\*x^4 + x^8),x]

[Out] x/(2\*(1 + x^4)) - ArcTan[1 - Sqrt[2]\*x]/(4\*Sqrt[2]) + ArcTan[1 + Sqrt[2]\*x]/(4\*Sqrt[2]) - Log[1 - Sqrt[2]\*x + x^2]/(8\*Sqrt[2]) + Log[1 + Sqrt[2]\*x + x^2]/(8\*Sqrt[2])

### Rule 28

Int[(u\_)\*((a\_) + (c\_)\*(x\_)^(n2\_)) + (b\_)\*(x\_)^(n\_)]^(p\_), x\_Symbol] :> Dist[1/c^p, Int[u\*(b/2 + c\*x^n)^(2\*p), x], x] /; FreeQ[{a, b, c, n}, x] && EqQ[n2, 2\*n] && EqQ[b^2 - 4\*a\*c, 0] && IntegerQ[p]

### Rule 385

Int[((a\_) + (b\_)\*(x\_)^(n\_)]^(p\_)\*((c\_) + (d\_)\*(x\_)^(n\_)), x\_Symbol] :> -Simp[((b\*c - a\*d)\*x\*(a + b\*x^n)^(p + 1))/(a\*b\*n\*(p + 1)), x] - Dist[(a\*d - b\*c\*(n\*(p + 1) + 1))/(a\*b\*n\*(p + 1)), Int[(a + b\*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b\*c - a\*d, 0] && (LtQ[p, -1] || ILtQ[1/n + p, 0])

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] := With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1-x^4}{1+2x^4+x^8} dx &= \int \frac{1-x^4}{(1+x^4)^2} dx \\
&= \frac{x}{2(1+x^4)} + \frac{1}{2} \int \frac{1}{1+x^4} dx \\
&= \frac{x}{2(1+x^4)} + \frac{1}{4} \int \frac{1-x^2}{1+x^4} dx + \frac{1}{4} \int \frac{1+x^2}{1+x^4} dx \\
&= \frac{x}{2(1+x^4)} + \frac{1}{8} \int \frac{1}{1-\sqrt{2}x+x^2} dx + \frac{1}{8} \int \frac{1}{1+\sqrt{2}x+x^2} dx - \frac{\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2} dx}{8\sqrt{2}} - \frac{\int \frac{\sqrt{2}-2x}{-1+\sqrt{2}x-x^2} dx}{8\sqrt{2}} \\
&= \frac{x}{2(1+x^4)} - \frac{\log(1-\sqrt{2}x+x^2)}{8\sqrt{2}} + \frac{\log(1+\sqrt{2}x+x^2)}{8\sqrt{2}} + \frac{\text{Subst}\left(\int \frac{1}{-1-x^2} dx, x, 1-\sqrt{2}x\right)}{4\sqrt{2}} - \frac{\text{Subst}\left(\int \frac{1}{-1-x^2} dx, x, 1+\sqrt{2}x\right)}{4\sqrt{2}} \\
&= \frac{x}{2(1+x^4)} - \frac{\tan^{-1}(1-\sqrt{2}x)}{4\sqrt{2}} + \frac{\tan^{-1}(1+\sqrt{2}x)}{4\sqrt{2}} - \frac{\log(1-\sqrt{2}x+x^2)}{8\sqrt{2}} + \frac{\log(1+\sqrt{2}x+x^2)}{8\sqrt{2}}
\end{aligned}$$

**Mathematica [A]** time = 0.0636589, size = 90, normalized size = 0.93

$$\frac{1}{16} \left( \frac{8x}{x^4+1} - \sqrt{2} \log(x^2 - \sqrt{2}x + 1) + \sqrt{2} \log(x^2 + \sqrt{2}x + 1) - 2\sqrt{2} \tan^{-1}(1 - \sqrt{2}x) + 2\sqrt{2} \tan^{-1}(\sqrt{2}x + 1) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(1 - x^4)/(1 + 2\*x^4 + x^8), x]

[Out] ((8\*x)/(1 + x^4) - 2\*Sqrt[2]\*ArcTan[1 - Sqrt[2]\*x] + 2\*Sqrt[2]\*ArcTan[1 + Sqrt[2]\*x] - Sqrt[2]\*Log[1 - Sqrt[2]\*x + x^2] + Sqrt[2]\*Log[1 + Sqrt[2]\*x + x^2])/16

**Maple [A]** time = 0.008, size = 68, normalized size = 0.7

$$\frac{x}{2x^4+2} + \frac{\arctan(1+x\sqrt{2})\sqrt{2}}{8} + \frac{\arctan(-1+x\sqrt{2})\sqrt{2}}{8} + \frac{\sqrt{2}}{16} \ln\left(\frac{1+x^2+x\sqrt{2}}{1+x^2-x\sqrt{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-x^4+1)/(x^8+2\*x^4+1), x)

[Out]  $\frac{1}{2}x/(x^4+1)+\frac{1}{8}\arctan(1+x\sqrt{2})\sqrt{2}+\frac{1}{8}\arctan(-1+x\sqrt{2})\sqrt{2}+\frac{1}{16}\sqrt{2}\ln\left(\frac{(1+x^2+x\sqrt{2})}{(1+x^2-x\sqrt{2})}\right)$

**Maxima [A]** time = 1.48283, size = 111, normalized size = 1.14

$$\frac{1}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(2x+\sqrt{2})\right)+\frac{1}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(2x-\sqrt{2})\right)+\frac{1}{16}\sqrt{2}\log(x^2+\sqrt{2}x+1)-\frac{1}{16}\sqrt{2}\log(x^2-\sqrt{2}x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^4+1)/(x^8+2\*x^4+1),x, algorithm="maxima")

[Out]  $\frac{1}{8}\sqrt{2}\arctan(1/2\sqrt{2}(2x+\sqrt{2}))+\frac{1}{8}\sqrt{2}\arctan(1/2\sqrt{2}(2x-\sqrt{2}))+\frac{1}{16}\sqrt{2}\log(x^2+\sqrt{2}x+1)-\frac{1}{16}\sqrt{2}\log(x^2-\sqrt{2}x+1)+\frac{1}{2}x/(x^4+1)$

**Fricas [A]** time = 1.31755, size = 371, normalized size = 3.82

$$\frac{4\sqrt{2}(x^4+1)\arctan\left(-\sqrt{2}x+\sqrt{2}\sqrt{x^2+\sqrt{2}x+1}-1\right)+4\sqrt{2}(x^4+1)\arctan\left(-\sqrt{2}x+\sqrt{2}\sqrt{x^2-\sqrt{2}x+1}+1\right)-\sqrt{2}(x^4+1)\log(x^2+\sqrt{2}x+1)+\sqrt{2}(x^4+1)\log(x^2-\sqrt{2}x+1)-8x}{16(x^4+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^4+1)/(x^8+2\*x^4+1),x, algorithm="fricas")

[Out]  $-\frac{1}{16}(4\sqrt{2}(x^4+1)\arctan(-\sqrt{2}x+\sqrt{2}\sqrt{x^2+\sqrt{2}x+1}-1)+4\sqrt{2}(x^4+1)\arctan(-\sqrt{2}x+\sqrt{2}\sqrt{x^2-\sqrt{2}x+1}+1)-\sqrt{2}(x^4+1)\log(x^2+\sqrt{2}x+1)+\sqrt{2}(x^4+1)\log(x^2-\sqrt{2}x+1)-8x)/(x^4+1)$

**Sympy [A]** time = 0.170657, size = 82, normalized size = 0.85

$$\frac{x}{2x^4+2}-\frac{\sqrt{2}\log(x^2-\sqrt{2}x+1)}{16}+\frac{\sqrt{2}\log(x^2+\sqrt{2}x+1)}{16}+\frac{\sqrt{2}\operatorname{atan}(\sqrt{2}x-1)}{8}+\frac{\sqrt{2}\operatorname{atan}(\sqrt{2}x+1)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**4+1)/(x**8+2*x**4+1),x)
```

```
[Out] x/(2*x**4 + 2) - sqrt(2)*log(x**2 - sqrt(2)*x + 1)/16 + sqrt(2)*log(x**2 +
sqrt(2)*x + 1)/16 + sqrt(2)*atan(sqrt(2)*x - 1)/8 + sqrt(2)*atan(sqrt(2)*x
+ 1)/8
```

**Giac [A]** time = 1.10954, size = 111, normalized size = 1.14

$$\frac{1}{8} \sqrt{2} \arctan\left(\frac{1}{2} \sqrt{2}(2x + \sqrt{2})\right) + \frac{1}{8} \sqrt{2} \arctan\left(\frac{1}{2} \sqrt{2}(2x - \sqrt{2})\right) + \frac{1}{16} \sqrt{2} \log(x^2 + \sqrt{2}x + 1) - \frac{1}{16} \sqrt{2} \log(x^2 - \sqrt{2}x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)/(x^8+2*x^4+1),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 1/8*sqrt(2)*arctan(1/2*sq
rt(2)*(2*x - sqrt(2))) + 1/16*sqrt(2)*log(x^2 + sqrt(2)*x + 1) - 1/16*sqrt(
2)*log(x^2 - sqrt(2)*x + 1) + 1/2*x/(x^4 + 1)
```

### 3.23 $\int \frac{1-x^4}{1+x^4+x^8} dx$

**Optimal.** Leaf size=140

$$\frac{1}{8} \log(x^2 - x + 1) - \frac{1}{8} \log(x^2 + x + 1) - \frac{1}{8} \sqrt{3} \log(x^2 - \sqrt{3}x + 1) + \frac{1}{8} \sqrt{3} \log(x^2 + \sqrt{3}x + 1) - \frac{1}{4} \sqrt{3} \tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right) + \frac{1}{4}$$

[Out]  $-(\text{Sqrt}[3] \cdot \text{ArcTan}[(1 - 2x)/\text{Sqrt}[3]])/4 + \text{ArcTan}[\text{Sqrt}[3] - 2x]/4 + (\text{Sqrt}[3] \cdot \text{ArcTan}[(1 + 2x)/\text{Sqrt}[3]])/4 - \text{ArcTan}[\text{Sqrt}[3] + 2x]/4 + \text{Log}[1 - x + x^2]/8 - \text{Log}[1 + x + x^2]/8 - (\text{Sqrt}[3] \cdot \text{Log}[1 - \text{Sqrt}[3]x + x^2])/8 + (\text{Sqrt}[3] \cdot \text{Log}[1 + \text{Sqrt}[3]x + x^2])/8$

**Rubi [A]** time = 0.0987737, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 6, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {1421, 1169, 634, 618, 204, 628}

$$\frac{1}{8} \log(x^2 - x + 1) - \frac{1}{8} \log(x^2 + x + 1) - \frac{1}{8} \sqrt{3} \log(x^2 - \sqrt{3}x + 1) + \frac{1}{8} \sqrt{3} \log(x^2 + \sqrt{3}x + 1) - \frac{1}{4} \sqrt{3} \tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right) + \frac{1}{4}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(1 - x^4)/(1 + x^4 + x^8), x]$

[Out]  $-(\text{Sqrt}[3] \cdot \text{ArcTan}[(1 - 2x)/\text{Sqrt}[3]])/4 + \text{ArcTan}[\text{Sqrt}[3] - 2x]/4 + (\text{Sqrt}[3] \cdot \text{ArcTan}[(1 + 2x)/\text{Sqrt}[3]])/4 - \text{ArcTan}[\text{Sqrt}[3] + 2x]/4 + \text{Log}[1 - x + x^2]/8 - \text{Log}[1 + x + x^2]/8 - (\text{Sqrt}[3] \cdot \text{Log}[1 - \text{Sqrt}[3]x + x^2])/8 + (\text{Sqrt}[3] \cdot \text{Log}[1 + \text{Sqrt}[3]x + x^2])/8$

#### Rule 1421

$\text{Int}[(d + (e \cdot x^n)/(a + (b \cdot x^n) + (c \cdot x^{n2})), x\_Symbol] :> \text{With}[\{q = \text{Rt}[(-2d)/e - b/c, 2]\}, \text{Dist}[e/(2c \cdot q), \text{Int}[(q - 2x^{(n/2)})/\text{Simp}[d/e + q \cdot x^{(n/2)} - x^n, x], x], x] + \text{Dist}[e/(2c \cdot q), \text{Int}[(q + 2x^{(n/2)})/\text{Simp}[d/e - q \cdot x^{(n/2)} - x^n, x], x], x]] /; \text{FreeQ}\{a, b, c, d, e, x\} \&\& \text{EqQ}[n2, 2n] \&\& \text{NeQ}[b^2 - 4a \cdot c, 0] \&\& \text{EqQ}[c \cdot d^2 - a \cdot e^2, 0] \&\& \text{IGtQ}[n/2, 0] \&\& !\text{GtQ}[b^2 - 4a \cdot c, 0]$

#### Rule 1169

$\text{Int}[(d + (e \cdot x^2)/(a + (b \cdot x^2) + (c \cdot x^4)), x\_Symbol] :> \text{With}[\{q = \text{Rt}[a/c, 2]\}, \text{With}[\{r = \text{Rt}[2q - b/c, 2]\}, \text{Dist}[1/(2c \cdot q \cdot r), \text{Int}$



```
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
(d - e*q)*x)/(q + r*x + x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

### Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

### Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

### Rule 204

```
Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

### Rule 628

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

### Rubi steps

$$\begin{aligned}
\int \frac{1-x^4}{1+x^4+x^8} dx &= -\left(\frac{1}{2} \int \frac{1+2x^2}{-1-x^2-x^4} dx\right) - \frac{1}{2} \int \frac{1-2x^2}{-1+x^2-x^4} dx \\
&= \frac{1}{4} \int \frac{1+x}{1-x+x^2} dx + \frac{1}{4} \int \frac{1-x}{1+x+x^2} dx + \frac{\int \frac{\sqrt{3}-3x}{1-\sqrt{3}x+x^2} dx}{4\sqrt{3}} + \frac{\int \frac{\sqrt{3}+3x}{1+\sqrt{3}x+x^2} dx}{4\sqrt{3}} \\
&= \frac{1}{8} \int \frac{-1+2x}{1-x+x^2} dx - \frac{1}{8} \int \frac{1+2x}{1+x+x^2} dx - \frac{1}{8} \int \frac{1}{1-\sqrt{3}x+x^2} dx - \frac{1}{8} \int \frac{1}{1+\sqrt{3}x+x^2} dx + \frac{3}{8} \int \frac{1}{1-x^2} dx \\
&= \frac{1}{8} \log(1-x+x^2) - \frac{1}{8} \log(1+x+x^2) - \frac{1}{8} \sqrt{3} \log(1-\sqrt{3}x+x^2) + \frac{1}{8} \sqrt{3} \log(1+\sqrt{3}x+x^2) + \frac{1}{4} \operatorname{Su} \\
&= -\frac{1}{4} \sqrt{3} \tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right) + \frac{1}{4} \tan^{-1}(\sqrt{3}-2x) + \frac{1}{4} \sqrt{3} \tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right) - \frac{1}{4} \tan^{-1}(\sqrt{3}+2x) + \frac{1}{8} \log(1-x^2)
\end{aligned}$$

**Mathematica [C]** time = 0.177305, size = 129, normalized size = 0.92

$$\frac{1}{8} \left( \log(x^2-x+1) - \log(x^2+x+1) - 2\sqrt{-2-2i\sqrt{3}} \tan^{-1}\left(\frac{1-i\sqrt{3}}{2}x\right) - 2\sqrt{-2+2i\sqrt{3}} \tan^{-1}\left(\frac{1+i\sqrt{3}}{2}x\right) + 2\sqrt{1-x^2} \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[(1 - x^4)/(1 + x^4 + x^8), x]

[Out] (-2\*Sqrt[-2 - (2\*I)\*Sqrt[3]]\*ArcTan[((1 - I\*Sqrt[3])\*x)/2] - 2\*Sqrt[-2 + (2\*I)\*Sqrt[3]]\*ArcTan[((1 + I\*Sqrt[3])\*x)/2] + 2\*Sqrt[3]\*ArcTan[(-1 + 2\*x)/Sqrt[3]] + 2\*Sqrt[3]\*ArcTan[(1 + 2\*x)/Sqrt[3]] + Log[1 - x + x^2] - Log[1 + x + x^2])/8

**Maple [A]** time = 0.012, size = 109, normalized size = 0.8

$$-\frac{\ln(x^2+x+1)}{8} + \frac{\sqrt{3}}{4} \arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right) + \frac{\ln(x^2-x+1)}{8} + \frac{\sqrt{3}}{4} \arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right) + \frac{\ln(1+x^2+x\sqrt{3})\sqrt{3}}{8} - \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-x^4+1)/(x^8+x^4+1), x)

[Out] -1/8\*ln(x^2+x+1)+1/4\*arctan(1/3\*(1+2\*x)\*3^(1/2))\*3^(1/2)+1/8\*ln(x^2-x+1)+1/4\*3^(1/2)\*arctan(1/3\*(2\*x-1)\*3^(1/2))+1/8\*ln(1+x^2+x\*3^(1/2))\*3^(1/2)-1/4\*a

$\text{rctan}(2*x+3^{(1/2)})-1/8*\ln(1+x^2-x*3^{(1/2)})*3^{(1/2)}-1/4*\arctan(2*x-3^{(1/2)})$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{1}{4}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{1}{4}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-\frac{1}{2}\int\frac{2x^2-1}{x^4-x^2+1}dx-\frac{1}{8}\log(x^2+x+1)+\frac{1}{8}\log(x^2-x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^4+1)/(x^8+x^4+1),x, algorithm="maxima")

[Out] 1/4\*sqrt(3)\*arctan(1/3\*sqrt(3)\*(2\*x + 1)) + 1/4\*sqrt(3)\*arctan(1/3\*sqrt(3)\*(2\*x - 1)) - 1/2\*integrate((2\*x^2 - 1)/(x^4 - x^2 + 1), x) - 1/8\*log(x^2 + x + 1) + 1/8\*log(x^2 - x + 1)

**Fricas [A]** time = 1.36462, size = 444, normalized size = 3.17

$$\frac{1}{4}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{1}{4}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{1}{8}\sqrt{3}\log(x^2+\sqrt{3}x+1)-\frac{1}{8}\sqrt{3}\log(x^2-\sqrt{3}x+1)+$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^4+1)/(x^8+x^4+1),x, algorithm="fricas")

[Out] 1/4\*sqrt(3)\*arctan(1/3\*sqrt(3)\*(2\*x + 1)) + 1/4\*sqrt(3)\*arctan(1/3\*sqrt(3)\*(2\*x - 1)) + 1/8\*sqrt(3)\*log(x^2 + sqrt(3)\*x + 1) - 1/8\*sqrt(3)\*log(x^2 - sqrt(3)\*x + 1) + 1/2\*arctan(-2\*x + sqrt(3) + 2\*sqrt(x^2 - sqrt(3)\*x + 1)) + 1/2\*arctan(-2\*x - sqrt(3) + 2\*sqrt(x^2 + sqrt(3)\*x + 1)) - 1/8\*log(x^2 + x + 1) + 1/8\*log(x^2 - x + 1)

**Sympy [C]** time = 0.612263, size = 148, normalized size = 1.06

$$-\left(-\frac{1}{8}-\frac{\sqrt{3}i}{8}\right)\log\left(x+1024\left(-\frac{1}{8}-\frac{\sqrt{3}i}{8}\right)^5\right)-\left(-\frac{1}{8}+\frac{\sqrt{3}i}{8}\right)\log\left(x+1024\left(-\frac{1}{8}+\frac{\sqrt{3}i}{8}\right)^5\right)-\left(\frac{1}{8}-\frac{\sqrt{3}i}{8}\right)\log\left(x+1024\left(\frac{1}{8}-\frac{\sqrt{3}i}{8}\right)^5\right)-\left(\frac{1}{8}+\frac{\sqrt{3}i}{8}\right)\log\left(x+1024\left(\frac{1}{8}+\frac{\sqrt{3}i}{8}\right)^5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**4+1)/(x**8+x**4+1),x)
```

```
[Out] -(-1/8 - sqrt(3)*I/8)*log(x + 1024*(-1/8 - sqrt(3)*I/8)**5) - (-1/8 + sqrt(
3)*I/8)*log(x + 1024*(-1/8 + sqrt(3)*I/8)**5) - (1/8 - sqrt(3)*I/8)*log(x +
1024*(1/8 - sqrt(3)*I/8)**5) - (1/8 + sqrt(3)*I/8)*log(x + 1024*(1/8 + sqrt
t(3)*I/8)**5) - RootSum(256*_t**4 - 16*_t**2 + 1, Lambda(_t, _t*log(1024*_t
**5 + x)))
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int -\frac{x^4 - 1}{x^8 + x^4 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)/(x^8+x^4+1),x, algorithm="giac")
```

```
[Out] integrate(-(x^4 - 1)/(x^8 + x^4 + 1), x)
```

$$3.24 \quad \int \frac{1-x^4}{1+x^8} dx$$

**Optimal.** Leaf size=347

$$\frac{1}{8}\sqrt{\frac{1}{2}(2-\sqrt{2})}\log\left(x^2-\sqrt{2-\sqrt{2}}x+1\right)-\frac{1}{8}\sqrt{\frac{1}{2}(2-\sqrt{2})}\log\left(x^2+\sqrt{2-\sqrt{2}}x+1\right)-\frac{1}{8}\sqrt{\frac{1}{2}(2+\sqrt{2})}\log\left(x^2-\sqrt{2+\sqrt{2}}x+1\right)+\frac{1}{8}\sqrt{\frac{1}{2}(2+\sqrt{2})}\log\left(x^2+\sqrt{2+\sqrt{2}}x+1\right)$$

```
[Out] -ArcTan[(Sqrt[2 - Sqrt[2]] - 2*x)/Sqrt[2 + Sqrt[2]]]/(4*Sqrt[2 - Sqrt[2]])
+ ArcTan[(Sqrt[2 + Sqrt[2]] - 2*x)/Sqrt[2 - Sqrt[2]]]/(4*Sqrt[2 + Sqrt[2]])
+ ArcTan[(Sqrt[2 - Sqrt[2]] + 2*x)/Sqrt[2 + Sqrt[2]]]/(4*Sqrt[2 - Sqrt[2]])
) - ArcTan[(Sqrt[2 + Sqrt[2]] + 2*x)/Sqrt[2 - Sqrt[2]]]/(4*Sqrt[2 + Sqrt[2]])
+ (Sqrt[(2 - Sqrt[2])/2]*Log[1 - Sqrt[2 - Sqrt[2]]*x + x^2])/8 - (Sqrt[(2 - Sqrt[2])/2]*Log[1 + Sqrt[2 - Sqrt[2]]*x + x^2])/8 - (Sqrt[(2 + Sqrt[2])/2]*Log[1 - Sqrt[2 + Sqrt[2]]*x + x^2])/8 + (Sqrt[(2 + Sqrt[2])/2]*Log[1 + Sqrt[2 + Sqrt[2]]*x + x^2])/8
```

---

**Rubi [A]** time = 0.270287, antiderivative size = 347, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 6, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.4$ , Rules used = {1414, 1169, 634, 618, 204, 628}

$$\frac{1}{8}\sqrt{\frac{1}{2}(2-\sqrt{2})}\log\left(x^2-\sqrt{2-\sqrt{2}}x+1\right)-\frac{1}{8}\sqrt{\frac{1}{2}(2-\sqrt{2})}\log\left(x^2+\sqrt{2-\sqrt{2}}x+1\right)-\frac{1}{8}\sqrt{\frac{1}{2}(2+\sqrt{2})}\log\left(x^2-\sqrt{2+\sqrt{2}}x+1\right)+\frac{1}{8}\sqrt{\frac{1}{2}(2+\sqrt{2})}\log\left(x^2+\sqrt{2+\sqrt{2}}x+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^4)/(1 + x^8), x]
```

```
[Out] -ArcTan[(Sqrt[2 - Sqrt[2]] - 2*x)/Sqrt[2 + Sqrt[2]]]/(4*Sqrt[2 - Sqrt[2]])
+ ArcTan[(Sqrt[2 + Sqrt[2]] - 2*x)/Sqrt[2 - Sqrt[2]]]/(4*Sqrt[2 + Sqrt[2]])
+ ArcTan[(Sqrt[2 - Sqrt[2]] + 2*x)/Sqrt[2 + Sqrt[2]]]/(4*Sqrt[2 - Sqrt[2]])
) - ArcTan[(Sqrt[2 + Sqrt[2]] + 2*x)/Sqrt[2 - Sqrt[2]]]/(4*Sqrt[2 + Sqrt[2]])
+ (Sqrt[(2 - Sqrt[2])/2]*Log[1 - Sqrt[2 - Sqrt[2]]*x + x^2])/8 - (Sqrt[(2 - Sqrt[2])/2]*Log[1 + Sqrt[2 - Sqrt[2]]*x + x^2])/8 - (Sqrt[(2 + Sqrt[2])/2]*Log[1 - Sqrt[2 + Sqrt[2]]*x + x^2])/8 + (Sqrt[(2 + Sqrt[2])/2]*Log[1 + Sqrt[2 + Sqrt[2]]*x + x^2])/8
```

Rule 1414

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (c_.)*(x_)^(n2_)), x_Symbol] := With[{
q = Rt[-2*d*e, 2]}, Dist[d/(2*a), Int[(d - q*x^(n/2))/(d - q*x^(n/2) - e*x^
n), x], x] + Dist[d/(2*a), Int[(d + q*x^(n/2))/(d + q*x^(n/2) - e*x^n), x],
x]] /; FreeQ[{a, c, d, e}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 - a*e^2, 0] &&
IGtQ[n/2, 0] && NegQ[d*e]
```

### Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
(d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

### Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

### Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[In
t[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

### Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

### Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

### Rubi steps

$$\begin{aligned}
\int \frac{1-x^4}{1+x^8} dx &= \frac{1}{2} \int \frac{1-\sqrt{2}x^2}{1-\sqrt{2}x^2+x^4} dx + \frac{1}{2} \int \frac{1+\sqrt{2}x^2}{1+\sqrt{2}x^2+x^4} dx \\
&= \frac{\int \frac{\sqrt{2-\sqrt{2}}-(1-\sqrt{2})x}{1-\sqrt{2-\sqrt{2}x+x^2}} dx}{4\sqrt{2-\sqrt{2}}} + \frac{\int \frac{\sqrt{2-\sqrt{2}}+(1-\sqrt{2})x}{1+\sqrt{2-\sqrt{2}x+x^2}} dx}{4\sqrt{2-\sqrt{2}}} + \frac{\int \frac{\sqrt{2+\sqrt{2}}-(1+\sqrt{2})x}{1-\sqrt{2+\sqrt{2}x+x^2}} dx}{4\sqrt{2+\sqrt{2}}} + \frac{\int \frac{\sqrt{2+\sqrt{2}}+(1+\sqrt{2})x}{1+\sqrt{2+\sqrt{2}x+x^2}} dx}{4\sqrt{2+\sqrt{2}}} \\
&= -\left(\frac{1}{8}\sqrt{3-2\sqrt{2}} \int \frac{1}{1-\sqrt{2+\sqrt{2}x+x^2}} dx\right) - \frac{1}{8}\sqrt{3-2\sqrt{2}} \int \frac{1}{1+\sqrt{2+\sqrt{2}x+x^2}} dx + \frac{(1-\sqrt{2}) \int \frac{\sqrt{2-\sqrt{2}}}{1+\sqrt{2-\sqrt{2}x+x^2}} dx}{8\sqrt{2-\sqrt{2}}} \\
&= \frac{1}{8}\sqrt{1-\frac{1}{\sqrt{2}}} \log\left(1-\sqrt{2-\sqrt{2}x+x^2}\right) - \frac{1}{8}\sqrt{1-\frac{1}{\sqrt{2}}} \log\left(1+\sqrt{2-\sqrt{2}x+x^2}\right) - \frac{1}{8}\sqrt{1+\frac{1}{\sqrt{2}}} \log\left(1-\sqrt{2+\sqrt{2}x+x^2}\right) \\
&= -\frac{1}{4}\sqrt{\frac{1}{2}(2+\sqrt{2})} \tan^{-1}\left(\frac{\sqrt{2-\sqrt{2}}-2x}{\sqrt{2+\sqrt{2}}}\right) + \frac{1}{4}\sqrt{\frac{1}{2}(2-\sqrt{2})} \tan^{-1}\left(\frac{\sqrt{2+\sqrt{2}}-2x}{\sqrt{2-\sqrt{2}}}\right) + \frac{1}{4}\sqrt{\frac{1}{2}(2+\sqrt{2})} \tan^{-1}\left(\frac{\sqrt{2-\sqrt{2}}-2x}{\sqrt{2+\sqrt{2}}}\right)
\end{aligned}$$

**Mathematica [A]** time = 0.164174, size = 257, normalized size = 0.74

$$\frac{1}{8} \left( -\left(\sin\left(\frac{\pi}{8}\right) + \cos\left(\frac{\pi}{8}\right)\right) \log\left(x^2 - 2x \cos\left(\frac{\pi}{8}\right) + 1\right) + \left(\sin\left(\frac{\pi}{8}\right) + \cos\left(\frac{\pi}{8}\right)\right) \log\left(x^2 + 2x \cos\left(\frac{\pi}{8}\right) + 1\right) + \left(\sin\left(\frac{\pi}{8}\right) - \cos\left(\frac{\pi}{8}\right)\right) \log\left(x^2 - 2x \cos\left(\frac{\pi}{8}\right) + 1\right) + \left(\sin\left(\frac{\pi}{8}\right) - \cos\left(\frac{\pi}{8}\right)\right) \log\left(x^2 + 2x \cos\left(\frac{\pi}{8}\right) + 1\right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(1 - x^4)/(1 + x^8), x]

[Out] (2\*ArcTan[Cot[Pi/8] - x\*Csc[Pi/8]]\*(Cos[Pi/8] - Sin[Pi/8]) + Log[1 + x^2 - 2\*x\*Sin[Pi/8]]\*(Cos[Pi/8] - Sin[Pi/8]) + 2\*ArcTan[(x + Cos[Pi/8])\*Csc[Pi/8]]\*(-Cos[Pi/8] + Sin[Pi/8]) + Log[1 + x^2 + 2\*x\*Sin[Pi/8]]\*(-Cos[Pi/8] + Sin[Pi/8]) + 2\*ArcTan[Sec[Pi/8]\*(x + Sin[Pi/8])]\*(Cos[Pi/8] + Sin[Pi/8]) + 2\*ArcTan[x\*Sec[Pi/8] - Tan[Pi/8]]\*(Cos[Pi/8] + Sin[Pi/8]) - Log[1 + x^2 - 2\*x\*Cos[Pi/8]]\*(Cos[Pi/8] + Sin[Pi/8]) + Log[1 + x^2 + 2\*x\*Cos[Pi/8]]\*(Cos[Pi/8] + Sin[Pi/8]))/8

**Maple [C]** time = 0.006, size = 29, normalized size = 0.1

$$\frac{1}{8} \sum_{R=\text{RootOf}(-Z^8+1)} \frac{(-R^4+1) \ln(x-R)}{-R^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^4+1)/(x^8+1),x)
```

```
[Out] 1/8*sum((-_R^4+1)/_R^7*ln(x-_R),_R=RootOf(_Z^8+1))
```

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^4 - 1}{x^8 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)/(x^8+1),x, algorithm="maxima")
```

```
[Out] -integrate((x^4 - 1)/(x^8 + 1), x)
```

**Fricas [B]** time = 1.46893, size = 3092, normalized size = 8.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)/(x^8+1),x, algorithm="fricas")
```

```
[Out] -1/8*(sqrt(sqrt(2) + 2) + sqrt(-sqrt(2) + 2))*arctan(-(2*x - 2*sqrt(x^2 + x
*sqrt(-sqrt(2) + 2) + 1) + sqrt(-sqrt(2) + 2))/sqrt(sqrt(2) + 2)) - 1/8*(sq
rt(sqrt(2) + 2) + sqrt(-sqrt(2) + 2))*arctan(-(2*x - 2*sqrt(x^2 - x*sqrt(-s
qrt(2) + 2) + 1) - sqrt(-sqrt(2) + 2))/sqrt(sqrt(2) + 2)) + 1/8*(sqrt(sqrt(
2) + 2) - sqrt(-sqrt(2) + 2))*arctan(-(2*x - 2*sqrt(x^2 + x*sqrt(sqrt(2) +
2) + 1) + sqrt(sqrt(2) + 2))/sqrt(-sqrt(2) + 2)) + 1/8*(sqrt(sqrt(2) + 2) -
sqrt(-sqrt(2) + 2))*arctan(-(2*x - 2*sqrt(x^2 - x*sqrt(sqrt(2) + 2) + 1) -
sqrt(sqrt(2) + 2))/sqrt(-sqrt(2) + 2)) - 1/8*sqrt(2)*sqrt(sqrt(2) + 2)*arc
tan(-(2*sqrt(2)*x - 2*sqrt(2)*sqrt(x^2 + 1/2*sqrt(2)*x*sqrt(sqrt(2) + 2) -
1/2*sqrt(2)*x*sqrt(-sqrt(2) + 2) + 1) + sqrt(sqrt(2) + 2) - sqrt(-sqrt(2) +
2))/(sqrt(sqrt(2) + 2) + sqrt(-sqrt(2) + 2))) - 1/8*sqrt(2)*sqrt(sqrt(2) +
2)*arctan(-(2*sqrt(2)*x - 2*sqrt(2)*sqrt(x^2 - 1/2*sqrt(2)*x*sqrt(sqrt(2)
+ 2) + 1/2*sqrt(2)*x*sqrt(-sqrt(2) + 2) + 1) - sqrt(sqrt(2) + 2) + sqrt(-sq
rt(2) + 2))/(sqrt(sqrt(2) + 2) + sqrt(-sqrt(2) + 2))) - 1/8*sqrt(2)*sqrt(-s
```



```

qrt(2) + 2)*arctan((2*sqrt(2)*x - 2*sqrt(2)*sqrt(x^2 + 1/2*sqrt(2)*x*sqrt(s
qrt(2) + 2) + 1/2*sqrt(2)*x*sqrt(-sqrt(2) + 2) + 1) + sqrt(sqrt(2) + 2) + s
qrt(-sqrt(2) + 2))/(sqrt(sqrt(2) + 2) - sqrt(-sqrt(2) + 2))) - 1/8*sqrt(2)*
sqrt(-sqrt(2) + 2)*arctan((2*sqrt(2)*x - 2*sqrt(2)*sqrt(x^2 - 1/2*sqrt(2)*x
*sqrt(sqrt(2) + 2) - 1/2*sqrt(2)*x*sqrt(-sqrt(2) + 2) + 1) - sqrt(sqrt(2) +
2) - sqrt(-sqrt(2) + 2))/(sqrt(sqrt(2) + 2) - sqrt(-sqrt(2) + 2))) + 1/32*
sqrt(2)*sqrt(sqrt(2) + 2)*log(x^2 + 1/2*sqrt(2)*x*sqrt(sqrt(2) + 2) + 1/2*s
qrt(2)*x*sqrt(-sqrt(2) + 2) + 1) - 1/32*sqrt(2)*sqrt(-sqrt(2) + 2)*log(x^2
+ 1/2*sqrt(2)*x*sqrt(sqrt(2) + 2) - 1/2*sqrt(2)*x*sqrt(-sqrt(2) + 2) + 1) +
1/32*sqrt(2)*sqrt(-sqrt(2) + 2)*log(x^2 - 1/2*sqrt(2)*x*sqrt(sqrt(2) + 2)
+ 1/2*sqrt(2)*x*sqrt(-sqrt(2) + 2) + 1) - 1/32*sqrt(2)*sqrt(sqrt(2) + 2)*lo
g(x^2 - 1/2*sqrt(2)*x*sqrt(sqrt(2) + 2) - 1/2*sqrt(2)*x*sqrt(-sqrt(2) + 2)
+ 1) + 1/32*(sqrt(sqrt(2) + 2) + sqrt(-sqrt(2) + 2))*log(x^2 + x*sqrt(sqrt(
2) + 2) + 1) - 1/32*(sqrt(sqrt(2) + 2) + sqrt(-sqrt(2) + 2))*log(x^2 - x*sq
rt(sqrt(2) + 2) + 1) - 1/32*(sqrt(sqrt(2) + 2) - sqrt(-sqrt(2) + 2))*log(x^
2 + x*sqrt(-sqrt(2) + 2) + 1) + 1/32*(sqrt(sqrt(2) + 2) - sqrt(-sqrt(2) + 2
))*log(x^2 - x*sqrt(-sqrt(2) + 2) + 1)

```

**Sympy [A]** time = 1.12129, size = 20, normalized size = 0.06

$$-\text{RootSum}\left(1048576t^8 + 1, \left(t \mapsto t \log(4096t^5 - 4t + x)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x\*\*4+1)/(x\*\*8+1),x)

[Out] -RootSum(1048576\*\_t\*\*8 + 1, Lambda(\_t, \_t\*log(4096\*\_t\*\*5 - 4\*\_t + x)))

**Giac [A]** time = 1.19706, size = 333, normalized size = 0.96

$$\frac{1}{8}\sqrt{2\sqrt{2}+4}\arctan\left(\frac{2x+\sqrt{-\sqrt{2}+2}}{\sqrt{\sqrt{2}+2}}\right)+\frac{1}{8}\sqrt{2\sqrt{2}+4}\arctan\left(\frac{2x-\sqrt{-\sqrt{2}+2}}{\sqrt{\sqrt{2}+2}}\right)-\frac{1}{8}\sqrt{-2\sqrt{2}+4}\arctan\left(\frac{2x+\sqrt{\sqrt{2}-2}}{\sqrt{-\sqrt{2}-2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^4+1)/(x^8+1),x, algorithm="giac")

[Out] 1/8\*sqrt(2\*sqrt(2) + 4)\*arctan((2\*x + sqrt(-sqrt(2) + 2))/sqrt(sqrt(2) + 2)) + 1/8\*sqrt(2\*sqrt(2) + 4)\*arctan((2\*x - sqrt(-sqrt(2) + 2))/sqrt(sqrt(2) + 4)) - 1/8\*sqrt(-2\*sqrt(2) + 4)\*arctan((2\*x + sqrt(sqrt(2) - 2))/sqrt(-sqrt(2) - 2))

$$\begin{aligned}
& + 2)) - 1/8*\sqrt{-2*\sqrt{2} + 4}*\arctan((2*x + \sqrt{\sqrt{2} + 2})/\sqrt{-\sqrt{2} + 2}) - 1/8*\sqrt{-2*\sqrt{2} + 4}*\arctan((2*x - \sqrt{\sqrt{2} + 2})/\sqrt{-\sqrt{2} + 2}) + 1/16*\sqrt{2*\sqrt{2} + 4}*\log(x^2 + x*\sqrt{\sqrt{2} + 2} + 1) - 1/16*\sqrt{2*\sqrt{2} + 4}*\log(x^2 - x*\sqrt{\sqrt{2} + 2} + 1) - 1/16*\sqrt{-2*\sqrt{2} + 4}*\log(x^2 + x*\sqrt{-\sqrt{2} + 2} + 1) + 1/16*\sqrt{-2*\sqrt{2} + 4}*\log(x^2 - x*\sqrt{-\sqrt{2} + 2} + 1)
\end{aligned}$$

$$3.25 \quad \int \frac{1-x^4}{1-x^4+x^8} dx$$

**Optimal.** Leaf size=355

$$\frac{1}{8}\sqrt{\frac{1}{3}(2-\sqrt{3})}\log\left(x^2-\sqrt{2-\sqrt{3}}x+1\right)-\frac{1}{8}\sqrt{\frac{1}{3}(2-\sqrt{3})}\log\left(x^2+\sqrt{2-\sqrt{3}}x+1\right)-\frac{1}{8}\sqrt{\frac{1}{3}(2+\sqrt{3})}\log\left(x^2-\sqrt{2+\sqrt{3}}x+1\right)+\frac{1}{8}\sqrt{\frac{1}{3}(2+\sqrt{3})}\log\left(x^2+\sqrt{2+\sqrt{3}}x+1\right)$$

[Out] -ArcTan[(Sqrt[2 - Sqrt[3]] - 2\*x)/Sqrt[2 + Sqrt[3]]]/(4\*Sqrt[3\*(2 - Sqrt[3])]) + ArcTan[(Sqrt[2 + Sqrt[3]] - 2\*x)/Sqrt[2 - Sqrt[3]]]/(4\*Sqrt[3\*(2 + Sqrt[3])]) + ArcTan[(Sqrt[2 - Sqrt[3]] + 2\*x)/Sqrt[2 + Sqrt[3]]]/(4\*Sqrt[3\*(2 - Sqrt[3])]) - ArcTan[(Sqrt[2 + Sqrt[3]] + 2\*x)/Sqrt[2 - Sqrt[3]]]/(4\*Sqrt[3\*(2 + Sqrt[3])]) + (Sqrt[(2 - Sqrt[3])/3]\*Log[1 - Sqrt[2 - Sqrt[3]]\*x + x^2])/8 - (Sqrt[(2 - Sqrt[3])/3]\*Log[1 + Sqrt[2 - Sqrt[3]]\*x + x^2])/8 - (Sqrt[(2 + Sqrt[3])/3]\*Log[1 - Sqrt[2 + Sqrt[3]]\*x + x^2])/8 + (Sqrt[(2 + Sqrt[3])/3]\*Log[1 + Sqrt[2 + Sqrt[3]]\*x + x^2])/8

---

**Rubi [A]** time = 0.277298, antiderivative size = 355, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 6, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.3$ , Rules used = {1421, 1169, 634, 618, 204, 628}

$$\frac{1}{8}\sqrt{\frac{1}{3}(2-\sqrt{3})}\log\left(x^2-\sqrt{2-\sqrt{3}}x+1\right)-\frac{1}{8}\sqrt{\frac{1}{3}(2-\sqrt{3})}\log\left(x^2+\sqrt{2-\sqrt{3}}x+1\right)-\frac{1}{8}\sqrt{\frac{1}{3}(2+\sqrt{3})}\log\left(x^2-\sqrt{2+\sqrt{3}}x+1\right)+\frac{1}{8}\sqrt{\frac{1}{3}(2+\sqrt{3})}\log\left(x^2+\sqrt{2+\sqrt{3}}x+1\right)$$

Antiderivative was successfully verified.

[In] Int[(1 - x^4)/(1 - x^4 + x^8), x]

[Out] -ArcTan[(Sqrt[2 - Sqrt[3]] - 2\*x)/Sqrt[2 + Sqrt[3]]]/(4\*Sqrt[3\*(2 - Sqrt[3])]) + ArcTan[(Sqrt[2 + Sqrt[3]] - 2\*x)/Sqrt[2 - Sqrt[3]]]/(4\*Sqrt[3\*(2 + Sqrt[3])]) + ArcTan[(Sqrt[2 - Sqrt[3]] + 2\*x)/Sqrt[2 + Sqrt[3]]]/(4\*Sqrt[3\*(2 - Sqrt[3])]) - ArcTan[(Sqrt[2 + Sqrt[3]] + 2\*x)/Sqrt[2 - Sqrt[3]]]/(4\*Sqrt[3\*(2 + Sqrt[3])]) + (Sqrt[(2 - Sqrt[3])/3]\*Log[1 - Sqrt[2 - Sqrt[3]]\*x + x^2])/8 - (Sqrt[(2 - Sqrt[3])/3]\*Log[1 + Sqrt[2 - Sqrt[3]]\*x + x^2])/8 - (Sqrt[(2 + Sqrt[3])/3]\*Log[1 - Sqrt[2 + Sqrt[3]]\*x + x^2])/8 + (Sqrt[(2 + Sqrt[3])/3]\*Log[1 + Sqrt[2 + Sqrt[3]]\*x + x^2])/8

Rule 1421

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] := With[{q = Rt[(-2*d)/e - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x^(
n/2))/Simp[d/e + q*x^(n/2) - x^n, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*
x^(n/2))/Simp[d/e - q*x^(n/2) - x^n, x], x], x]] /; FreeQ[{a, b, c, d, e},
x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && IGtQ[
n/2, 0] && !GtQ[b^2 - 4*a*c, 0]
```

### Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
(d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

### Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

### Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

### Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

### Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

### Rubi steps

$$\begin{aligned}
\int \frac{1-x^4}{1-x^4+x^8} dx &= -\frac{\int \frac{\sqrt{3}+2x^2}{-1-\sqrt{3}x^2-x^4} dx}{2\sqrt{3}} - \frac{\int \frac{\sqrt{3}-2x^2}{-1+\sqrt{3}x^2-x^4} dx}{2\sqrt{3}} \\
&= \frac{\int \frac{\sqrt{3(2-\sqrt{3})-(-2+\sqrt{3})x}}{1-\sqrt{2-\sqrt{3}x+x^2}} dx}{4\sqrt{3}(2-\sqrt{3})} + \frac{\int \frac{\sqrt{3(2-\sqrt{3})+(-2+\sqrt{3})x}}{1+\sqrt{2-\sqrt{3}x+x^2}} dx}{4\sqrt{3}(2-\sqrt{3})} \\
&\quad + \frac{\int \frac{\sqrt{3(2+\sqrt{3})-(-2+\sqrt{3})x}}{1-\sqrt{2+\sqrt{3}x+x^2}} dx}{4\sqrt{3}(2+\sqrt{3})} + \frac{\int \frac{\sqrt{3(2+\sqrt{3})+(-2+\sqrt{3})x}}{1+\sqrt{2+\sqrt{3}x+x^2}} dx}{4\sqrt{3}(2+\sqrt{3})} \\
&= -\left( \frac{1}{8}\sqrt{\frac{1}{3}}(7-4\sqrt{3}) \int \frac{1}{1-\sqrt{2+\sqrt{3}x+x^2}} dx \right) - \frac{1}{8}\sqrt{\frac{1}{3}}(7-4\sqrt{3}) \int \frac{1}{1+\sqrt{2+\sqrt{3}x+x^2}} dx + \frac{1}{8}\sqrt{\frac{1}{3}}(7-4\sqrt{3}) \int \frac{1}{1-\sqrt{2-\sqrt{3}x+x^2}} dx \\
&\quad + \frac{1}{8}\sqrt{\frac{1}{3}}(7-4\sqrt{3}) \int \frac{1}{1+\sqrt{2-\sqrt{3}x+x^2}} dx \\
&= \frac{1}{8}\sqrt{\frac{1}{3}}(2-\sqrt{3}) \log\left(1-\sqrt{2-\sqrt{3}x+x^2}\right) - \frac{1}{8}\sqrt{\frac{2}{3}-\frac{1}{\sqrt{3}}} \log\left(1+\sqrt{2-\sqrt{3}x+x^2}\right) - \frac{1}{8}\sqrt{\frac{1}{3}}(2+\sqrt{3}) \log\left(1-\sqrt{2+\sqrt{3}x+x^2}\right) \\
&\quad + \frac{1}{8}\sqrt{\frac{2}{3}-\frac{1}{\sqrt{3}}} \log\left(1+\sqrt{2+\sqrt{3}x+x^2}\right) \\
&= -\frac{1}{4}\sqrt{\frac{1}{3}}(2+\sqrt{3}) \tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}-2x}}{\sqrt{2+\sqrt{3}}}\right) + \frac{1}{4}\sqrt{\frac{1}{3}}(2-\sqrt{3}) \tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}-2x}}{\sqrt{2-\sqrt{3}}}\right) + \frac{1}{4}\sqrt{\frac{1}{3}}(2+\sqrt{3}) \tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}-2x}}{\sqrt{2+\sqrt{3}}}\right) \\
&\quad - \frac{1}{4}\sqrt{\frac{1}{3}}(2-\sqrt{3}) \tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}-2x}}{\sqrt{2-\sqrt{3}}}\right)
\end{aligned}$$

**Mathematica [C]** time = 0.0153069, size = 57, normalized size = 0.16

$$-\frac{1}{4}\text{RootSum}\left[\#1^8 - \#1^4 + 1 \&, \frac{\#1^4 \log(x - \#1) - \log(x - \#1)}{2\#1^7 - \#1^3} \&\right]$$

Antiderivative was successfully verified.

[In] Integrate[(1 - x^4)/(1 - x^4 + x^8), x]

[Out] -RootSum[1 - #1^4 + #1^8 &, (-Log[x - #1] + Log[x - #1]\*#1^4)/(-#1^3 + 2\*#1^7) & ]/4

**Maple [C]** time = 0.006, size = 44, normalized size = 0.1

$$\frac{1}{4} \sum_{_R=\text{RootOf}(Z^8-Z^4+1)} \frac{(-_R^4+1) \ln(x-_R)}{2\_R^7 - \_R^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^4+1)/(x^8-x^4+1),x)
```

```
[Out] 1/4*sum((-_R^4+1)/(2*_R^7-_R^3)*ln(x-_R),_R=RootOf(_Z^8-_Z^4+1))
```

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^4 - 1}{x^8 - x^4 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)/(x^8-x^4+1),x, algorithm="maxima")
```

```
[Out] -integrate((x^4 - 1)/(x^8 - x^4 + 1), x)
```

**Fricas [B]** time = 1.51969, size = 2313, normalized size = 6.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)/(x^8-x^4+1),x, algorithm="fricas")
```

```
[Out] 1/48*sqrt(6)*(sqrt(3)*sqrt(2) - 2*sqrt(2))*sqrt(sqrt(3) + 2)*log(12*x^2 + 2
*sqrt(6)*(2*sqrt(3)*sqrt(2)*x - 3*sqrt(2)*x)*sqrt(sqrt(3) + 2) + 12) - 1/48
*sqrt(6)*(sqrt(3)*sqrt(2) - 2*sqrt(2))*sqrt(sqrt(3) + 2)*log(12*x^2 - 2*sqrt
(6)*(2*sqrt(3)*sqrt(2)*x - 3*sqrt(2)*x)*sqrt(sqrt(3) + 2) + 12) + 1/96*sqrt
(6)*(sqrt(3)*sqrt(2) + 2*sqrt(2))*sqrt(-4*sqrt(3) + 8)*log(12*x^2 + sqrt(6)
)*(2*sqrt(3)*sqrt(2)*x + 3*sqrt(2)*x)*sqrt(-4*sqrt(3) + 8) + 12) - 1/96*sqrt
(6)*(sqrt(3)*sqrt(2) + 2*sqrt(2))*sqrt(-4*sqrt(3) + 8)*log(12*x^2 - sqrt(6)
)*(2*sqrt(3)*sqrt(2)*x + 3*sqrt(2)*x)*sqrt(-4*sqrt(3) + 8) + 12) + 1/12*sqrt
(6)*sqrt(2)*sqrt(sqrt(3) + 2)*arctan(1/6*sqrt(6)*sqrt(12*x^2 + 2*sqrt(6)*(
2*sqrt(3)*sqrt(2)*x - 3*sqrt(2)*x)*sqrt(sqrt(3) + 2) + 12)*(sqrt(3)*sqrt(2)
- 2*sqrt(2))*sqrt(sqrt(3) + 2) + 1/3*sqrt(6)*(2*sqrt(3)*sqrt(2)*x - 3*sqrt
(2)*x)*sqrt(sqrt(3) + 2) - sqrt(3) + 2) + 1/12*sqrt(6)*sqrt(2)*sqrt(sqrt(3)
+ 2)*arctan(1/6*sqrt(6)*sqrt(12*x^2 - 2*sqrt(6)*(2*sqrt(3)*sqrt(2)*x - 3*sqrt
(2)*x)*sqrt(sqrt(3) + 2) + 12)*(sqrt(3)*sqrt(2) - 2*sqrt(2))*sqrt(sqrt(3)
) + 2) + 1/3*sqrt(6)*(2*sqrt(3)*sqrt(2)*x - 3*sqrt(2)*x)*sqrt(sqrt(3) + 2)
+ sqrt(3) - 2) + 1/24*sqrt(6)*sqrt(2)*sqrt(-4*sqrt(3) + 8)*arctan(1/12*sqrt
```

```
(6)*sqrt(12*x^2 + sqrt(6)*(2*sqrt(3)*sqrt(2)*x + 3*sqrt(2)*x)*sqrt(-4*sqrt(3) + 8) + 12)*(sqrt(3)*sqrt(2) + 2*sqrt(2))*sqrt(-4*sqrt(3) + 8) - 1/6*sqrt(6)*(2*sqrt(3)*sqrt(2)*x + 3*sqrt(2)*x)*sqrt(-4*sqrt(3) + 8) - sqrt(3) - 2 + 1/24*sqrt(6)*sqrt(2)*sqrt(-4*sqrt(3) + 8)*arctan(1/12*sqrt(6)*sqrt(12*x^2 - sqrt(6)*(2*sqrt(3)*sqrt(2)*x + 3*sqrt(2)*x)*sqrt(-4*sqrt(3) + 8) + 12)*(sqrt(3)*sqrt(2) + 2*sqrt(2))*sqrt(-4*sqrt(3) + 8) - 1/6*sqrt(6)*(2*sqrt(3)*sqrt(2)*x + 3*sqrt(2)*x)*sqrt(-4*sqrt(3) + 8) + sqrt(3) + 2)
```

**Sympy [A]** time = 1.25847, size = 26, normalized size = 0.07

$$-\text{RootSum}\left(5308416t^8 - 2304t^4 + 1, (t \mapsto t \log(9216t^5 - 8t + x))\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**4+1)/(x**8-x**4+1),x)
```

```
[Out] -RootSum(5308416*_t**8 - 2304*_t**4 + 1, Lambda(_t, _t*log(9216*_t**5 - 8*_t + x)))
```

**Giac [A]** time = 1.17265, size = 342, normalized size = 0.96

$$\frac{1}{24} \left( \sqrt{6} + 3\sqrt{2} \right) \arctan\left( \frac{4x + \sqrt{6} - \sqrt{2}}{\sqrt{6} + \sqrt{2}} \right) + \frac{1}{24} \left( \sqrt{6} + 3\sqrt{2} \right) \arctan\left( \frac{4x - \sqrt{6} + \sqrt{2}}{\sqrt{6} + \sqrt{2}} \right) + \frac{1}{24} \left( \sqrt{6} - 3\sqrt{2} \right) \arctan\left( \frac{4x + \sqrt{6} + \sqrt{2}}{\sqrt{6} - \sqrt{2}} \right) + \frac{1}{24} \left( \sqrt{6} - 3\sqrt{2} \right) \arctan\left( \frac{4x - \sqrt{6} - \sqrt{2}}{\sqrt{6} - \sqrt{2}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)/(x^8-x^4+1),x, algorithm="giac")
```

```
[Out] 1/24*(sqrt(6) + 3*sqrt(2))*arctan((4*x + sqrt(6) - sqrt(2))/(sqrt(6) + sqrt(2))) + 1/24*(sqrt(6) + 3*sqrt(2))*arctan((4*x - sqrt(6) + sqrt(2))/(sqrt(6) + sqrt(2))) + 1/24*(sqrt(6) - 3*sqrt(2))*arctan((4*x + sqrt(6) + sqrt(2))/(sqrt(6) - sqrt(2))) + 1/24*(sqrt(6) - 3*sqrt(2))*arctan((4*x - sqrt(6) - sqrt(2))/(sqrt(6) - sqrt(2))) + 1/48*(sqrt(6) + 3*sqrt(2))*log(x^2 + 1/2*x*(sqrt(6) + sqrt(2)) + 1) - 1/48*(sqrt(6) + 3*sqrt(2))*log(x^2 - 1/2*x*(sqrt(6) + sqrt(2)) + 1) + 1/48*(sqrt(6) - 3*sqrt(2))*log(x^2 + 1/2*x*(sqrt(6) - sqrt(2)) + 1) - 1/48*(sqrt(6) - 3*sqrt(2))*log(x^2 - 1/2*x*(sqrt(6) - sqrt(2)) + 1)
```

$$3.26 \quad \int \frac{1-x^4}{1-2x^4+x^8} dx$$

**Optimal.** Leaf size=13

$$\frac{1}{2} \tan^{-1}(x) + \frac{1}{2} \tanh^{-1}(x)$$

[Out] ArcTan[x]/2 + ArcTanh[x]/2

**Rubi [A]** time = 0.0046864, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$ , Rules used = {28, 21, 212, 206, 203}

$$\frac{1}{2} \tan^{-1}(x) + \frac{1}{2} \tanh^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[(1 - x^4)/(1 - 2\*x^4 + x^8), x]

[Out] ArcTan[x]/2 + ArcTanh[x]/2

### Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

### Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
&& EqQ[b*c - a*d, 0] && IntegerQ[m] && (!IntegerQ[n] || SimplerQ[c + d*x,
a + b*x])
```

### Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] && !GtQ
[a/b, 0]
```



Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{1-x^4}{1-2x^4+x^8} dx &= \int \frac{1-x^4}{(-1+x^4)^2} dx \\ &= - \int \frac{1}{-1+x^4} dx \\ &= \frac{1}{2} \int \frac{1}{1-x^2} dx + \frac{1}{2} \int \frac{1}{1+x^2} dx \\ &= \frac{1}{2} \tan^{-1}(x) + \frac{1}{2} \tanh^{-1}(x) \end{aligned}$$

**Mathematica [A]** time = 0.0046195, size = 25, normalized size = 1.92

$$-\frac{1}{4} \log(1-x) + \frac{1}{4} \log(x+1) + \frac{1}{2} \tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^4)/(1 - 2*x^4 + x^8), x]
```

```
[Out] ArcTan[x]/2 - Log[1 - x]/4 + Log[1 + x]/4
```

**Maple [A]** time = 0.003, size = 10, normalized size = 0.8

$$\frac{\arctan(x)}{2} + \frac{\operatorname{Artanh}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((-x^4+1)/(x^8-2*x^4+1),x)`

[Out] `1/2*arctan(x)+1/2*arctanh(x)`

**Maxima [A]** time = 1.47873, size = 23, normalized size = 1.77

$$\frac{1}{2} \arctan(x) + \frac{1}{4} \log(x+1) - \frac{1}{4} \log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-x^4+1)/(x^8-2*x^4+1),x, algorithm="maxima")`

[Out] `1/2*arctan(x) + 1/4*log(x + 1) - 1/4*log(x - 1)`

**Fricas [A]** time = 1.30202, size = 66, normalized size = 5.08

$$\frac{1}{2} \arctan(x) + \frac{1}{4} \log(x+1) - \frac{1}{4} \log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-x^4+1)/(x^8-2*x^4+1),x, algorithm="fricas")`

[Out] `1/2*arctan(x) + 1/4*log(x + 1) - 1/4*log(x - 1)`

**Sympy [B]** time = 0.123423, size = 17, normalized size = 1.31

$$-\frac{\log(x-1)}{4} + \frac{\log(x+1)}{4} + \frac{\operatorname{atan}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-x**4+1)/(x**8-2*x**4+1),x)`

[Out] `-log(x - 1)/4 + log(x + 1)/4 + atan(x)/2`

---

**Giac [B]** time = 1.12839, size = 26, normalized size = 2.

$$\frac{1}{2} \arctan(x) + \frac{1}{4} \log(|x + 1|) - \frac{1}{4} \log(|x - 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^4+1)/(x^8-2\*x^4+1),x, algorithm="giac")

[Out] 1/2\*arctan(x) + 1/4\*log(abs(x + 1)) - 1/4\*log(abs(x - 1))

$$3.27 \quad \int \frac{1-x^4}{1-3x^4+x^8} dx$$

**Optimal.** Leaf size=129

$$\frac{\tan^{-1}\left(\sqrt{\frac{2}{\sqrt{5}-1}}x\right)}{\sqrt{10(\sqrt{5}-1)}} + \frac{\tan^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right)}{\sqrt{10(1+\sqrt{5})}} + \frac{\tanh^{-1}\left(\sqrt{\frac{2}{\sqrt{5}-1}}x\right)}{\sqrt{10(\sqrt{5}-1)}} + \frac{\tanh^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right)}{\sqrt{10(1+\sqrt{5})}}$$

[Out] ArcTan[Sqrt[2/(-1 + Sqrt[5])]\*x]/Sqrt[10\*(-1 + Sqrt[5])] + ArcTan[Sqrt[2/(1 + Sqrt[5])]\*x]/Sqrt[10\*(1 + Sqrt[5])] + ArcTanh[Sqrt[2/(-1 + Sqrt[5])]\*x]/Sqrt[10\*(-1 + Sqrt[5])] + ArcTanh[Sqrt[2/(1 + Sqrt[5])]\*x]/Sqrt[10\*(1 + Sqrt[5])]

**Rubi [A]** time = 0.117834, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$ , Rules used = {1419, 1093, 207, 203}

$$\frac{\tan^{-1}\left(\sqrt{\frac{2}{\sqrt{5}-1}}x\right)}{\sqrt{10(\sqrt{5}-1)}} + \frac{\tan^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right)}{\sqrt{10(1+\sqrt{5})}} + \frac{\tanh^{-1}\left(\sqrt{\frac{2}{\sqrt{5}-1}}x\right)}{\sqrt{10(\sqrt{5}-1)}} + \frac{\tanh^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right)}{\sqrt{10(1+\sqrt{5})}}$$

Antiderivative was successfully verified.

[In] Int[(1 - x^4)/(1 - 3\*x^4 + x^8), x]

[Out] ArcTan[Sqrt[2/(-1 + Sqrt[5])]\*x]/Sqrt[10\*(-1 + Sqrt[5])] + ArcTan[Sqrt[2/(1 + Sqrt[5])]\*x]/Sqrt[10\*(1 + Sqrt[5])] + ArcTanh[Sqrt[2/(-1 + Sqrt[5])]\*x]/Sqrt[10\*(-1 + Sqrt[5])] + ArcTanh[Sqrt[2/(1 + Sqrt[5])]\*x]/Sqrt[10\*(1 + Sqrt[5])]

### Rule 1419

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e +
q*x^(n/2) + x^n, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x^(n/2) +
x^n, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4
*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && IGtQ[n/2, 0] && (GtQ[(2*d)/e - b/c, 0]
|| (!LtQ[(2*d)/e - b/c, 0] && EqQ[d, e*Rt[a/c, 2]]))
```

Rule 1093

Int[((a\_) + (b\_)\*(x\_)^2 + (c\_)\*(x\_)^4)^(-1), x\_Symbol] := With[{q = Rt[b^2 - 4\*a\*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c\*x^2), x], x] - Dist[c/q, Int[1/(b/2 + q/2 + c\*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0] && PosQ[b^2 - 4\*a\*c]

Rule 207

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTanh[(Rt[b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 203

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTan[(Rt[b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rubi steps

$$\begin{aligned} \int \frac{1-x^4}{1-3x^4+x^8} dx &= -\left(\frac{1}{2} \int \frac{1}{-1-x^2+x^4} dx\right) - \frac{1}{2} \int \frac{1}{-1+x^2+x^4} dx \\ &= -\frac{\int \frac{1}{-\frac{1}{2}-\frac{\sqrt{5}}{2}+x^2} dx}{2\sqrt{5}} - \frac{\int \frac{1}{\frac{1}{2}-\frac{\sqrt{5}}{2}+x^2} dx}{2\sqrt{5}} + \frac{\int \frac{1}{-\frac{1}{2}+\frac{\sqrt{5}}{2}+x^2} dx}{2\sqrt{5}} + \frac{\int \frac{1}{\frac{1}{2}+\frac{\sqrt{5}}{2}+x^2} dx}{2\sqrt{5}} \\ &= \frac{\tan^{-1}\left(\sqrt{\frac{2}{-1+\sqrt{5}}}x\right)}{\sqrt{10(-1+\sqrt{5})}} + \frac{\tan^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right)}{\sqrt{10(1+\sqrt{5})}} + \frac{\tanh^{-1}\left(\sqrt{\frac{2}{-1+\sqrt{5}}}x\right)}{\sqrt{10(-1+\sqrt{5})}} + \frac{\tanh^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right)}{\sqrt{10(1+\sqrt{5})}} \end{aligned}$$

**Mathematica [A]** time = 0.0772217, size = 129, normalized size = 1.

$$\frac{\tan^{-1}\left(\sqrt{\frac{2}{\sqrt{5}-1}}x\right)}{\sqrt{10(\sqrt{5}-1)}} + \frac{\tan^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right)}{\sqrt{10(1+\sqrt{5})}} + \frac{\tanh^{-1}\left(\sqrt{\frac{2}{\sqrt{5}-1}}x\right)}{\sqrt{10(\sqrt{5}-1)}} + \frac{\tanh^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right)}{\sqrt{10(1+\sqrt{5})}}$$

Antiderivative was successfully verified.

[In] Integrate[(1 - x^4)/(1 - 3\*x^4 + x^8), x]

[Out] ArcTan[Sqrt[2/(-1 + Sqrt[5])]x]/Sqrt[10\*(-1 + Sqrt[5])] + ArcTan[Sqrt[2/(1 + Sqrt[5])]x]/Sqrt[10\*(1 + Sqrt[5])] + ArcTanh[Sqrt[2/(-1 + Sqrt[5])]x]/Sqrt[10\*(-1 + Sqrt[5])] + ArcTanh[Sqrt[2/(1 + Sqrt[5])]x]/Sqrt[10\*(1 + Sqrt[5])]

**Maple [A]** time = 0.02, size = 110, normalized size = 0.9

$$\frac{\sqrt{5}}{5\sqrt{2+2\sqrt{5}}}\operatorname{Arctanh}\left(2\frac{x}{\sqrt{2+2\sqrt{5}}}\right) + \frac{\sqrt{5}}{5\sqrt{-2+2\sqrt{5}}}\arctan\left(2\frac{x}{\sqrt{-2+2\sqrt{5}}}\right) + \frac{\sqrt{5}}{5\sqrt{-2+2\sqrt{5}}}\operatorname{Arctanh}\left(2\frac{x}{\sqrt{-2+2\sqrt{5}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-x^4+1)/(x^8-3\*x^4+1),x)

[Out] 1/5\*5^(1/2)/(2+2\*5^(1/2))^(1/2)\*arctanh(2\*x/(2+2\*5^(1/2))^(1/2))+1/5\*5^(1/2)/(-2+2\*5^(1/2))^(1/2)\*arctan(2\*x/(-2+2\*5^(1/2))^(1/2))+1/5\*5^(1/2)/(-2+2\*5^(1/2))^(1/2)\*arctanh(2\*x/(-2+2\*5^(1/2))^(1/2))+1/5\*5^(1/2)/(2+2\*5^(1/2))^(1/2)\*arctan(2\*x/(2+2\*5^(1/2))^(1/2))

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^4 - 1}{x^8 - 3x^4 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^4+1)/(x^8-3\*x^4+1),x, algorithm="maxima")

[Out] -integrate((x^4 - 1)/(x^8 - 3\*x^4 + 1), x)

**Fricas [B]** time = 1.41056, size = 905, normalized size = 7.02

$$-\frac{1}{10}\sqrt{10}\sqrt{\sqrt{5}+1}\arctan\left(\frac{1}{20}\sqrt{10}\sqrt{5}\sqrt{2}\sqrt{2x^2+\sqrt{5}-1}\sqrt{\sqrt{5}+1}-\frac{1}{10}\sqrt{10}\sqrt{5}x\sqrt{\sqrt{5}+1}\right)-\frac{1}{10}\sqrt{10}\sqrt{\sqrt{5}-1}\arctan\left(\frac{1}{20}\sqrt{10}\sqrt{5}\sqrt{2}\sqrt{2x^2+\sqrt{5}-1}\sqrt{\sqrt{5}-1}+\frac{1}{10}\sqrt{10}\sqrt{5}x\sqrt{\sqrt{5}-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^4+1)/(x^8-3\*x^4+1),x, algorithm="fricas")

[Out]  $-1/10\sqrt{10}\sqrt{\sqrt{5} + 1}\arctan(1/20\sqrt{10}\sqrt{5}\sqrt{2}\sqrt{2x^2 + \sqrt{5} - 1}\sqrt{\sqrt{5} + 1}) - 1/10\sqrt{10}\sqrt{5}x\sqrt{\sqrt{5} + 1}) - 1/10\sqrt{10}\sqrt{\sqrt{5} - 1}\arctan(1/20\sqrt{10}\sqrt{5}\sqrt{2}\sqrt{2x^2 + \sqrt{5} + 1}\sqrt{\sqrt{5} - 1}) - 1/10\sqrt{10}\sqrt{5}x\sqrt{\sqrt{5} - 1}) + 1/40\sqrt{10}\sqrt{\sqrt{5} - 1}\log(\sqrt{10}(\sqrt{5} + 5)\sqrt{\sqrt{5} - 1} + 20x) - 1/40\sqrt{10}\sqrt{\sqrt{5} - 1}\log(-\sqrt{10}(\sqrt{5} + 5)\sqrt{\sqrt{5} - 1} + 20x) - 1/40\sqrt{10}\sqrt{\sqrt{5} + 1}\log(\sqrt{10}\sqrt{\sqrt{5} + 1}(\sqrt{5} - 5) + 20x) + 1/40\sqrt{10}\sqrt{\sqrt{5} + 1}\log(-\sqrt{10}\sqrt{\sqrt{5} + 1}(\sqrt{5} - 5) + 20x)$

**Sympy [A]** time = 0.879065, size = 51, normalized size = 0.4

$-\text{RootSum}(6400t^4 - 80t^2 - 1, (t \mapsto t \log(25600t^5 - 16t + x))) - \text{RootSum}(6400t^4 + 80t^2 - 1, (t \mapsto t \log(25600t^5 - 16t + x)))$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x\*\*4+1)/(x\*\*8-3\*x\*\*4+1),x)

[Out]  $-\text{RootSum}(6400*_t**4 - 80*_t**2 - 1, \text{Lambda}(_t, _t*\log(25600*_t**5 - 16*_t + x))) - \text{RootSum}(6400*_t**4 + 80*_t**2 - 1, \text{Lambda}(_t, _t*\log(25600*_t**5 - 16*_t + x)))$

**Giac [A]** time = 1.20993, size = 198, normalized size = 1.53

$$\frac{1}{20} \sqrt{10\sqrt{5} - 10} \arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5} + \frac{1}{2}}}\right) + \frac{1}{20} \sqrt{10\sqrt{5} + 10} \arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5} - \frac{1}{2}}}\right) + \frac{1}{40} \sqrt{10\sqrt{5} - 10} \log\left(\left|x + \sqrt{\frac{1}{2}\sqrt{5} + 1}\right|\right) - \frac{1}{40} \sqrt{10\sqrt{5} - 10} \log\left(\left|x - \sqrt{\frac{1}{2}\sqrt{5} + 1}\right|\right) + \frac{1}{40} \sqrt{10\sqrt{5} + 10} \log\left(\left|x + \sqrt{\frac{1}{2}\sqrt{5} - 1}\right|\right) - \frac{1}{40} \sqrt{10\sqrt{5} + 10} \log\left(\left|x - \sqrt{\frac{1}{2}\sqrt{5} - 1}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^4+1)/(x^8-3\*x^4+1),x, algorithm="giac")

[Out]  $1/20\sqrt{10}\sqrt{5} - 10)\arctan(x/\sqrt{1/2\sqrt{5} + 1/2}) + 1/20\sqrt{10}\sqrt{5} + 10)\arctan(x/\sqrt{1/2\sqrt{5} - 1/2}) + 1/40\sqrt{10}\sqrt{5} - 10)\log(\text{abs}(x + \sqrt{1/2\sqrt{5} + 1/2})) - 1/40\sqrt{10}\sqrt{5} - 10)\log(\text{abs}(x - \sqrt{1/2\sqrt{5} + 1/2})) + 1/40\sqrt{10}\sqrt{5} + 10)\log(\text{abs}(x + \sqrt{1/2\sqrt{5} - 1/2})) - 1/40\sqrt{10}\sqrt{5} + 10)\log(\text{abs}(x - \sqrt{1/2\sqrt{5} - 1/2}))$

$\sqrt{5 - 1/2})$



$$3.28 \quad \int \frac{1-x^4}{1-4x^4+x^8} dx$$

**Optimal.** Leaf size=165

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt{\sqrt{3}-1}}\right)}{2\sqrt[4]{2}\sqrt{3}(\sqrt{3}-1)} + \frac{\tan^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt{1+\sqrt{3}}}\right)}{2\sqrt[4]{2}\sqrt{3}(1+\sqrt{3})} + \frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt{\sqrt{3}-1}}\right)}{2\sqrt[4]{2}\sqrt{3}(\sqrt{3}-1)} + \frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt{1+\sqrt{3}}}\right)}{2\sqrt[4]{2}\sqrt{3}(1+\sqrt{3})}$$

[Out] ArcTan[(2^(1/4)\*x)/Sqrt[-1 + Sqrt[3]]]/(2\*2^(1/4)\*Sqrt[3\*(-1 + Sqrt[3])]) + ArcTan[(2^(1/4)\*x)/Sqrt[1 + Sqrt[3]]]/(2\*2^(1/4)\*Sqrt[3\*(1 + Sqrt[3])]) + ArcTanh[(2^(1/4)\*x)/Sqrt[-1 + Sqrt[3]]]/(2\*2^(1/4)\*Sqrt[3\*(-1 + Sqrt[3])]) + ArcTanh[(2^(1/4)\*x)/Sqrt[1 + Sqrt[3]]]/(2\*2^(1/4)\*Sqrt[3\*(1 + Sqrt[3])])

**Rubi [A]** time = 0.104054, antiderivative size = 165, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$ , Rules used = {1419, 1093, 207, 203}

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt{\sqrt{3}-1}}\right)}{2\sqrt[4]{2}\sqrt{3}(\sqrt{3}-1)} + \frac{\tan^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt{1+\sqrt{3}}}\right)}{2\sqrt[4]{2}\sqrt{3}(1+\sqrt{3})} + \frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt{\sqrt{3}-1}}\right)}{2\sqrt[4]{2}\sqrt{3}(\sqrt{3}-1)} + \frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt{1+\sqrt{3}}}\right)}{2\sqrt[4]{2}\sqrt{3}(1+\sqrt{3})}$$

Antiderivative was successfully verified.

[In] Int[(1 - x^4)/(1 - 4\*x^4 + x^8),x]

[Out] ArcTan[(2^(1/4)\*x)/Sqrt[-1 + Sqrt[3]]]/(2\*2^(1/4)\*Sqrt[3\*(-1 + Sqrt[3])]) + ArcTan[(2^(1/4)\*x)/Sqrt[1 + Sqrt[3]]]/(2\*2^(1/4)\*Sqrt[3\*(1 + Sqrt[3])]) + ArcTanh[(2^(1/4)\*x)/Sqrt[-1 + Sqrt[3]]]/(2\*2^(1/4)\*Sqrt[3\*(-1 + Sqrt[3])]) + ArcTanh[(2^(1/4)\*x)/Sqrt[1 + Sqrt[3]]]/(2\*2^(1/4)\*Sqrt[3\*(1 + Sqrt[3])])

### Rule 1419

Int[((d\_) + (e\_.)\*(x\_)^(n\_))/((a\_) + (b\_.)\*(x\_)^(n\_) + (c\_.)\*(x\_)^(n2\_)), x\_Symbol] := With[{q = Rt[(2\*d)/e - b/c, 2]}, Dist[e/(2\*c), Int[1/Simp[d/e + q\*x^(n/2) + x^n, x], x], x] + Dist[e/(2\*c), Int[1/Simp[d/e - q\*x^(n/2) + x^n, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[n2, 2\*n] && NeQ[b^2 - 4\*a\*c, 0] && EqQ[c\*d^2 - a\*e^2, 0] && IGtQ[n/2, 0] && (GtQ[(2\*d)/e - b/c, 0] || (!LtQ[(2\*d)/e - b/c, 0] && EqQ[d, e\*Rt[a/c, 2]]))

Rule 1093

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{1-x^4}{1-4x^4+x^8} dx &= -\left(\frac{1}{2} \int \frac{1}{-1-\sqrt{2}x^2+x^4} dx\right) - \frac{1}{2} \int \frac{1}{-1+\sqrt{2}x^2+x^4} dx \\ &= -\frac{\int \frac{1}{-\sqrt{\frac{3}{2}-\frac{1}{\sqrt{2}}+x^2}} dx}{2\sqrt{6}} + \frac{\int \frac{1}{\sqrt{\frac{3}{2}-\frac{1}{\sqrt{2}}+x^2}} dx}{2\sqrt{6}} - \frac{\int \frac{1}{-\sqrt{\frac{3}{2}+\frac{1}{\sqrt{2}}+x^2}} dx}{2\sqrt{6}} + \frac{\int \frac{1}{\sqrt{\frac{3}{2}+\frac{1}{\sqrt{2}}+x^2}} dx}{2\sqrt{6}} \\ &= \frac{\tan^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt{-1+\sqrt{3}}}\right)}{2\sqrt[4]{2}\sqrt{3}(-1+\sqrt{3})} + \frac{\tan^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt{1+\sqrt{3}}}\right)}{2\sqrt[4]{2}\sqrt{3}(1+\sqrt{3})} + \frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt{-1+\sqrt{3}}}\right)}{2\sqrt[4]{2}\sqrt{3}(-1+\sqrt{3})} + \frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt{1+\sqrt{3}}}\right)}{2\sqrt[4]{2}\sqrt{3}(1+\sqrt{3})} \end{aligned}$$

**Mathematica [C]** time = 0.0136134, size = 55, normalized size = 0.33

$$-\frac{1}{8}\text{RootSum}\left[\#1^8 - 4\#1^4 + 1\&, \frac{\#1^4 \log(x - \#1) - \log(x - \#1)}{\#1^7 - 2\#1^3}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^4)/(1 - 4*x^4 + x^8), x]
```

[Out]  $-\text{RootSum}[1 - 4\#1^4 + \#1^8 \& , (-\text{Log}[x - \#1] + \text{Log}[x - \#1]\#1^4)/(-2\#1^3 + \#1^7) \& ]/8$

**Maple [C]** time = 0.007, size = 42, normalized size = 0.3

$$\frac{1}{8} \sum_{_R=\text{RootOf}(_Z^8-4_Z^4+1)} \frac{(-_R^4 + 1) \ln(x - _R)}{_R^7 - 2_R^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((-x^4+1)/(x^8-4x^4+1), x)$

[Out]  $1/8*\text{sum}((-_R^4+1)/(_R^7-2*_R^3)*\ln(x-_R), _R=\text{RootOf}(_Z^8-4*_Z^4+1))$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^4 - 1}{x^8 - 4x^4 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((-x^4+1)/(x^8-4x^4+1), x, \text{algorithm}="maxima")$

[Out]  $-\text{integrate}((x^4 - 1)/(x^8 - 4x^4 + 1), x)$

**Fricas [B]** time = 1.41732, size = 1004, normalized size = 6.08

$$-\frac{1}{6} \sqrt{6}(-\sqrt{3} + 2)^{\frac{1}{4}} \arctan\left(\frac{1}{6} \sqrt{6} \sqrt{x^2 + (\sqrt{3} + 2)\sqrt{-\sqrt{3} + 2}(\sqrt{3} + 3)}(-\sqrt{3} + 2)^{\frac{3}{4}} - \frac{1}{6} \sqrt{6}(\sqrt{3}x + 3x)(-\sqrt{3} + 2)^{\frac{3}{4}}\right) + \frac{1}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((-x^4+1)/(x^8-4x^4+1), x, \text{algorithm}="fricas")$

[Out]  $-1/6*\text{sqrt}(6)*(-\text{sqrt}(3) + 2)^{(1/4)}*\arctan(1/6*\text{sqrt}(6)*\text{sqrt}(x^2 + (\text{sqrt}(3) + 2)*\text{sqrt}(-\text{sqrt}(3) + 2))*(\text{sqrt}(3) + 3)*(-\text{sqrt}(3) + 2)^{(3/4)} - 1/6*\text{sqrt}(6)*(sq$

$$\begin{aligned} & \text{rt}(3)*x + 3*x)*(-\sqrt{3} + 2)^{(3/4)} + 1/6*\sqrt{6}*(\sqrt{3} + 2)^{(1/4)}*\arctan(1/6*(\sqrt{6}*\sqrt{x^2 - \sqrt{3} + 2}*(\sqrt{3} - 2))*\sqrt{\sqrt{3} + 2}*(\sqrt{3} - 3) - \sqrt{6}*(\sqrt{3}*x - 3*x)*\sqrt{\sqrt{3} + 2})*(\sqrt{3} + 2)^{(1/4)}) - 1/24*\sqrt{6}*(\sqrt{3} + 2)^{(1/4)}*\log(\sqrt{6}*(\sqrt{3} + 2)^{(1/4)}*(\sqrt{3} - 3) + 6*x) + 1/24*\sqrt{6}*(\sqrt{3} + 2)^{(1/4)}*\log(-\sqrt{6}*(\sqrt{3} + 2)^{(1/4)}*(\sqrt{3} - 3) + 6*x) + 1/24*\sqrt{6}*(-\sqrt{3} + 2)^{(1/4)}*\log(\sqrt{6}*(\sqrt{3} + 3)*(-\sqrt{3} + 2)^{(1/4)} + 6*x) - 1/24*\sqrt{6}*(-\sqrt{3} + 2)^{(1/4)}*\log(-\sqrt{6}*(\sqrt{3} + 3)*(-\sqrt{3} + 2)^{(1/4)} + 6*x) \end{aligned}$$

**Sympy [A]** time = 0.181679, size = 26, normalized size = 0.16

$$-\text{RootSum}\left(84934656t^8 - 36864t^4 + 1, \left(t \mapsto t \log(36864t^5 - 20t + x)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x\*\*4+1)/(x\*\*8-4\*x\*\*4+1),x)

[Out] -RootSum(84934656\*\_t\*\*8 - 36864\*\_t\*\*4 + 1, Lambda(\_t, \_t\*log(36864\*\_t\*\*5 - 20\*\_t + x)))

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int -\frac{x^4 - 1}{x^8 - 4x^4 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^4+1)/(x^8-4\*x^4+1),x, algorithm="giac")

[Out] integrate(-(x^4 - 1)/(x^8 - 4\*x^4 + 1), x)

$$3.29 \quad \int \frac{1-x^4}{1-5x^4+x^8} dx$$

**Optimal.** Leaf size=169

$$\frac{\tan^{-1}\left(\sqrt{\frac{2}{\sqrt{7}-\sqrt{3}}}x\right)}{\sqrt{14}(\sqrt{7}-\sqrt{3})} + \frac{\tan^{-1}\left(\sqrt{\frac{2}{\sqrt{3}+\sqrt{7}}}x\right)}{\sqrt{14}(\sqrt{3}+\sqrt{7})} + \frac{\tanh^{-1}\left(\sqrt{\frac{2}{\sqrt{7}-\sqrt{3}}}x\right)}{\sqrt{14}(\sqrt{7}-\sqrt{3})} + \frac{\tanh^{-1}\left(\sqrt{\frac{2}{\sqrt{3}+\sqrt{7}}}x\right)}{\sqrt{14}(\sqrt{3}+\sqrt{7})}$$

[Out] ArcTan[Sqrt[2/(-Sqrt[3] + Sqrt[7]])\*x]/Sqrt[14\*(-Sqrt[3] + Sqrt[7])] + ArcTan[Sqrt[2/(Sqrt[3] + Sqrt[7]])\*x]/Sqrt[14\*(Sqrt[3] + Sqrt[7])] + ArcTanh[Sqrt[2/(-Sqrt[3] + Sqrt[7]])\*x]/Sqrt[14\*(-Sqrt[3] + Sqrt[7])] + ArcTanh[Sqrt[2/(Sqrt[3] + Sqrt[7]])\*x]/Sqrt[14\*(Sqrt[3] + Sqrt[7])]

**Rubi [A]** time = 0.142447, antiderivative size = 169, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$ , Rules used = {1419, 1093, 207, 203}

$$\frac{\tan^{-1}\left(\sqrt{\frac{2}{\sqrt{7}-\sqrt{3}}}x\right)}{\sqrt{14}(\sqrt{7}-\sqrt{3})} + \frac{\tan^{-1}\left(\sqrt{\frac{2}{\sqrt{3}+\sqrt{7}}}x\right)}{\sqrt{14}(\sqrt{3}+\sqrt{7})} + \frac{\tanh^{-1}\left(\sqrt{\frac{2}{\sqrt{7}-\sqrt{3}}}x\right)}{\sqrt{14}(\sqrt{7}-\sqrt{3})} + \frac{\tanh^{-1}\left(\sqrt{\frac{2}{\sqrt{3}+\sqrt{7}}}x\right)}{\sqrt{14}(\sqrt{3}+\sqrt{7})}$$

Antiderivative was successfully verified.

[In] Int[(1 - x^4)/(1 - 5\*x^4 + x^8),x]

[Out] ArcTan[Sqrt[2/(-Sqrt[3] + Sqrt[7]])\*x]/Sqrt[14\*(-Sqrt[3] + Sqrt[7])] + ArcTan[Sqrt[2/(Sqrt[3] + Sqrt[7]])\*x]/Sqrt[14\*(Sqrt[3] + Sqrt[7])] + ArcTanh[Sqrt[2/(-Sqrt[3] + Sqrt[7]])\*x]/Sqrt[14\*(-Sqrt[3] + Sqrt[7])] + ArcTanh[Sqrt[2/(Sqrt[3] + Sqrt[7]])\*x]/Sqrt[14\*(Sqrt[3] + Sqrt[7])]

### Rule 1419

Int[((d\_) + (e\_.)\*(x\_)^(n\_))/((a\_) + (b\_.)\*(x\_)^(n\_) + (c\_.)\*(x\_)^(2n\_)), x\_Symbol] :> With[{q = Rt[(2\*d)/e - b/c, 2]}, Dist[e/(2\*c), Int[1/Simp[d/e + q\*x^(n/2) + x^n, x], x], x] + Dist[e/(2\*c), Int[1/Simp[d/e - q\*x^(n/2) + x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[n2, 2\*n] && NeQ[b^2 - 4\*a\*c, 0] && EqQ[c\*d^2 - a\*e^2, 0] && IGtQ[n/2, 0] && (GtQ[(2\*d)/e - b/c, 0] || (!LtQ[(2\*d)/e - b/c, 0] && EqQ[d, e\*Rt[a/c, 2]]))

Rule 1093

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{1-x^4}{1-5x^4+x^8} dx &= -\left(\frac{1}{2} \int \frac{1}{-1-\sqrt{3}x^2+x^4} dx\right) - \frac{1}{2} \int \frac{1}{-1+\sqrt{3}x^2+x^4} dx \\ &= -\frac{\int \frac{1}{-\frac{\sqrt{3}}{2}-\frac{\sqrt{7}}{2}+x^2} dx}{2\sqrt{7}} - \frac{\int \frac{1}{\frac{\sqrt{3}}{2}-\frac{\sqrt{7}}{2}+x^2} dx}{2\sqrt{7}} + \frac{\int \frac{1}{-\frac{\sqrt{3}}{2}+\frac{\sqrt{7}}{2}+x^2} dx}{2\sqrt{7}} + \frac{\int \frac{1}{\frac{\sqrt{3}}{2}+\frac{\sqrt{7}}{2}+x^2} dx}{2\sqrt{7}} \\ &= \frac{\tan^{-1}\left(\sqrt{\frac{2}{-\sqrt{3}+\sqrt{7}}}x\right)}{\sqrt{14(-\sqrt{3}+\sqrt{7})}} + \frac{\tan^{-1}\left(\sqrt{\frac{2}{\sqrt{3}+\sqrt{7}}}x\right)}{\sqrt{14(\sqrt{3}+\sqrt{7})}} + \frac{\tanh^{-1}\left(\sqrt{\frac{2}{-\sqrt{3}+\sqrt{7}}}x\right)}{\sqrt{14(-\sqrt{3}+\sqrt{7})}} + \frac{\tanh^{-1}\left(\sqrt{\frac{2}{\sqrt{3}+\sqrt{7}}}x\right)}{\sqrt{14(\sqrt{3}+\sqrt{7})}} \end{aligned}$$

**Mathematica [C]** time = 0.0133492, size = 57, normalized size = 0.34

$$-\frac{1}{4}\text{RootSum}\left[\#1^8 - 5\#1^4 + 1 \&, \frac{\#1^4 \log(x - \#1) - \log(x - \#1)}{2\#1^7 - 5\#1^3} \&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^4)/(1 - 5*x^4 + x^8), x]
```

[Out]  $-\text{RootSum}[1 - 5\#1^4 + \#1^8 \& , (-\text{Log}[x - \#1] + \text{Log}[x - \#1]\#\#1^4)/(-5\#\#1^3 + 2\#\#1^7) \& ]/4$

**Maple [C]** time = 0.006, size = 44, normalized size = 0.3

$$\frac{1}{4} \sum_{_R=\text{RootOf}(-Z^8-5Z^4+1)} \frac{(-_R^4 + 1) \ln(x - _R)}{2\_R^7 - 5\_R^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((-x^4+1)/(x^8-5x^4+1), x)$

[Out]  $1/4*\text{sum}((-_R^4+1)/(2*_R^7-5*_R^3)*\ln(x-_R), _R=\text{RootOf}(-Z^8-5*_Z^4+1))$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^4 - 1}{x^8 - 5x^4 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((-x^4+1)/(x^8-5x^4+1), x, \text{algorithm}="maxima")$

[Out]  $-\text{integrate}((x^4 - 1)/(x^8 - 5x^4 + 1), x)$

**Fricas [B]** time = 1.55648, size = 1751, normalized size = 10.36

$$-\frac{1}{14} \sqrt{14} \sqrt{\sqrt{2} \sqrt{-\sqrt{7} \sqrt{3}} + 5} \arctan \left( \frac{1}{112} \sqrt{14} \sqrt{4x^2 + (\sqrt{7} \sqrt{3} \sqrt{2} + 5 \sqrt{2}) \sqrt{-\sqrt{7} \sqrt{3}} + 5} (\sqrt{7} \sqrt{3} \sqrt{2} + 7 \sqrt{2}) \sqrt{-\sqrt{7} \sqrt{3}} + \dots \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((-x^4+1)/(x^8-5x^4+1), x, \text{algorithm}="fricas")$

[Out]  $-1/14*\text{sqrt}(14)*\text{sqrt}(\text{sqrt}(2)*\text{sqrt}(-\text{sqrt}(7)*\text{sqrt}(3) + 5))*\text{arctan}(1/112*\text{sqrt}(14)*\text{sqrt}(4*x^2 + (\text{sqrt}(7)*\text{sqrt}(3)*\text{sqrt}(2) + 5*\text{sqrt}(2))*\text{sqrt}(-\text{sqrt}(7)*\text{sqrt}(3))$

+ 5))\*(sqrt(7)\*sqrt(3)\*sqrt(2) + 7\*sqrt(2))\*sqrt(-sqrt(7)\*sqrt(3) + 5)\*sqrt(sqrt(2)\*sqrt(-sqrt(7)\*sqrt(3) + 5)) - 1/56\*sqrt(14)\*(sqrt(7)\*sqrt(3)\*sqrt(2)\*x + 7\*sqrt(2)\*x)\*sqrt(-sqrt(7)\*sqrt(3) + 5)\*sqrt(sqrt(2)\*sqrt(-sqrt(7)\*sqrt(3) + 5))) + 1/14\*sqrt(14)\*sqrt(sqrt(2)\*sqrt(sqrt(7)\*sqrt(3) + 5))\*arctan(1/112\*(sqrt(14)\*sqrt(4\*x^2 - (sqrt(7)\*sqrt(3)\*sqrt(2) - 5\*sqrt(2))\*sqrt(sqrt(7)\*sqrt(3) + 5))\*(sqrt(7)\*sqrt(3)\*sqrt(2) - 7\*sqrt(2))\*sqrt(sqrt(7)\*sqrt(3) + 5) - 2\*sqrt(14)\*(sqrt(7)\*sqrt(3)\*sqrt(2)\*x - 7\*sqrt(2)\*x)\*sqrt(sqrt(7)\*sqrt(3) + 5))\*sqrt(sqrt(2)\*sqrt(sqrt(7)\*sqrt(3) + 5))) - 1/56\*sqrt(14)\*sqrt(sqrt(2)\*sqrt(sqrt(7)\*sqrt(3) + 5))\*log(sqrt(14)\*(sqrt(7)\*sqrt(3) - 7)\*sqrt(sqrt(2)\*sqrt(sqrt(7)\*sqrt(3) + 5)) + 28\*x) + 1/56\*sqrt(14)\*sqrt(sqrt(2)\*sqrt(sqrt(7)\*sqrt(3) + 5))\*log(-sqrt(14)\*(sqrt(7)\*sqrt(3) - 7)\*sqrt(sqrt(2)\*sqrt(sqrt(7)\*sqrt(3) + 5)) + 28\*x) + 1/56\*sqrt(14)\*sqrt(sqrt(2)\*sqrt(-sqrt(7)\*sqrt(3) + 5))\*log(sqrt(14)\*(sqrt(7)\*sqrt(3) + 7)\*sqrt(sqrt(2)\*sqrt(-sqrt(7)\*sqrt(3) + 5)) + 28\*x) - 1/56\*sqrt(14)\*sqrt(sqrt(2)\*sqrt(-sqrt(7)\*sqrt(3) + 5))\*log(-sqrt(14)\*(sqrt(7)\*sqrt(3) + 7)\*sqrt(sqrt(2)\*sqrt(-sqrt(7)\*sqrt(3) + 5)) + 28\*x)

**Sympy [A]** time = 0.185546, size = 26, normalized size = 0.15

$$-\text{RootSum}\left(157351936t^8 - 62720t^4 + 1, \left(t \mapsto t \log\left(50176t^5 - 24t + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x\*\*4+1)/(x\*\*8-5\*x\*\*4+1),x)

[Out] -RootSum(157351936\*\_t\*\*8 - 62720\*\_t\*\*4 + 1, Lambda(\_t, \_t\*log(50176\*\_t\*\*5 - 24\*\_t + x)))

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4 - 1}{x^8 - 5x^4 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^4+1)/(x^8-5\*x^4+1),x, algorithm="giac")

[Out] integrate(-(x^4 - 1)/(x^8 - 5\*x^4 + 1), x)



$$3.30 \quad \int \frac{1-x^4}{1-6x^4+x^8} dx$$

**Optimal.** Leaf size=125

$$\frac{\tan^{-1}\left(\frac{x}{\sqrt{\sqrt{2}-1}}\right)}{4\sqrt{2}(\sqrt{2}-1)} + \frac{\tan^{-1}\left(\frac{x}{\sqrt{1+\sqrt{2}}}\right)}{4\sqrt{2}(1+\sqrt{2})} + \frac{\tanh^{-1}\left(\frac{x}{\sqrt{\sqrt{2}-1}}\right)}{4\sqrt{2}(\sqrt{2}-1)} + \frac{\tanh^{-1}\left(\frac{x}{\sqrt{1+\sqrt{2}}}\right)}{4\sqrt{2}(1+\sqrt{2})}$$

[Out] ArcTan[x/Sqrt[-1 + Sqrt[2]]]/(4\*Sqrt[2\*(-1 + Sqrt[2])]) + ArcTan[x/Sqrt[1 + Sqrt[2]]]/(4\*Sqrt[2\*(1 + Sqrt[2])]) + ArcTanh[x/Sqrt[-1 + Sqrt[2]]]/(4\*Sqrt[2\*(-1 + Sqrt[2])]) + ArcTanh[x/Sqrt[1 + Sqrt[2]]]/(4\*Sqrt[2\*(1 + Sqrt[2])])

**Rubi [A]** time = 0.0674187, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$ , Rules used = {1419, 1093, 207, 203}

$$\frac{\tan^{-1}\left(\frac{x}{\sqrt{\sqrt{2}-1}}\right)}{4\sqrt{2}(\sqrt{2}-1)} + \frac{\tan^{-1}\left(\frac{x}{\sqrt{1+\sqrt{2}}}\right)}{4\sqrt{2}(1+\sqrt{2})} + \frac{\tanh^{-1}\left(\frac{x}{\sqrt{\sqrt{2}-1}}\right)}{4\sqrt{2}(\sqrt{2}-1)} + \frac{\tanh^{-1}\left(\frac{x}{\sqrt{1+\sqrt{2}}}\right)}{4\sqrt{2}(1+\sqrt{2})}$$

Antiderivative was successfully verified.

[In] Int[(1 - x^4)/(1 - 6\*x^4 + x^8),x]

[Out] ArcTan[x/Sqrt[-1 + Sqrt[2]]]/(4\*Sqrt[2\*(-1 + Sqrt[2])]) + ArcTan[x/Sqrt[1 + Sqrt[2]]]/(4\*Sqrt[2\*(1 + Sqrt[2])]) + ArcTanh[x/Sqrt[-1 + Sqrt[2]]]/(4\*Sqrt[2\*(-1 + Sqrt[2])]) + ArcTanh[x/Sqrt[1 + Sqrt[2]]]/(4\*Sqrt[2\*(1 + Sqrt[2])])

### Rule 1419

Int[((d\_) + (e\_.)\*(x\_)^(n\_))/((a\_) + (b\_.)\*(x\_)^(n\_) + (c\_.)\*(x\_)^(2n\_)), x\_Symbol] := With[{q = Rt[(2\*d)/e - b/c, 2]}, Dist[e/(2\*c), Int[1/Simp[d/e + q\*x^(n/2) + x^n, x], x], x] + Dist[e/(2\*c), Int[1/Simp[d/e - q\*x^(n/2) + x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[n2, 2\*n] && NeQ[b^2 - 4\*a\*c, 0] && EqQ[c\*d^2 - a\*e^2, 0] && IGtQ[n/2, 0] && (GtQ[(2\*d)/e - b/c, 0] || (!LtQ[(2\*d)/e - b/c, 0] && EqQ[d, e\*Rt[a/c, 2]]))

Rule 1093

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{1-x^4}{1-6x^4+x^8} dx &= -\left(\frac{1}{2} \int \frac{1}{-1-2x^2+x^4} dx\right) - \frac{1}{2} \int \frac{1}{-1+2x^2+x^4} dx \\ &= -\frac{\int \frac{1}{-1-\sqrt{2}+x^2} dx}{4\sqrt{2}} - \frac{\int \frac{1}{1-\sqrt{2}+x^2} dx}{4\sqrt{2}} + \frac{\int \frac{1}{-1+\sqrt{2}+x^2} dx}{4\sqrt{2}} + \frac{\int \frac{1}{1+\sqrt{2}+x^2} dx}{4\sqrt{2}} \\ &= \frac{\tan^{-1}\left(\frac{x}{\sqrt{-1+\sqrt{2}}}\right)}{4\sqrt{2}(-1+\sqrt{2})} + \frac{\tan^{-1}\left(\frac{x}{\sqrt{1+\sqrt{2}}}\right)}{4\sqrt{2}(1+\sqrt{2})} + \frac{\tanh^{-1}\left(\frac{x}{\sqrt{-1+\sqrt{2}}}\right)}{4\sqrt{2}(-1+\sqrt{2})} + \frac{\tanh^{-1}\left(\frac{x}{\sqrt{1+\sqrt{2}}}\right)}{4\sqrt{2}(1+\sqrt{2})} \end{aligned}$$

**Mathematica [A]** time = 0.0541279, size = 114, normalized size = 0.91

$$\frac{\sqrt{1+\sqrt{2}} \tan^{-1}\left(\frac{x}{\sqrt{\sqrt{2}-1}}\right) + \sqrt{\sqrt{2}-1} \tan^{-1}\left(\frac{x}{\sqrt{1+\sqrt{2}}}\right) + \sqrt{1+\sqrt{2}} \tanh^{-1}\left(\frac{x}{\sqrt{\sqrt{2}-1}}\right) + \sqrt{\sqrt{2}-1} \tanh^{-1}\left(\frac{x}{\sqrt{1+\sqrt{2}}}\right)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^4)/(1 - 6*x^4 + x^8),x]
```

[Out]  $(\sqrt{1 + \sqrt{2}} \operatorname{ArcTan}[x/\sqrt{-1 + \sqrt{2}}] + \sqrt{-1 + \sqrt{2}} \operatorname{ArcTan}[x/\sqrt{1 + \sqrt{2}}] + \sqrt{1 + \sqrt{2}} \operatorname{ArcTanh}[x/\sqrt{-1 + \sqrt{2}}] + \sqrt{-1 + \sqrt{2}} \operatorname{ArcTanh}[x/\sqrt{1 + \sqrt{2}}]) / (4\sqrt{2})$

**Maple [A]** time = 0.026, size = 90, normalized size = 0.7

$$\frac{\sqrt{2}}{8\sqrt{\sqrt{2}-1}} \operatorname{Artanh}\left(\frac{x}{\sqrt{\sqrt{2}-1}}\right) + \frac{\sqrt{2}}{8\sqrt{1+\sqrt{2}}} \arctan\left(\frac{x}{\sqrt{1+\sqrt{2}}}\right) + \frac{\sqrt{2}}{8\sqrt{1+\sqrt{2}}} \operatorname{Artanh}\left(\frac{x}{\sqrt{1+\sqrt{2}}}\right) + \frac{\sqrt{2}}{8\sqrt{\sqrt{2}-1}} \arctan\left(\frac{x}{\sqrt{\sqrt{2}-1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((-x^4+1)/(x^8-6*x^4+1),x)`

[Out]  $1/8*2^{(1/2)}/(2^{(1/2)}-1)^{(1/2)}*\operatorname{arctanh}(x/(2^{(1/2)}-1)^{(1/2)})+1/8*2^{(1/2)}/(1+2^{(1/2)})^{(1/2)}*\operatorname{arctan}(x/(1+2^{(1/2)})^{(1/2)})+1/8*2^{(1/2)}/(1+2^{(1/2)})^{(1/2)}*\operatorname{artanh}(x/(1+2^{(1/2)})^{(1/2)})+1/8*2^{(1/2)}/(2^{(1/2)}-1)^{(1/2)}*\operatorname{arctan}(x/(2^{(1/2)}-1)^{(1/2)})$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^4 - 1}{x^8 - 6x^4 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-x^4+1)/(x^8-6*x^4+1),x, algorithm="maxima")`

[Out] `-integrate((x^4 - 1)/(x^8 - 6*x^4 + 1), x)`

**Fricas [B]** time = 1.3468, size = 686, normalized size = 5.49

$$-\frac{1}{4}\sqrt{2}\sqrt{\sqrt{2}+1}\arctan\left(-x\sqrt{\sqrt{2}+1}+\sqrt{x^2+\sqrt{2}-1}\sqrt{\sqrt{2}+1}\right)-\frac{1}{4}\sqrt{2}\sqrt{\sqrt{2}-1}\arctan\left(-x\sqrt{\sqrt{2}-1}+\sqrt{x^2+\sqrt{2}+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^4+1)/(x^8-6\*x^4+1),x, algorithm="fricas")

[Out]  $-1/4*\sqrt{2}*\sqrt{\sqrt{2} + 1}*\arctan(-x*\sqrt{\sqrt{2} + 1} + \sqrt{x^2 + \sqrt{2} - 1})*\sqrt{\sqrt{2} + 1}) - 1/4*\sqrt{2}*\sqrt{\sqrt{2} - 1}*\arctan(-x*\sqrt{\sqrt{2} - 1} + \sqrt{x^2 + \sqrt{2} + 1})*\sqrt{\sqrt{2} - 1}) + 1/16*\sqrt{2}*\sqrt{\sqrt{2} - 1}*\log((\sqrt{2} + 1)*\sqrt{\sqrt{2} - 1} + x) - 1/16*\sqrt{2}*\sqrt{\sqrt{2} - 1}*\log(-(\sqrt{2} + 1)*\sqrt{\sqrt{2} - 1} + x) + 1/16*\sqrt{2}*\sqrt{\sqrt{2} + 1}*\log(\sqrt{\sqrt{2} + 1}*(\sqrt{2} - 1) + x) - 1/16*\sqrt{2}*\sqrt{\sqrt{2} + 1}*\log(-\sqrt{\sqrt{2} + 1}*(\sqrt{2} - 1) + x)$

**Sympy [A]** time = 0.868287, size = 51, normalized size = 0.41

$-\text{RootSum}(16384t^4 - 256t^2 - 1, (t \mapsto t \log(65536t^5 - 28t + x))) - \text{RootSum}(16384t^4 + 256t^2 - 1, (t \mapsto t \log(65536t^5 - 28t + x)))$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x\*\*4+1)/(x\*\*8-6\*x\*\*4+1),x)

[Out]  $-\text{RootSum}(16384*_t**4 - 256*_t**2 - 1, \text{Lambda}(_t, _t*\log(65536*_t**5 - 28*_t + x))) - \text{RootSum}(16384*_t**4 + 256*_t**2 - 1, \text{Lambda}(_t, _t*\log(65536*_t**5 - 28*_t + x)))$

**Giac [A]** time = 1.2046, size = 182, normalized size = 1.46

$\frac{1}{8}\sqrt{2\sqrt{2}-2}\arctan\left(\frac{x}{\sqrt{\sqrt{2}+1}}\right) + \frac{1}{8}\sqrt{2\sqrt{2}+2}\arctan\left(\frac{x}{\sqrt{\sqrt{2}-1}}\right) + \frac{1}{16}\sqrt{2\sqrt{2}-2}\log\left(\left|x + \sqrt{\sqrt{2}+1}\right|\right) - \frac{1}{16}\sqrt{2\sqrt{2}+2}\log\left(\left|x - \sqrt{\sqrt{2}-1}\right|\right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^4+1)/(x^8-6\*x^4+1),x, algorithm="giac")

[Out]  $1/8*\sqrt{2*\sqrt{2} - 2}*\arctan(x/\sqrt{\sqrt{2} + 1}) + 1/8*\sqrt{2*\sqrt{2} + 2}*\arctan(x/\sqrt{\sqrt{2} - 1}) + 1/16*\sqrt{2*\sqrt{2} - 2}*\log(\text{abs}(x + \sqrt{\sqrt{2} + 1})) - 1/16*\sqrt{2*\sqrt{2} - 2}*\log(\text{abs}(x - \sqrt{\sqrt{2} + 1})) + 1/16*\sqrt{2*\sqrt{2} + 2}*\log(\text{abs}(x + \sqrt{\sqrt{2} - 1})) - 1/16*\sqrt{2*\sqrt{2} + 2}*\log(\text{abs}(x - \sqrt{\sqrt{2} - 1}))$

$$3.31 \quad \int \frac{-1+\sqrt{3}+2x^4}{1-x^4+x^8} dx$$

**Optimal.** Leaf size=135

$$-\frac{\log\left(x^2 - \sqrt{2 - \sqrt{3}x} + 1\right)}{2\sqrt{2}} + \frac{\log\left(x^2 + \sqrt{2 - \sqrt{3}x} + 1\right)}{2\sqrt{2}} - \frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}-2x}}{\sqrt{2-\sqrt{3}}}\right)}{\sqrt{2}} + \frac{\tan^{-1}\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{\sqrt{2}}$$

[Out] -(ArcTan[(Sqrt[2 + Sqrt[3]] - 2\*x)/Sqrt[2 - Sqrt[3]]]/Sqrt[2]) + ArcTan[(Sqrt[2 + Sqrt[3]] + 2\*x)/Sqrt[2 - Sqrt[3]]]/Sqrt[2] - Log[1 - Sqrt[2 - Sqrt[3]]]\*x + x^2/(2\*Sqrt[2]) + Log[1 + Sqrt[2 - Sqrt[3]]]\*x + x^2/(2\*Sqrt[2])

**Rubi [A]** time = 0.123538, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.24$ , Rules used = {1423, 1161, 618, 204, 1164, 628}

$$-\frac{\log\left(x^2 - \sqrt{2 - \sqrt{3}x} + 1\right)}{2\sqrt{2}} + \frac{\log\left(x^2 + \sqrt{2 - \sqrt{3}x} + 1\right)}{2\sqrt{2}} - \frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}-2x}}{\sqrt{2-\sqrt{3}}}\right)}{\sqrt{2}} + \frac{\tan^{-1}\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[(-1 + Sqrt[3] + 2\*x^4)/(1 - x^4 + x^8), x]

[Out] -(ArcTan[(Sqrt[2 + Sqrt[3]] - 2\*x)/Sqrt[2 - Sqrt[3]]]/Sqrt[2]) + ArcTan[(Sqrt[2 + Sqrt[3]] + 2\*x)/Sqrt[2 - Sqrt[3]]]/Sqrt[2] - Log[1 - Sqrt[2 - Sqrt[3]]]\*x + x^2/(2\*Sqrt[2]) + Log[1 + Sqrt[2 - Sqrt[3]]]\*x + x^2/(2\*Sqrt[2])

#### Rule 1423

Int[((d\_) + (e\_.)\*(x\_)^(n\_))/((a\_) + (b\_.)\*(x\_)^(n\_) + (c\_.)\*(x\_)^(n2\_)), x\_Symbol] :> With[{q = Rt[a/c, 2]}, With[{r = Rt[2\*q - b/c, 2]}, Dist[1/(2\*c\*q\*r), Int[(d\*r - (d - e\*q)\*x^(n/2))/(q - r\*x^(n/2) + x^n), x], x] + Dist[1/(2\*c\*q\*r), Int[(d\*r + (d - e\*q)\*x^(n/2))/(q + r\*x^(n/2) + x^n), x], x]] / ; FreeQ[{a, b, c, d, e}, x] && EqQ[n2, 2\*n] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && IGtQ[n/2, 0] && NegQ[b^2 - 4\*a\*c]

#### Rule 1161

Int[((d\_) + (e\_.)\*(x\_)^2)/((a\_) + (b\_.)\*(x\_)^2 + (c\_.)\*(x\_)^4), x\_Symbol] :> With[{q = Rt[(2\*d)/e - b/c, 2]}, Dist[e/(2\*c), Int[1/Simp[d/e + q\*x + x^2

```
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (GtQ[(2*d)/e - b/c, 0] || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2], 0]))
```

### Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

### Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

### Rule 1164

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[(-2*d)/e - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && !GtQ[b^2 - 4*a*c, 0]
```

### Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

### Rubi steps

$$\begin{aligned}
\int \frac{-1 + \sqrt{3} + 2x^4}{1 - x^4 + x^8} dx &= \frac{\int \frac{\sqrt{3}(-1+\sqrt{3})+(3-\sqrt{3})x^2}{1-\sqrt{3}x^2+x^4} dx}{2\sqrt{3}} + \frac{\int \frac{\sqrt{3}(-1+\sqrt{3})+(-3+\sqrt{3})x^2}{1+\sqrt{3}x^2+x^4} dx}{2\sqrt{3}} \\
&= -\frac{\int \frac{\sqrt{2-\sqrt{3}}+2x}{-1-\sqrt{2-\sqrt{3}}x-x^2} dx}{2\sqrt{2}} - \frac{\int \frac{\sqrt{2-\sqrt{3}}-2x}{-1+\sqrt{2-\sqrt{3}}x-x^2} dx}{2\sqrt{2}} + \frac{1}{4}(-1 + \sqrt{3}) \int \frac{1}{1 - \sqrt{2 + \sqrt{3}}x + x^2} dx + \frac{1}{4}(-1 + \sqrt{3}) \int \frac{1}{1 + \sqrt{2 + \sqrt{3}}x + x^2} dx \\
&= -\frac{\log\left(1 - \sqrt{2 - \sqrt{3}}x + x^2\right)}{2\sqrt{2}} + \frac{\log\left(1 + \sqrt{2 - \sqrt{3}}x + x^2\right)}{2\sqrt{2}} + \frac{1}{2}(1 - \sqrt{3}) \text{Subst}\left(\int \frac{1}{-2 + \sqrt{3} - x^2} dx\right) + \frac{1}{2}(1 + \sqrt{3}) \text{Subst}\left(\int \frac{1}{2 + \sqrt{3} - x^2} dx\right) \\
&= -\frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)}{\sqrt{2}} + \frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}+2x}{\sqrt{2-\sqrt{3}}}\right)}{\sqrt{2}} - \frac{\log\left(1 - \sqrt{2 - \sqrt{3}}x + x^2\right)}{2\sqrt{2}} + \frac{\log\left(1 + \sqrt{2 - \sqrt{3}}x + x^2\right)}{2\sqrt{2}}
\end{aligned}$$

**Mathematica [C]** time = 0.035876, size = 71, normalized size = 0.53

$$\frac{1}{4} \text{RootSum}\left[\#1^8 - \#1^4 + 1 \&, \frac{2\#1^4 \log(x - \#1) + \sqrt{3} \log(x - \#1) - \log(x - \#1)}{2\#1^7 - \#1^3} \&\right]$$

Antiderivative was successfully verified.

[In] Integrate[(-1 + Sqrt[3] + 2\*x^4)/(1 - x^4 + x^8), x]

[Out] RootSum[1 - #1^4 + #1^8 &, (-Log[x - #1] + Sqrt[3]\*Log[x - #1] + 2\*Log[x - #1]\*#1^4)/(-#1^3 + 2\*#1^7) & ]/4

**Maple [C]** time = 0.051, size = 47, normalized size = 0.4

$$\frac{1}{4} \sum_{_R=\text{RootOf}(_Z^8-_Z^4+1)} \frac{(-1 + 2\_R^4 + \sqrt{3}) \ln(x - _R)}{2\_R^7 - _R^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-1+2\*x^4+3^(1/2))/(x^8-x^4+1), x)

[Out] 1/4\*sum(1/(2\*\_R^7-\_R^3)\*(-1+2\*\_R^4+3^(1/2))\*ln(x-\_R),\_R=RootOf(\_Z^8-\_Z^4+1))

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{2x^4 + \sqrt{3} - 1}{x^8 - x^4 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1+2\*x^4+3^(1/2))/(x^8-x^4+1),x, algorithm="maxima")

[Out] integrate((2\*x^4 + sqrt(3) - 1)/(x^8 - x^4 + 1), x)

**Fricas [A]** time = 1.40833, size = 315, normalized size = 2.33

$$\frac{1}{2} \sqrt{2} \arctan\left(\frac{1}{2}(\sqrt{3}\sqrt{2} + \sqrt{2})x^3 - \sqrt{2}x\right) + \frac{1}{2} \sqrt{2} \arctan\left(\frac{1}{2}(\sqrt{3}\sqrt{2} + \sqrt{2})x\right) + \frac{1}{4} \sqrt{2} \log\left(-\frac{(\sqrt{3}\sqrt{2} - \sqrt{2})x + 2x^2 + 2}{(\sqrt{3}\sqrt{2} - \sqrt{2})x - 2x^2 - 2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1+2\*x^4+3^(1/2))/(x^8-x^4+1),x, algorithm="fricas")

[Out] 1/2\*sqrt(2)\*arctan(1/2\*(sqrt(3)\*sqrt(2) + sqrt(2))\*x^3 - sqrt(2)\*x) + 1/2\*sqrt(2)\*arctan(1/2\*(sqrt(3)\*sqrt(2) + sqrt(2))\*x) + 1/4\*sqrt(2)\*log(-((sqrt(3)\*sqrt(2) - sqrt(2))\*x + 2\*x^2 + 2)/((sqrt(3)\*sqrt(2) - sqrt(2))\*x - 2\*x^2 - 2))

**Sympy [A]** time = 1.00303, size = 136, normalized size = 1.01

$$\frac{\sqrt{2} \left( 2 \operatorname{atan}\left(\frac{x(\sqrt{6}+2\sqrt{2})}{1+\sqrt{3}}\right) + 2 \operatorname{atan}\left(\frac{x^3(\sqrt{6}+2\sqrt{2})}{1+\sqrt{3}} - \sqrt{2}x\right) \right)}{4} - \frac{\sqrt{2} \log\left(x^2 - \frac{\sqrt{2}x(2+2\sqrt{3})}{4(\sqrt{3}+2)} + 1\right)}{4} + \frac{\sqrt{2} \log\left(x^2 + \frac{\sqrt{2}x(2+2\sqrt{3})}{4(\sqrt{3}+2)} + 1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1+2\*x\*\*4+3\*\*(1/2))/(x\*\*8-x\*\*4+1),x)



```
[Out] sqrt(2)*(2*atan(x*(sqrt(6) + 2*sqrt(2))/(1 + sqrt(3))) + 2*atan(x**3*(sqrt(6) + 2*sqrt(2))/(1 + sqrt(3)) - sqrt(2)*x))/4 - sqrt(2)*log(x**2 - sqrt(2)*x*(2 + 2*sqrt(3))/(4*(sqrt(3) + 2)) + 1)/4 + sqrt(2)*log(x**2 + sqrt(2)*x*(2 + 2*sqrt(3))/(4*(sqrt(3) + 2)) + 1)/4
```

**Giac [A]** time = 1.16231, size = 144, normalized size = 1.07

$$\frac{1}{2} \sqrt{2} \arctan\left(\frac{4x + \sqrt{6} + \sqrt{2}}{\sqrt{6} - \sqrt{2}}\right) + \frac{1}{2} \sqrt{2} \arctan\left(\frac{4x - \sqrt{6} - \sqrt{2}}{\sqrt{6} - \sqrt{2}}\right) + \frac{1}{4} \sqrt{2} \log\left(x^2 + \frac{1}{2}x(\sqrt{6} - \sqrt{2}) + 1\right) - \frac{1}{4} \sqrt{2} \log\left(x^2 - \frac{1}{2}x(\sqrt{6} - \sqrt{2}) + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+2*x^4+3^(1/2))/(x^8-x^4+1),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*arctan((4*x + sqrt(6) + sqrt(2))/(sqrt(6) - sqrt(2))) + 1/2*sqrt(2)*arctan((4*x - sqrt(6) - sqrt(2))/(sqrt(6) - sqrt(2))) + 1/4*sqrt(2)*log(x^2 + 1/2*x*(sqrt(6) - sqrt(2)) + 1) - 1/4*sqrt(2)*log(x^2 - 1/2*x*(sqrt(6) - sqrt(2)) + 1)
```

$$3.32 \quad \int \frac{1+(1+\sqrt{3})x^4}{1-x^4+x^8} dx$$

**Optimal.** Leaf size=164

$$-\frac{1}{4}\sqrt{2+\sqrt{3}}\log\left(x^2-\sqrt{2-\sqrt{3}}x+1\right)+\frac{1}{4}\sqrt{2+\sqrt{3}}\log\left(x^2+\sqrt{2-\sqrt{3}}x+1\right)-\frac{1}{2}\sqrt{2+\sqrt{3}}\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}-2x}}{\sqrt{2-\sqrt{3}}}\right)+\frac{1}{2}\sqrt{2+\sqrt{3}}\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}+2x}}{\sqrt{2-\sqrt{3}}}\right)$$

```
[Out] -(Sqrt[2 + Sqrt[3]]*ArcTan[(Sqrt[2 + Sqrt[3]] - 2*x)/Sqrt[2 - Sqrt[3]]])/2
+ (Sqrt[2 + Sqrt[3]]*ArcTan[(Sqrt[2 + Sqrt[3]] + 2*x)/Sqrt[2 - Sqrt[3]]])/2
- (Sqrt[2 + Sqrt[3]]*Log[1 - Sqrt[2 - Sqrt[3]]*x + x^2])/4 + (Sqrt[2 + Sqr
t[3]]*Log[1 + Sqrt[2 - Sqrt[3]]*x + x^2])/4
```

---

**Rubi [A]** time = 0.0947788, antiderivative size = 164, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$ , Rules used = {1423, 1161, 618, 204, 1164, 628}

$$-\frac{1}{4}\sqrt{2+\sqrt{3}}\log\left(x^2-\sqrt{2-\sqrt{3}}x+1\right)+\frac{1}{4}\sqrt{2+\sqrt{3}}\log\left(x^2+\sqrt{2-\sqrt{3}}x+1\right)-\frac{1}{2}\sqrt{2+\sqrt{3}}\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}-2x}}{\sqrt{2-\sqrt{3}}}\right)+\frac{1}{2}\sqrt{2+\sqrt{3}}\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}+2x}}{\sqrt{2-\sqrt{3}}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + (1 + Sqrt[3])*x^4)/(1 - x^4 + x^8), x]
```

```
[Out] -(Sqrt[2 + Sqrt[3]]*ArcTan[(Sqrt[2 + Sqrt[3]] - 2*x)/Sqrt[2 - Sqrt[3]]])/2
+ (Sqrt[2 + Sqrt[3]]*ArcTan[(Sqrt[2 + Sqrt[3]] + 2*x)/Sqrt[2 - Sqrt[3]]])/2
- (Sqrt[2 + Sqrt[3]]*Log[1 - Sqrt[2 - Sqrt[3]]*x + x^2])/4 + (Sqrt[2 + Sqr
t[3]]*Log[1 + Sqrt[2 - Sqrt[3]]*x + x^2])/4
```

### Rule 1423

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c
*q*r), Int[(d*r - (d - e*q)*x^(n/2))/(q - r*x^(n/2) + x^n), x], x] + Dist[1
/(2*c*q*r), Int[(d*r + (d - e*q)*x^(n/2))/(q + r*x^(n/2) + x^n), x], x]] /
; FreeQ[{a, b, c, d, e}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[c
*d^2 - b*d*e + a*e^2, 0] && IGtQ[n/2, 0] && NegQ[b^2 - 4*a*c]
```

### Rule 1161

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (b_)*(x_)^2 + (c_)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || (!LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

### Rule 618

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

### Rule 204

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

### Rule 1164

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (b_)*(x_)^2 + (c_)*(x_)^4), x_Symbol] :
> With[{q = Rt[(-2*d)/e - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] && !GtQ[b^2 - 4*a*c, 0]
```

### Rule 628

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

### Rubi steps

$$\begin{aligned}
\int \frac{1 + (1 + \sqrt{3})x^4}{1 - x^4 + x^8} dx &= \frac{\int \frac{\sqrt{3} + \sqrt{3}x^2}{1 - \sqrt{3}x^2 + x^4} dx}{2\sqrt{3}} + \frac{\int \frac{\sqrt{3} - \sqrt{3}x^2}{1 + \sqrt{3}x^2 + x^4} dx}{2\sqrt{3}} \\
&= \frac{1}{4} \int \frac{1}{1 - \sqrt{2 + \sqrt{3}}x + x^2} dx + \frac{1}{4} \int \frac{1}{1 + \sqrt{2 + \sqrt{3}}x + x^2} dx - \frac{1}{4} \sqrt{2 + \sqrt{3}} \int \frac{\sqrt{2 - \sqrt{3}} + 2x}{-1 - \sqrt{2 - \sqrt{3}}x - x^2} dx \\
&= -\frac{1}{4} \sqrt{2 + \sqrt{3}} \log\left(1 - \sqrt{2 - \sqrt{3}}x + x^2\right) + \frac{1}{4} \sqrt{2 + \sqrt{3}} \log\left(1 + \sqrt{2 - \sqrt{3}}x + x^2\right) - \frac{1}{2} \text{Subst}\left(\int \frac{1}{-2 - \sqrt{2 - \sqrt{3}}x - x^2} dx, \frac{\sqrt{2 - \sqrt{3}} + 2x}{-1 - \sqrt{2 - \sqrt{3}}x - x^2}\right) \\
&= -\frac{\tan^{-1}\left(\frac{\sqrt{2 + \sqrt{3}} - 2x}{\sqrt{2 - \sqrt{3}}}\right)}{2\sqrt{2 - \sqrt{3}}} + \frac{\tan^{-1}\left(\frac{\sqrt{2 + \sqrt{3}} + 2x}{\sqrt{2 - \sqrt{3}}}\right)}{2\sqrt{2 - \sqrt{3}}} - \frac{1}{4} \sqrt{2 + \sqrt{3}} \log\left(1 - \sqrt{2 - \sqrt{3}}x + x^2\right) + \frac{1}{4} \sqrt{2 + \sqrt{3}} \log\left(1 + \sqrt{2 - \sqrt{3}}x + x^2\right)
\end{aligned}$$

**Mathematica [C]** time = 0.0373736, size = 72, normalized size = 0.44

$$\frac{1}{4} \text{RootSum}\left[\#1^8 - \#1^4 + 1 \&, \frac{\sqrt{3}\#1^4 \log(x - \#1) + \#1^4 \log(x - \#1) + \log(x - \#1)}{2\#1^7 - \#1^3} \&\right]$$

Antiderivative was successfully verified.

[In] Integrate[(1 + (1 + Sqrt[3])\*x^4)/(1 - x^4 + x^8), x]

[Out] RootSum[1 - #1^4 + #1^8 &, (Log[x - #1] + Log[x - #1]\*#1^4 + Sqrt[3]\*Log[x - #1]\*#1^4)/(-#1^3 + 2\*#1^7) & ]/4

**Maple [C]** time = 0.039, size = 62, normalized size = 0.4

$$\frac{1}{8} \sum_{_R = \text{RootOf}(_Z^8 - _Z^4 + 1)} \frac{(2\_R^4 + 2\sqrt{3}\_R^4 + (1 + \sqrt{3})(\sqrt{3} - 1)) \ln(x - _R)}{2\_R^7 - _R^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1+x^4\*(1+3^(1/2)))/(x^8-x^4+1), x)

[Out] 1/8\*sum(1/(2\*\_R^7-\_R^3)\*(2\*\_R^4+2\*3^(1/2)\*\_R^4+(1+3^(1/2))\*(3^(1/2)-1))\*ln(x-\_R),\_R=RootOf(\_Z^8-\_Z^4+1))

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4(\sqrt{3}+1)+1}{x^8-x^4+1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+x^4\*(1+3^(1/2)))/(x^8-x^4+1),x, algorithm="maxima")

[Out] integrate((x^4\*(sqrt(3) + 1) + 1)/(x^8 - x^4 + 1), x)

---

**Fricas [A]** time = 1.38947, size = 355, normalized size = 2.16

$$\frac{1}{2} \sqrt{\sqrt{3}+2} \arctan\left(x^3 \sqrt{\sqrt{3}+2} - x \sqrt{\sqrt{3}+2}(\sqrt{3}-1)\right) + \frac{1}{2} \sqrt{\sqrt{3}+2} \arctan\left(x \sqrt{\sqrt{3}+2}\right) + \frac{1}{4} \sqrt{\sqrt{3}+2} \log\left(-\frac{x \sqrt{\sqrt{3}}}{x \sqrt{\sqrt{3}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+x^4\*(1+3^(1/2)))/(x^8-x^4+1),x, algorithm="fricas")

[Out] 1/2\*sqrt(sqrt(3) + 2)\*arctan(x^3\*sqrt(sqrt(3) + 2) - x\*sqrt(sqrt(3) + 2)\*(sqrt(3) - 1)) + 1/2\*sqrt(sqrt(3) + 2)\*arctan(x\*sqrt(sqrt(3) + 2)) + 1/4\*sqrt(sqrt(3) + 2)\*log(-(x\*sqrt(sqrt(3) + 2)\*(sqrt(3) - 2) - x^2 - 1)/(x\*sqrt(sqrt(3) + 2)\*(sqrt(3) - 2) + x^2 + 1))

---

**Sympy [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: PolynomialError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+x\*\*4\*(1+3\*\*(1/2)))/(x\*\*8-x\*\*4+1),x)

[Out] Exception raised: PolynomialError

---

**Giac [A]** time = 1.14206, size = 166, normalized size = 1.01

$$\frac{1}{4}(\sqrt{6} + \sqrt{2}) \arctan\left(\frac{4x + \sqrt{6} + \sqrt{2}}{\sqrt{6} - \sqrt{2}}\right) + \frac{1}{4}(\sqrt{6} + \sqrt{2}) \arctan\left(\frac{4x - \sqrt{6} - \sqrt{2}}{\sqrt{6} - \sqrt{2}}\right) + \frac{1}{8}(\sqrt{6} + \sqrt{2}) \log\left(x^2 + \frac{1}{2}x(\sqrt{6} - \sqrt{2})\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+x^4\*(1+3^(1/2)))/(x^8-x^4+1),x, algorithm="giac")

[Out] 1/4\*(sqrt(6) + sqrt(2))\*arctan((4\*x + sqrt(6) + sqrt(2))/(sqrt(6) - sqrt(2))) + 1/4\*(sqrt(6) + sqrt(2))\*arctan((4\*x - sqrt(6) - sqrt(2))/(sqrt(6) - sqrt(2))) + 1/8\*(sqrt(6) + sqrt(2))\*log(x^2 + 1/2\*x\*(sqrt(6) - sqrt(2)) + 1) - 1/8\*(sqrt(6) + sqrt(2))\*log(x^2 - 1/2\*x\*(sqrt(6) - sqrt(2)) + 1)

$$3.33 \quad \int \frac{3-2\sqrt{3}+(-3+\sqrt{3})x^4}{1-x^4+x^8} dx$$

**Optimal.** Leaf size=180

$$\frac{1}{4}\sqrt{3(2-\sqrt{3})}\log\left(x^2-\sqrt{2-\sqrt{3}}x+1\right)-\frac{1}{4}\sqrt{3(2-\sqrt{3})}\log\left(x^2+\sqrt{2-\sqrt{3}}x+1\right)+\frac{1}{2}\sqrt{3(2-\sqrt{3})}\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-\sqrt{2-\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)$$

[Out] (Sqrt[3\*(2 - Sqrt[3])]\*ArcTan[(Sqrt[2 + Sqrt[3]] - 2\*x)/Sqrt[2 - Sqrt[3]]])/2 - (Sqrt[3\*(2 - Sqrt[3])]\*ArcTan[(Sqrt[2 + Sqrt[3]] + 2\*x)/Sqrt[2 - Sqrt[3]]])/2 + (Sqrt[3\*(2 - Sqrt[3])]\*Log[1 - Sqrt[2 - Sqrt[3]]\*x + x^2])/4 - (Sqrt[3\*(2 - Sqrt[3])]\*Log[1 + Sqrt[2 - Sqrt[3]]\*x + x^2])/4

**Rubi [A]** time = 0.12236, antiderivative size = 180, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 33,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$ , Rules used = {1423, 1161, 618, 204, 1164, 628}

$$\frac{1}{4}\sqrt{3(2-\sqrt{3})}\log\left(x^2-\sqrt{2-\sqrt{3}}x+1\right)-\frac{1}{4}\sqrt{3(2-\sqrt{3})}\log\left(x^2+\sqrt{2-\sqrt{3}}x+1\right)+\frac{1}{2}\sqrt{3(2-\sqrt{3})}\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-\sqrt{2-\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)$$

Antiderivative was successfully verified.

[In] Int[(3 - 2\*Sqrt[3] + (-3 + Sqrt[3])\*x^4)/(1 - x^4 + x^8), x]

[Out] (Sqrt[3\*(2 - Sqrt[3])]\*ArcTan[(Sqrt[2 + Sqrt[3]] - 2\*x)/Sqrt[2 - Sqrt[3]]])/2 - (Sqrt[3\*(2 - Sqrt[3])]\*ArcTan[(Sqrt[2 + Sqrt[3]] + 2\*x)/Sqrt[2 - Sqrt[3]]])/2 + (Sqrt[3\*(2 - Sqrt[3])]\*Log[1 - Sqrt[2 - Sqrt[3]]\*x + x^2])/4 - (Sqrt[3\*(2 - Sqrt[3])]\*Log[1 + Sqrt[2 - Sqrt[3]]\*x + x^2])/4

### Rule 1423

Int[((d\_) + (e\_)\*(x\_)^(n\_))/((a\_) + (b\_)\*(x\_)^(n\_) + (c\_)\*(x\_)^(n2\_)), x\_Symbol] :> With[{q = Rt[a/c, 2]}, With[{r = Rt[2\*q - b/c, 2]}, Dist[1/(2\*c\*q\*r), Int[(d\*r - (d - e\*q)\*x^(n/2))/(q - r\*x^(n/2) + x^n), x], x] + Dist[1/(2\*c\*q\*r), Int[(d\*r + (d - e\*q)\*x^(n/2))/(q + r\*x^(n/2) + x^n), x], x]] / ; FreeQ[{a, b, c, d, e}, x] && EqQ[n2, 2\*n] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && IGtQ[n/2, 0] && NegQ[b^2 - 4\*a\*c]

### Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

### Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :=> Dist[-2, Subst[In
t[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

### Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :=> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

### Rule 1164

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(-2*d)/e - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - a*e^2, 0] && !GtQ[b^2 - 4*a*c, 0]
```

### Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :=> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

### Rubi steps



$$\begin{aligned}
\int \frac{3 - 2\sqrt{3} + (-3 + \sqrt{3})x^4}{1 - x^4 + x^8} dx &= \int \frac{\sqrt{3}(3-2\sqrt{3})+(-6+3\sqrt{3})x^2}{1-\sqrt{3}x^2+x^4} dx + \int \frac{\sqrt{3}(3-2\sqrt{3})+(6-3\sqrt{3})x^2}{1+\sqrt{3}x^2+x^4} dx \\
&= \frac{1}{4}\sqrt{3}(2-\sqrt{3}) \int \frac{\sqrt{2-\sqrt{3}}+2x}{-1-\sqrt{2-\sqrt{3}}x-x^2} dx + \frac{1}{4}\sqrt{3}(2-\sqrt{3}) \int \frac{\sqrt{2-\sqrt{3}}-2x}{-1+\sqrt{2-\sqrt{3}}x-x^2} dx \\
&= \frac{1}{4}\sqrt{3}(2-\sqrt{3}) \log\left(1-\sqrt{2-\sqrt{3}}x+x^2\right) - \frac{1}{4}\sqrt{3}(2-\sqrt{3}) \log\left(1+\sqrt{2-\sqrt{3}}x+x^2\right) - \\
&= \frac{1}{2}\sqrt{6-3\sqrt{3}} \tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right) - \frac{1}{2}\sqrt{6-3\sqrt{3}} \tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}+2x}{\sqrt{2-\sqrt{3}}}\right) + \frac{1}{4}\sqrt{3}(2-\sqrt{3}) \log\left(\frac{1-\sqrt{2-\sqrt{3}}x+x^2}{1+\sqrt{2-\sqrt{3}}x+x^2}\right)
\end{aligned}$$

**Mathematica [C]** time = 0.0460575, size = 89, normalized size = 0.49

$$\frac{1}{4} \text{RootSum}\left[\#1^8 - \#1^4 + 1 \&, \frac{\sqrt{3}\#1^4 \log(x - \#1) - 3\#1^4 \log(x - \#1) - 2\sqrt{3} \log(x - \#1) + 3 \log(x - \#1)}{2\#1^7 - \#1^3} \&\right]$$

Antiderivative was successfully verified.

[In] Integrate[(3 - 2\*Sqrt[3] + (-3 + Sqrt[3]))\*x^4)/(1 - x^4 + x^8), x]

[Out] RootSum[1 - #1^4 + #1^8 &, (3\*Log[x - #1] - 2\*Sqrt[3]\*Log[x - #1] - 3\*Log[x - #1]\*#1^4 + Sqrt[3]\*Log[x - #1]\*#1^4)/(-#1^3 + 2\*#1^7) & ]/4

**Maple [C]** time = 0.01, size = 62, normalized size = 0.3

$$\frac{1}{8} \sum_{_R=\text{RootOf}(_Z^8-_Z^4+1)} \frac{(-6\_R^4 + 2\sqrt{3}\_R^4 + (-3 + \sqrt{3})(\sqrt{3} - 1)) \ln(x - _R)}{2\_R^7 - _R^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3+x^4\*(-3+3^(1/2))-2\*3^(1/2))/(x^8-x^4+1), x)

[Out] 1/8\*sum(1/(2\*\_R^7-\_R^3)\*(-6\*\_R^4+2\*3^(1/2)\*\_R^4+(-3+3^(1/2))\*(3^(1/2)-1))\*ln(x-\_R), \_R=RootOf(\_Z^8-\_Z^4+1))

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4(\sqrt{3}-3)-2\sqrt{3}+3}{x^8-x^4+1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3+x^4\*(-3+3^(1/2))-2\*3^(1/2))/(x^8-x^4+1),x, algorithm="maxima")

[Out] integrate((x^4\*(sqrt(3) - 3) - 2\*sqrt(3) + 3)/(x^8 - x^4 + 1), x)

---

**Fricas [A]** time = 1.43674, size = 436, normalized size = 2.42

$$-\frac{1}{2}\sqrt{-3\sqrt{3}+6}\arctan\left(\frac{1}{3}x^3(2\sqrt{3}+3)\sqrt{-3\sqrt{3}+6}-\frac{1}{3}x(\sqrt{3}+3)\sqrt{-3\sqrt{3}+6}\right)-\frac{1}{2}\sqrt{-3\sqrt{3}+6}\arctan\left(\frac{1}{3}x(2\sqrt{3}+3)\sqrt{-3\sqrt{3}+6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3+x^4\*(-3+3^(1/2))-2\*3^(1/2))/(x^8-x^4+1),x, algorithm="fricas")

[Out] -1/2\*sqrt(-3\*sqrt(3) + 6)\*arctan(1/3\*x^3\*(2\*sqrt(3) + 3)\*sqrt(-3\*sqrt(3) + 6) - 1/3\*x\*(sqrt(3) + 3)\*sqrt(-3\*sqrt(3) + 6)) - 1/2\*sqrt(-3\*sqrt(3) + 6)\*arctan(1/3\*x\*(2\*sqrt(3) + 3)\*sqrt(-3\*sqrt(3) + 6)) + 1/4\*sqrt(-3\*sqrt(3) + 6)\*log((3\*x^2 - sqrt(3)\*x\*sqrt(-3\*sqrt(3) + 6) + 3)/(3\*x^2 + sqrt(3)\*x\*sqrt(-3\*sqrt(3) + 6) + 3))

---

**Sympy [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: PolynomialError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3+x\*\*4\*(-3+3\*\*(1/2))-2\*3\*\*(1/2))/(x\*\*8-x\*\*4+1),x)

[Out] Exception raised: PolynomialError

---

**Giac [A]** time = 1.16023, size = 177, normalized size = 0.98

$$\frac{1}{4}(\sqrt{6} - 3\sqrt{2}) \arctan\left(\frac{4x + \sqrt{6} + \sqrt{2}}{\sqrt{6} - \sqrt{2}}\right) + \frac{1}{4}(\sqrt{6} - 3\sqrt{2}) \arctan\left(\frac{4x - \sqrt{6} - \sqrt{2}}{\sqrt{6} - \sqrt{2}}\right) + \frac{1}{8}(\sqrt{6} - 3\sqrt{2}) \log\left(x^2 + \frac{1}{2}x(\sqrt{6} - \sqrt{2}) + 1\right) - \frac{1}{8}(\sqrt{6} - 3\sqrt{2}) \log\left(x^2 - \frac{1}{2}x(\sqrt{6} - \sqrt{2}) + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3+x^4\*(-3+3^(1/2))-2\*3^(1/2))/(x^8-x^4+1),x, algorithm="giac")

[Out] 1/4\*(sqrt(6) - 3\*sqrt(2))\*arctan((4\*x + sqrt(6) + sqrt(2))/(sqrt(6) - sqrt(2))) + 1/4\*(sqrt(6) - 3\*sqrt(2))\*arctan((4\*x - sqrt(6) - sqrt(2))/(sqrt(6) - sqrt(2))) + 1/8\*(sqrt(6) - 3\*sqrt(2))\*log(x^2 + 1/2\*x\*(sqrt(6) - sqrt(2)) + 1) - 1/8\*(sqrt(6) - 3\*sqrt(2))\*log(x^2 - 1/2\*x\*(sqrt(6) - sqrt(2)) + 1)

$$3.34 \quad \int \frac{d + \frac{e}{x}}{c + \frac{a}{x^2}} dx$$

**Optimal.** Leaf size=49

$$-\frac{\sqrt{ad} \tan^{-1}\left(\frac{\sqrt{cx}}{\sqrt{a}}\right)}{c^{3/2}} + \frac{e \log(a + cx^2)}{2c} + \frac{dx}{c}$$

[Out] (d\*x)/c - (Sqrt[a]\*d\*ArcTan[(Sqrt[c]\*x)/Sqrt[a]])/c^(3/2) + (e\*Log[a + c\*x^2])/(2\*c)

**Rubi [A]** time = 0.0300692, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$ , Rules used = {1394, 774, 635, 205, 260}

$$-\frac{\sqrt{ad} \tan^{-1}\left(\frac{\sqrt{cx}}{\sqrt{a}}\right)}{c^{3/2}} + \frac{e \log(a + cx^2)}{2c} + \frac{dx}{c}$$

Antiderivative was successfully verified.

[In] Int[(d + e/x)/(c + a/x^2), x]

[Out] (d\*x)/c - (Sqrt[a]\*d\*ArcTan[(Sqrt[c]\*x)/Sqrt[a]])/c^(3/2) + (e\*Log[a + c\*x^2])/(2\*c)

### Rule 1394

Int[((a\_) + (c\_.)\*(x\_)^(n2\_.))^(p\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_))^(q\_.), x\_Symbol] := Int[x^(n\*(2\*p + q))\*(e + d/x^n)^q\*(c + a/x^(2\*n))^p, x] /; FreeQ[{a, c, d, e, n}, x] && EqQ[n2, 2\*n] && IntegersQ[p, q] && NegQ[n]

### Rule 774

Int[(((d\_.) + (e\_.)\*(x\_))\*((f\_.) + (g\_.)\*(x\_)))/((a\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Simp[(e\*g\*x)/c, x] + Dist[1/c, Int[(c\*d\*f - a\*e\*g + c\*(e\*f + d\*g)\*x)/(a + c\*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x]

### Rule 635

Int[((d\_) + (e\_.)\*(x\_))/((a\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Dist[d, Int[1/(a + c\*x^2), x], x] + Dist[e, Int[x/(a + c\*x^2), x], x] /; FreeQ[{a, c, d, e

}, x] && !NiceSqrtQ[-(a\*c)]

### Rule 205

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

### Rule 260

Int[(x\_)^(m\_.)/((a\_) + (b\_.)\*(x\_)^(n\_.)), x\_Symbol] := Simp[Log[RemoveContent[a + b\*x^n, x]]/(b\*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

### Rubi steps

$$\begin{aligned}
 \int \frac{d + \frac{e}{x}}{c + \frac{a}{x^2}} dx &= \int \frac{x(e + dx)}{a + cx^2} dx \\
 &= \frac{dx}{c} + \frac{\int \frac{-ad+ce x}{a+cx^2} dx}{c} \\
 &= \frac{dx}{c} - \frac{(ad) \int \frac{1}{a+cx^2} dx}{c} + e \int \frac{x}{a+cx^2} dx \\
 &= \frac{dx}{c} - \frac{\sqrt{ad} \tan^{-1}\left(\frac{\sqrt{cx}}{\sqrt{a}}\right)}{c^{3/2}} + \frac{e \log(a + cx^2)}{2c}
 \end{aligned}$$

**Mathematica [A]** time = 0.0233652, size = 49, normalized size = 1.

$$-\frac{\sqrt{ad} \tan^{-1}\left(\frac{\sqrt{cx}}{\sqrt{a}}\right)}{c^{3/2}} + \frac{e \log(a + cx^2)}{2c} + \frac{dx}{c}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e/x)/(c + a/x^2), x]

[Out] (d\*x)/c - (Sqrt[a]\*d\*ArcTan[(Sqrt[c]\*x)/Sqrt[a]])/c^(3/2) + (e\*Log[a + c\*x^2])/(2\*c)

**Maple [A]** time = 0.007, size = 43, normalized size = 0.9

$$\frac{dx}{c} + \frac{e \ln(cx^2 + a)}{2c} - \frac{ad}{c} \arctan\left(cx \frac{1}{\sqrt{ac}}\right) \frac{1}{\sqrt{ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d+e/x)/(c+a/x^2),x)

[Out] d\*x/c+1/2\*e\*ln(c\*x^2+a)/c-1/c\*a\*d/(a\*c)^(1/2)\*arctan(c\*x/(a\*c)^(1/2))

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e/x)/(c+a/x^2),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 1.27228, size = 238, normalized size = 4.86

$$\left[ \frac{d\sqrt{-\frac{a}{c}} \log\left(\frac{cx^2 - 2cx\sqrt{-\frac{a}{c}} - a}{cx^2 + a}\right) + 2dx + e \log(cx^2 + a)}{2c}, -\frac{2d\sqrt{\frac{a}{c}} \arctan\left(\frac{cx\sqrt{\frac{a}{c}}}{a}\right) - 2dx - e \log(cx^2 + a)}{2c} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e/x)/(c+a/x^2),x, algorithm="fricas")

[Out] [1/2\*(d\*sqrt(-a/c)\*log((c\*x^2 - 2\*c\*x\*sqrt(-a/c) - a)/(c\*x^2 + a)) + 2\*d\*x + e\*log(c\*x^2 + a))/c, -1/2\*(2\*d\*sqrt(a/c)\*arctan(c\*x\*sqrt(a/c)/a) - 2\*d\*x - e\*log(c\*x^2 + a))/c]

**Sympy [B]** time = 0.438742, size = 112, normalized size = 2.29

$$\left(\frac{e}{2c} - \frac{d\sqrt{-ac^3}}{2c^3}\right) \log\left(x + \frac{-2c\left(\frac{e}{2c} - \frac{d\sqrt{-ac^3}}{2c^3}\right) + e}{d}\right) + \left(\frac{e}{2c} + \frac{d\sqrt{-ac^3}}{2c^3}\right) \log\left(x + \frac{-2c\left(\frac{e}{2c} + \frac{d\sqrt{-ac^3}}{2c^3}\right) + e}{d}\right) + \frac{dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e/x)/(c+a/x\*\*2),x)

[Out] (e/(2\*c) - d\*sqrt(-a\*c\*\*3)/(2\*c\*\*3))\*log(x + (-2\*c\*(e/(2\*c) - d\*sqrt(-a\*c\*\*3)/(2\*c\*\*3)) + e)/d) + (e/(2\*c) + d\*sqrt(-a\*c\*\*3)/(2\*c\*\*3))\*log(x + (-2\*c\*(e/(2\*c) + d\*sqrt(-a\*c\*\*3)/(2\*c\*\*3)) + e)/d) + d\*x/c

**Giac [A]** time = 1.09732, size = 58, normalized size = 1.18

$$-\frac{ad \arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{acc}} + \frac{dx}{c} + \frac{e \log(cx^2 + a)}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e/x)/(c+a/x^2),x, algorithm="giac")

[Out] -a\*d\*arctan(c\*x/sqrt(a\*c))/(sqrt(a\*c)\*c) + d\*x/c + 1/2\*e\*log(c\*x^2 + a)/c

$$3.35 \quad \int \frac{d + \frac{e}{x}}{c + \frac{a}{x^2} + \frac{b}{x}} dx$$

**Optimal.** Leaf size=86

$$-\frac{(-2acd + b^2d - bce) \tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^2\sqrt{b^2-4ac}} - \frac{(bd - ce) \log(a + bx + cx^2)}{2c^2} + \frac{dx}{c}$$

[Out] (d\*x)/c - ((b^2\*d - 2\*a\*c\*d - b\*c\*e)\*ArcTanh[(b + 2\*c\*x)/Sqrt[b^2 - 4\*a\*c]])/(c^2\*Sqrt[b^2 - 4\*a\*c]) - ((b\*d - c\*e)\*Log[a + b\*x + c\*x^2])/(2\*c^2)

**Rubi [A]** time = 0.0812762, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$ , Rules used = {1393, 773, 634, 618, 206, 628}

$$-\frac{(-2acd + b^2d - bce) \tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^2\sqrt{b^2-4ac}} - \frac{(bd - ce) \log(a + bx + cx^2)}{2c^2} + \frac{dx}{c}$$

Antiderivative was successfully verified.

[In] Int[(d + e/x)/(c + a/x^2 + b/x), x]

[Out] (d\*x)/c - ((b^2\*d - 2\*a\*c\*d - b\*c\*e)\*ArcTanh[(b + 2\*c\*x)/Sqrt[b^2 - 4\*a\*c]])/(c^2\*Sqrt[b^2 - 4\*a\*c]) - ((b\*d - c\*e)\*Log[a + b\*x + c\*x^2])/(2\*c^2)

### Rule 1393

Int[((a\_) + (c\_)\*(x\_)^(n2\_)) + (b\_)\*(x\_)^(n\_)]^(p\_)\*((d\_) + (e\_)\*(x\_)^(n\_))^(q\_), x\_Symbol] :> Int[x^(n\*(2\*p + q))\*(e + d/x^n)^q\*(c + b/x^n + a/x^(2\*n))^p, x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2\*n] && IntegersQ[p, q] && NegQ[n]

### Rule 773

Int[(((d\_) + (e\_)\*(x\_))\*((f\_) + (g\_)\*(x\_)))/((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2), x\_Symbol] :> Simp[(e\*g\*x)/c, x] + Dist[1/c, Int[(c\*d\*f - a\*e\*g + (c\*e\*f + c\*d\*g - b\*e\*g)\*x]/(a + b\*x + c\*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4\*a\*c, 0]



Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{d + \frac{e}{x}}{c + \frac{a}{x^2} + \frac{b}{x}} dx &= \int \frac{x(e + dx)}{a + bx + cx^2} dx \\
&= \frac{dx}{c} + \frac{\int \frac{-ad + (-bd + ce)x}{a + bx + cx^2} dx}{c} \\
&= \frac{dx}{c} - \frac{(bd - ce) \int \frac{b + 2cx}{a + bx + cx^2} dx}{2c^2} + \frac{(b^2d - 2acd - bce) \int \frac{1}{a + bx + cx^2} dx}{2c^2} \\
&= \frac{dx}{c} - \frac{(bd - ce) \log(a + bx + cx^2)}{2c^2} - \frac{(b^2d - 2acd - bce) \text{Subst}\left(\int \frac{1}{b^2 - 4ac - x^2} dx, x, b + 2cx\right)}{c^2} \\
&= \frac{dx}{c} - \frac{(b^2d - 2acd - bce) \tanh^{-1}\left(\frac{b + 2cx}{\sqrt{b^2 - 4ac}}\right)}{c^2 \sqrt{b^2 - 4ac}} - \frac{(bd - ce) \log(a + bx + cx^2)}{2c^2}
\end{aligned}$$

**Mathematica [A]** time = 0.0898583, size = 86, normalized size = 1.

$$\frac{2(-2acd+b^2d-bce) \tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right) + (ce-bd) \log(a+x(b+cx)) + 2cdx}{2c^2 \sqrt{4ac-b^2}}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e/x)/(c + a/x^2 + b/x), x]

[Out] (2\*c\*d\*x + (2\*(b^2\*d - 2\*a\*c\*d - b\*c\*e)\*ArcTan[(b + 2\*c\*x)/Sqrt[-b^2 + 4\*a\*c]])/Sqrt[-b^2 + 4\*a\*c] + (-(b\*d) + c\*e)\*Log[a + x\*(b + c\*x)]/(2\*c^2)

**Maple [A]** time = 0.003, size = 161, normalized size = 1.9

$$\frac{dx}{c} - \frac{\ln(cx^2 + bx + a)bd}{2c^2} + \frac{\ln(cx^2 + bx + a)e}{2c} - 2 \frac{ad}{c\sqrt{4ac-b^2}} \arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right) + \frac{b^2d}{c^2} \arctan\left((2cx+b) \frac{1}{\sqrt{4ac-b^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d+e/x)/(c+a/x^2+b/x), x)

[Out] d\*x/c-1/2/c^2\*ln(c\*x^2+b\*x+a)\*b\*d+1/2/c\*ln(c\*x^2+b\*x+a)\*e-2/c/(4\*a\*c-b^2)^(1/2)\*arctan((2\*c\*x+b)/(4\*a\*c-b^2)^(1/2))\*a\*d+1/c^2/(4\*a\*c-b^2)^(1/2)\*arctan((2\*c\*x+b)/(4\*a\*c-b^2)^(1/2))\*b^2\*d-1/c/(4\*a\*c-b^2)^(1/2)\*arctan((2\*c\*x+b)/(4\*a\*c-b^2)^(1/2))\*b\*e

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e/x)/(c+a/x^2+b/x), x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 1.29934, size = 643, normalized size = 7.48

$$\frac{2(b^2c - 4ac^2)dx + (bce - (b^2 - 2ac)d)\sqrt{b^2 - 4ac} \log\left(\frac{2c^2x^2 + 2bcx + b^2 - 2ac + \sqrt{b^2 - 4ac}(2cx + b)}{cx^2 + bx + a}\right) - ((b^3 - 4abc)d - (b^2c - 4ac^2))}{2(b^2c^2 - 4ac^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e/x)/(c+a/x^2+b/x),x, algorithm="fricas")

[Out] [1/2\*(2\*(b^2\*c - 4\*a\*c^2)\*d\*x + (b\*c\*e - (b^2 - 2\*a\*c)\*d)\*sqrt(b^2 - 4\*a\*c) \*log((2\*c^2\*x^2 + 2\*b\*c\*x + b^2 - 2\*a\*c + sqrt(b^2 - 4\*a\*c)\*(2\*c\*x + b))/(c\*x^2 + b\*x + a)) - ((b^3 - 4\*a\*b\*c)\*d - (b^2\*c - 4\*a\*c^2)\*e)\*log(c\*x^2 + b\*x + a))/(b^2\*c^2 - 4\*a\*c^3), 1/2\*(2\*(b^2\*c - 4\*a\*c^2)\*d\*x + 2\*(b\*c\*e - (b^2 - 2\*a\*c)\*d)\*sqrt(-b^2 + 4\*a\*c)\*arctan(-sqrt(-b^2 + 4\*a\*c)\*(2\*c\*x + b)/(b^2 - 4\*a\*c)) - ((b^3 - 4\*a\*b\*c)\*d - (b^2\*c - 4\*a\*c^2)\*e)\*log(c\*x^2 + b\*x + a))/(b^2\*c^2 - 4\*a\*c^3)]

**Sympy [B]** time = 1.26763, size = 423, normalized size = 4.92

$$\left(-\frac{\sqrt{-4ac + b^2}(2acd - b^2d + bce)}{2c^2(4ac - b^2)} - \frac{bd - ce}{2c^2}\right) \log\left(x + \frac{-abd - 4ac^2\left(-\frac{\sqrt{-4ac + b^2}(2acd - b^2d + bce)}{2c^2(4ac - b^2)} - \frac{bd - ce}{2c^2}\right) + 2ace + b^2c\left(-\frac{\sqrt{-4ac + b^2}(2acd - b^2d + bce)}{2c^2(4ac - b^2)} - \frac{bd - ce}{2c^2}\right)}{2acd - b^2d + bce}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e/x)/(c+a/x\*\*2+b/x),x)

[Out] (-sqrt(-4\*a\*c + b\*\*2)\*(2\*a\*c\*d - b\*\*2\*d + b\*c\*e)/(2\*c\*\*2\*(4\*a\*c - b\*\*2)) - (b\*d - c\*e)/(2\*c\*\*2))\*log(x + (-a\*b\*d - 4\*a\*c\*\*2\*(-sqrt(-4\*a\*c + b\*\*2)\*(2\*a\*c\*d - b\*\*2\*d + b\*c\*e)/(2\*c\*\*2\*(4\*a\*c - b\*\*2)) - (b\*d - c\*e)/(2\*c\*\*2)) + 2\*a\*c\*e + b\*\*2\*c\*(-sqrt(-4\*a\*c + b\*\*2)\*(2\*a\*c\*d - b\*\*2\*d + b\*c\*e)/(2\*c\*\*2\*(4\*a\*c - b\*\*2)) - (b\*d - c\*e)/(2\*c\*\*2)))/(2\*a\*c\*d - b\*\*2\*d + b\*c\*e)) + (sqrt(-4\*a\*c + b\*\*2)\*(2\*a\*c\*d - b\*\*2\*d + b\*c\*e)/(2\*c\*\*2\*(4\*a\*c - b\*\*2)) - (b\*d - c\*e)/(2\*c\*\*2))\*log(x + (-a\*b\*d - 4\*a\*c\*\*2\*(sqrt(-4\*a\*c + b\*\*2)\*(2\*a\*c\*d - b\*\*2\*d + b\*c\*e)/(2\*c\*\*2\*(4\*a\*c - b\*\*2)) - (b\*d - c\*e)/(2\*c\*\*2)) + 2\*a\*c\*e + b\*\*2\*c\*(sqrt(-4\*a\*c + b\*\*2)\*(2\*a\*c\*d - b\*\*2\*d + b\*c\*e)/(2\*c\*\*2\*(4\*a\*c - b\*\*2)) - (b\*d - c\*e)/(2\*c\*\*2)))/(2\*a\*c\*d - b\*\*2\*d + b\*c\*e)) + d\*x/c

---

**Giac [A]** time = 1.10065, size = 115, normalized size = 1.34

$$\frac{dx}{c} - \frac{(bd - ce) \log(cx^2 + bx + a)}{2c^2} + \frac{(b^2d - 2acd - bce) \arctan\left(\frac{2cx+b}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e/x)/(c+a/x^2+b/x),x, algorithm="giac")

[Out] d\*x/c - 1/2\*(b\*d - c\*e)\*log(c\*x^2 + b\*x + a)/c^2 + (b^2\*d - 2\*a\*c\*d - b\*c\*e)\*arctan((2\*c\*x + b)/sqrt(-b^2 + 4\*a\*c))/(sqrt(-b^2 + 4\*a\*c)\*c^2)

$$3.36 \quad \int \frac{d + \frac{e}{x^2}}{c + \frac{a}{x^4}} dx$$

**Optimal.** Leaf size=253

$$\frac{(\sqrt{ad} + \sqrt{ce}) \log(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{cx} + \sqrt{a} + \sqrt{cx^2})}{4\sqrt{2}\sqrt[4]{ac^{5/4}}} - \frac{(\sqrt{ad} + \sqrt{ce}) \log(\sqrt{2}\sqrt[4]{a}\sqrt[4]{cx} + \sqrt{a} + \sqrt{cx^2})}{4\sqrt{2}\sqrt[4]{ac^{5/4}}} + \frac{(\sqrt{ad} - \sqrt{ce}) \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{cx} + \sqrt{a} + \sqrt{cx^2}}{2\sqrt{2}\sqrt[4]{ac^{5/4}}}\right)}{2\sqrt{2}\sqrt[4]{ac^{5/4}}}$$

```
[Out] (d*x)/c + ((Sqrt[a]*d - Sqrt[c]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])
/(2*Sqrt[2]*a^(1/4)*c^(5/4)) - ((Sqrt[a]*d - Sqrt[c]*e)*ArcTan[1 + (Sqrt[2]
*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(1/4)*c^(5/4)) + ((Sqrt[a]*d + Sqrt[c]*e
)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(1/4
)*c^(5/4)) - ((Sqrt[a]*d + Sqrt[c]*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4
)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(1/4)*c^(5/4))
```

**Rubi [A]** time = 0.210931, antiderivative size = 253, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.471$ , Rules used = {1394, 1280, 1168, 1162, 617, 204, 1165, 628}

$$\frac{(\sqrt{ad} + \sqrt{ce}) \log(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{cx} + \sqrt{a} + \sqrt{cx^2})}{4\sqrt{2}\sqrt[4]{ac^{5/4}}} - \frac{(\sqrt{ad} + \sqrt{ce}) \log(\sqrt{2}\sqrt[4]{a}\sqrt[4]{cx} + \sqrt{a} + \sqrt{cx^2})}{4\sqrt{2}\sqrt[4]{ac^{5/4}}} + \frac{(\sqrt{ad} - \sqrt{ce}) \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{cx} + \sqrt{a} + \sqrt{cx^2}}{2\sqrt{2}\sqrt[4]{ac^{5/4}}}\right)}{2\sqrt{2}\sqrt[4]{ac^{5/4}}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e/x^2)/(c + a/x^4), x]
```

```
[Out] (d*x)/c + ((Sqrt[a]*d - Sqrt[c]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])
/(2*Sqrt[2]*a^(1/4)*c^(5/4)) - ((Sqrt[a]*d - Sqrt[c]*e)*ArcTan[1 + (Sqrt[2]
*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(1/4)*c^(5/4)) + ((Sqrt[a]*d + Sqrt[c]*e
)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(1/4
)*c^(5/4)) - ((Sqrt[a]*d + Sqrt[c]*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4
)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(1/4)*c^(5/4))
```

### Rule 1394

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_.))^(q_.), x_Symb
ol] :> Int[x^(n*(2*p + q))*(e + d/x^n)^q*(c + a/x^(2*n))^p, x] /; FreeQ[{a,
c, d, e, n}, x] && EqQ[n2, 2*n] && IntegersQ[p, q] && NegQ[n]
```

Rule 1280

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] := Simp[(e*f*(f*x)^(m - 1)*(a + c*x^4)^(p + 1))/(c*(m + 4*p + 3)),
x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(a + c*x^4)^p*(a*e*(m -
1) - c*d*(m + 4*p + 3)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && GtQ[
m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m
])
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]]/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

### Rubi steps

$$\begin{aligned}
\int \frac{d + \frac{e}{x^2}}{c + \frac{a}{x^4}} dx &= \int \frac{x^2(e + dx^2)}{a + cx^4} dx \\
&= \frac{dx}{c} - \frac{\int \frac{ad - cex^2}{a + cx^4} dx}{c} \\
&= \frac{dx}{c} - \frac{\left(\frac{\sqrt{ad}}{\sqrt{c}} - e\right) \int \frac{\sqrt{a}\sqrt{c} + cx^2}{a + cx^4} dx}{2c} - \frac{\left(\frac{\sqrt{ad}}{\sqrt{c}} + e\right) \int \frac{\sqrt{a}\sqrt{c} - cx^2}{a + cx^4} dx}{2c} \\
&= \frac{dx}{c} - \frac{\left(\frac{\sqrt{ad}}{\sqrt{c}} - e\right) \int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}} - \frac{\sqrt{2}\sqrt[4]{ax}}{\sqrt[4]{c}} + x^2} dx}{4c} - \frac{\left(\frac{\sqrt{ad}}{\sqrt{c}} + e\right) \int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}} + \frac{\sqrt{2}\sqrt[4]{ax}}{\sqrt[4]{c}} + x^2} dx}{4c} + \frac{(\sqrt{ad} + \sqrt{ce}) \int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}} + 2x}{-\frac{\sqrt{a}}{\sqrt{c}} - \frac{\sqrt{2}\sqrt[4]{ax}}{\sqrt[4]{c}} - x^2} dx}{4\sqrt{2}\sqrt[4]{ac}^{5/4}} + \dots \\
&= \frac{dx}{c} + \frac{(\sqrt{ad} + \sqrt{ce}) \log(\sqrt{a} - \sqrt{2}\sqrt[4]{a}\sqrt[4]{cx} + \sqrt{cx^2})}{4\sqrt{2}\sqrt[4]{ac}^{5/4}} - \frac{(\sqrt{ad} + \sqrt{ce}) \log(\sqrt{a} + \sqrt{2}\sqrt[4]{a}\sqrt[4]{cx} + \sqrt{cx^2})}{4\sqrt{2}\sqrt[4]{ac}^{5/4}} - \frac{(\sqrt{ad} - \sqrt{ce}) \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{cx}}{\sqrt[4]{a}}\right)}{2\sqrt{2}\sqrt[4]{ac}^{5/4}} - \frac{(\sqrt{ad} - \sqrt{ce}) \tan^{-1}\left(1 + \frac{\sqrt{2}\sqrt[4]{cx}}{\sqrt[4]{a}}\right)}{2\sqrt{2}\sqrt[4]{ac}^{5/4}} + \frac{(\sqrt{ad} + \sqrt{ce}) \log(\sqrt{a} - \sqrt{2}\sqrt[4]{a}\sqrt[4]{cx} + \sqrt{cx^2})}{4\sqrt{2}\sqrt[4]{ac}^{5/4}} + \dots
\end{aligned}$$

**Mathematica [A]** time = 0.106166, size = 293, normalized size = 1.16

$$\frac{(a^{5/4}\sqrt{cd} + a^{3/4}ce) \log(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{cx} + \sqrt{a} + \sqrt{cx^2})}{4\sqrt{2}ac^{7/4}} - \frac{(a^{5/4}\sqrt{cd} + a^{3/4}ce) \log(\sqrt{2}\sqrt[4]{a}\sqrt[4]{cx} + \sqrt{a} + \sqrt{cx^2})}{4\sqrt{2}ac^{7/4}} + \frac{(a^{3/4}ce - a^{5/4}\sqrt{cd}) \log(\sqrt{a} - \sqrt{2}\sqrt[4]{a}\sqrt[4]{cx} + \sqrt{cx^2})}{4\sqrt{2}ac^{7/4}} - \frac{(a^{3/4}ce - a^{5/4}\sqrt{cd}) \log(\sqrt{a} + \sqrt{2}\sqrt[4]{a}\sqrt[4]{cx} + \sqrt{cx^2})}{4\sqrt{2}ac^{7/4}}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e/x^2)/(c + a/x^4), x]

[Out] (d\*x)/c + ((-(a^(5/4)\*Sqrt[c]\*d) + a^(3/4)\*c\*e)\*ArcTan[(-(Sqrt[2]\*a^(1/4)) + 2\*c^(1/4)\*x)/(Sqrt[2]\*a^(1/4))]/(2\*Sqrt[2]\*a\*c^(7/4)) + ((-(a^(5/4)\*Sqrt[c]\*d) + a^(3/4)\*c\*e)\*ArcTan[(Sqrt[2]\*a^(1/4) + 2\*c^(1/4)\*x)/(Sqrt[2]\*a^(1/4))]/(2\*Sqrt[2]\*a\*c^(7/4)) + ((a^(5/4)\*Sqrt[c]\*d + a^(3/4)\*c\*e)\*Log[Sqrt[a] - Sqrt[2]\*a^(1/4)\*c^(1/4)\*x + Sqrt[c]\*x^2]/(4\*Sqrt[2]\*a\*c^(7/4)) - ((a^(5/4)\*Sqrt[c]\*d + a^(3/4)\*c\*e)\*Log[Sqrt[a] + Sqrt[2]\*a^(1/4)\*c^(1/4)\*x + Sqrt[c]\*x^2]/(4\*Sqrt[2]\*a\*c^(7/4))

$5/4 * \text{Sqrt}[c] * d + a^{(3/4)} * c * e) * \text{Log}[\text{Sqrt}[a] + \text{Sqrt}[2] * a^{(1/4)} * c^{(1/4)} * x + \text{Sqrt}[c] * x^2]) / (4 * \text{Sqrt}[2] * a * c^{(7/4)})$

**Maple [A]** time = 0.01, size = 266, normalized size = 1.1

$$\frac{dx}{c} - \frac{d\sqrt{2}}{4c} \sqrt[4]{\frac{a}{c}} \arctan\left(x\sqrt{2} \frac{1}{\sqrt[4]{\frac{a}{c}}} - 1\right) - \frac{d\sqrt{2}}{8c} \sqrt[4]{\frac{a}{c}} \ln\left(\left(x^2 + \sqrt[4]{\frac{a}{c}} x\sqrt{2} + \sqrt{\frac{a}{c}}\right)\left(x^2 - \sqrt[4]{\frac{a}{c}} x\sqrt{2} + \sqrt{\frac{a}{c}}\right)^{-1}\right) - \frac{d\sqrt{2}}{4c} \sqrt[4]{\frac{a}{c}} \arctan\left(x\sqrt{2} \frac{1}{\sqrt[4]{\frac{a}{c}}} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d+e/x^2)/(c+a/x^4),x)

[Out]  $d*x/c - 1/4/c*d*(a/c)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(a/c)^{(1/4)}*x-1) - 1/8/c*d*(a/c)^{(1/4)}*2^{(1/2)}*\ln((x^2+(a/c)^{(1/4)}*x*2^{(1/2)}+(a/c)^{(1/2)})/(x^2-(a/c)^{(1/4)}*x*2^{(1/2)}+(a/c)^{(1/2)})) - 1/4/c*d*(a/c)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(a/c)^{(1/4)}*x+1) + 1/8/c*e/(a/c)^{(1/4)}*2^{(1/2)}*\ln((x^2-(a/c)^{(1/4)}*x*2^{(1/2)}+(a/c)^{(1/2)})/(x^2+(a/c)^{(1/4)}*x*2^{(1/2)}+(a/c)^{(1/2)})) + 1/4/c*e/(a/c)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(a/c)^{(1/4)}*x+1) + 1/4/c*e/(a/c)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(a/c)^{(1/4)}*x-1)$

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e/x^2)/(c+a/x^4),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [B]** time = 1.36182, size = 1489, normalized size = 5.89

$$c\sqrt{\frac{c^2\sqrt{-\frac{a^2d^4-2acd^2e^2+c^2e^4}{ac^5}}+2de}{c^2}} \log\left(-\left(a^2d^4-c^2e^4\right)x + \left(ac^4e\sqrt{-\frac{a^2d^4-2acd^2e^2+c^2e^4}{ac^5}} + a^2cd^3 - ac^2de^2\right)\sqrt{\frac{c^2\sqrt{-\frac{a^2d^4-2acd^2e^2+c^2e^4}{ac^5}}+2de}{c^2}}\right)$$



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e/x^2)/(c+a/x^4),x, algorithm="fricas")

[Out]  $\frac{1}{4} * (c * \sqrt{(c^2 * \sqrt{-(a^2 * d^4 - 2 * a * c * d^2 * e^2 + c^2 * e^4)} / (a * c^5)) + 2 * d * e} / c^2) * \log(-(a^2 * d^4 - c^2 * e^4) * x + (a * c^4 * e * \sqrt{-(a^2 * d^4 - 2 * a * c * d^2 * e^2 + c^2 * e^4)} / (a * c^5)) + a^2 * c * d^3 - a * c^2 * d * e^2) * \sqrt{(c^2 * \sqrt{-(a^2 * d^4 - 2 * a * c * d^2 * e^2 + c^2 * e^4)} / (a * c^5)) + 2 * d * e} / c^2) - c * \sqrt{(c^2 * \sqrt{-(a^2 * d^4 - 2 * a * c * d^2 * e^2 + c^2 * e^4)} / (a * c^5)) + 2 * d * e} / c^2) * \log(-(a^2 * d^4 - c^2 * e^4) * x - (a * c^4 * e * \sqrt{-(a^2 * d^4 - 2 * a * c * d^2 * e^2 + c^2 * e^4)} / (a * c^5)) + a^2 * c * d^3 - a * c^2 * d * e^2) * \sqrt{(c^2 * \sqrt{-(a^2 * d^4 - 2 * a * c * d^2 * e^2 + c^2 * e^4)} / (a * c^5)) + 2 * d * e} / c^2) - c * \sqrt{-(c^2 * \sqrt{-(a^2 * d^4 - 2 * a * c * d^2 * e^2 + c^2 * e^4)} / (a * c^5)) - 2 * d * e} / c^2) * \log(-(a^2 * d^4 - c^2 * e^4) * x + (a * c^4 * e * \sqrt{-(a^2 * d^4 - 2 * a * c * d^2 * e^2 + c^2 * e^4)} / (a * c^5)) - a^2 * c * d^3 + a * c^2 * d * e^2) * \sqrt{-(c^2 * \sqrt{-(a^2 * d^4 - 2 * a * c * d^2 * e^2 + c^2 * e^4)} / (a * c^5)) - 2 * d * e} / c^2) + c * \sqrt{-(c^2 * \sqrt{-(a^2 * d^4 - 2 * a * c * d^2 * e^2 + c^2 * e^4)} / (a * c^5)) - 2 * d * e} / c^2) * \log(-(a^2 * d^4 - c^2 * e^4) * x - (a * c^4 * e * \sqrt{-(a^2 * d^4 - 2 * a * c * d^2 * e^2 + c^2 * e^4)} / (a * c^5)) - a^2 * c * d^3 + a * c^2 * d * e^2) * \sqrt{-(c^2 * \sqrt{-(a^2 * d^4 - 2 * a * c * d^2 * e^2 + c^2 * e^4)} / (a * c^5)) - 2 * d * e} / c^2) + 4 * d * x / c$

**Sympy [A]** time = 0.838989, size = 109, normalized size = 0.43

$\text{RootSum}\left(256t^4ac^5 - 64t^2ac^3de + a^2d^4 + 2acd^2e^2 + c^2e^4, \left(t \mapsto t \log\left(x + \frac{-64t^3ac^4e - 4ta^2cd^3 + 12tac^2de^2}{a^2d^4 - c^2e^4}\right)\right)\right) + \frac{dx}{c}$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e/x\*\*2)/(c+a/x\*\*4),x)

[Out]  $\text{RootSum}(256 * _t^{**4} * a * c^{**5} - 64 * _t^{**2} * a * c^{**3} * d * e + a^{**2} * d^{**4} + 2 * a * c * d^{**2} * e^{**2} + c^{**2} * e^{**4}, \text{Lambda}(_t, _t * \log(x + (-64 * _t^{**3} * a * c^{**4} * e - 4 * _t * a^{**2} * c * d^{**3} + 12 * _t * a * c^{**2} * d * e^{**2}) / (a^{**2} * d^{**4} - c^{**2} * e^{**4})))) + d * x / c$

**Giac [A]** time = 1.10218, size = 347, normalized size = 1.37

$\frac{dx}{c} - \frac{\sqrt{2} \left( (ac^3)^{\frac{1}{4}} acd - (ac^3)^{\frac{3}{4}} e \right) \arctan\left( \frac{\sqrt{2} \left( 2x - \sqrt{2} \left( \frac{a}{c} \right)^{\frac{1}{4}} \right)}{2 \left( \frac{a}{c} \right)^{\frac{1}{4}}} \right)}{4ac^3} + \frac{\sqrt{2} \left( (ac^3)^{\frac{1}{4}} acd + (ac^3)^{\frac{3}{4}} e \right) \log\left( x^2 - \sqrt{2} x \left( \frac{a}{c} \right)^{\frac{1}{4}} + \sqrt{\frac{a}{c}} \right)}{8ac^3} - \frac{\sqrt{2} \left( (ac^3)^{\frac{1}{4}} acd - (ac^3)^{\frac{3}{4}} e \right)}{4ac^3}$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e/x^2)/(c+a/x^4),x, algorithm="giac")

[Out]  $d*x/c - 1/4*\sqrt{2}*((a*c^3)^{1/4}*a*c*d - (a*c^3)^{3/4}*e)*\arctan(1/2*\sqrt{2}*(2*x - \sqrt{2}*(a/c)^{1/4})/(a/c)^{1/4})/(a*c^3) + 1/8*\sqrt{2}*((a*c^3)^{1/4}*a*c*d + (a*c^3)^{3/4}*e)*\log(x^2 - \sqrt{2}*x*(a/c)^{1/4} + \sqrt{a/c})/(a*c^3) - 1/4*\sqrt{2}*((a*c^3)^{1/4}*a*c^3*d - (a*c^3)^{3/4}*c^2*e)*\arctan(1/2*\sqrt{2}*(2*x + \sqrt{2}*(a/c)^{1/4})/(a/c)^{1/4})/(a*c^5) - 1/8*\sqrt{2}*((a*c^3)^{1/4}*a*c^3*d + (a*c^3)^{3/4}*c^2*e)*\log(x^2 + \sqrt{2}*x*(a/c)^{1/4} + \sqrt{a/c})/(a*c^5)$

$$3.37 \quad \int \frac{d + \frac{e}{x^2}}{c + \frac{a}{x^4} + \frac{b}{x^2}} dx$$

**Optimal.** Leaf size=208

$$\frac{\left(-\frac{-2acd+b^2d-bce}{\sqrt{b^2-4ac}} + bd - ce\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\left(-\frac{-2acd+b^2d-bce}{\sqrt{b^2-4ac}} + bd - ce\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}c^{3/2}\sqrt{\sqrt{b^2-4ac}+b}} + \frac{dx}{c}$$

[Out] (d\*x)/c - ((b\*d - c\*e - (b^2\*d - 2\*a\*c\*d - b\*c\*e)/Sqrt[b^2 - 4\*a\*c])\*ArcTan[(Sqrt[2]\*Sqrt[c]\*x)/Sqrt[b - Sqrt[b^2 - 4\*a\*c]]]/(Sqrt[2]\*c^(3/2)\*Sqrt[b - Sqrt[b^2 - 4\*a\*c]]) - ((b\*d - c\*e + (b^2\*d - 2\*a\*c\*d - b\*c\*e)/Sqrt[b^2 - 4\*a\*c])\*ArcTan[(Sqrt[2]\*Sqrt[c]\*x)/Sqrt[b + Sqrt[b^2 - 4\*a\*c]]]/(Sqrt[2]\*c^(3/2)\*Sqrt[b + Sqrt[b^2 - 4\*a\*c]])

**Rubi [A]** time = 0.542967, antiderivative size = 208, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$ , Rules used = {1393, 1279, 1166, 205}

$$\frac{\left(-\frac{-2acd+b^2d-bce}{\sqrt{b^2-4ac}} + bd - ce\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\left(-\frac{-2acd+b^2d-bce}{\sqrt{b^2-4ac}} + bd - ce\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}c^{3/2}\sqrt{\sqrt{b^2-4ac}+b}} + \frac{dx}{c}$$

Antiderivative was successfully verified.

[In] Int[(d + e/x^2)/(c + a/x^4 + b/x^2), x]

[Out] (d\*x)/c - ((b\*d - c\*e - (b^2\*d - 2\*a\*c\*d - b\*c\*e)/Sqrt[b^2 - 4\*a\*c])\*ArcTan[(Sqrt[2]\*Sqrt[c]\*x)/Sqrt[b - Sqrt[b^2 - 4\*a\*c]]]/(Sqrt[2]\*c^(3/2)\*Sqrt[b - Sqrt[b^2 - 4\*a\*c]]) - ((b\*d - c\*e + (b^2\*d - 2\*a\*c\*d - b\*c\*e)/Sqrt[b^2 - 4\*a\*c])\*ArcTan[(Sqrt[2]\*Sqrt[c]\*x)/Sqrt[b + Sqrt[b^2 - 4\*a\*c]]]/(Sqrt[2]\*c^(3/2)\*Sqrt[b + Sqrt[b^2 - 4\*a\*c]])

**Rule 1393**

Int[((a\_) + (c\_.)\*(x\_)^(n2\_.) + (b\_.)\*(x\_)^(n\_.))^(p\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))^(q\_.), x\_Symbol] :> Int[x^(n\*(2\*p + q))\*(e + d/x^n)^q\*(c + b/x^n + a/x^(2\*n))^p, x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2\*n] && IntegersQ[

p, q] && NegQ[n]

### Rule 1279

Int[((f\_)\*(x\_))^(m\_)\*((d\_) + (e\_)\*(x\_)^2)\*((a\_) + (b\_)\*(x\_)^2 + (c\_)\*(x\_)^4)^(p\_), x\_Symbol] :> Simp[(e\*f\*(f\*x)^(m - 1)\*(a + b\*x^2 + c\*x^4)^(p + 1))/(c\*(m + 4\*p + 3)), x] - Dist[f^2/(c\*(m + 4\*p + 3)), Int[(f\*x)^(m - 2)\*(a + b\*x^2 + c\*x^4)^p\*Simp[a\*e\*(m - 1) + (b\*e\*(m + 2\*p + 1) - c\*d\*(m + 4\*p + 3))\*x^2, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4\*a\*c, 0] && GtQ[m, 1] && NeQ[m + 4\*p + 3, 0] && IntegerQ[2\*p] && (IntegerQ[p] || IntegerQ[m])

### Rule 1166

Int[((d\_) + (e\_)\*(x\_)^2)/((a\_) + (b\_)\*(x\_)^2 + (c\_)\*(x\_)^4), x\_Symbol] :> With[{q = Rt[b^2 - 4\*a\*c, 2]}, Dist[e/2 + (2\*c\*d - b\*e)/(2\*q), Int[1/(b/2 - q/2 + c\*x^2), x], x] + Dist[e/2 - (2\*c\*d - b\*e)/(2\*q), Int[1/(b/2 + q/2 + c\*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - a\*e^2, 0] && PosQ[b^2 - 4\*a\*c]

### Rule 205

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

### Rubi steps

$$\begin{aligned}
 \int \frac{d + \frac{e}{x^2}}{c + \frac{a}{x^4} + \frac{b}{x^2}} dx &= \int \frac{x^2(e + dx^2)}{a + bx^2 + cx^4} dx \\
 &= \frac{dx}{c} - \frac{\int \frac{ad + (bd - ce)x^2}{a + bx^2 + cx^4} dx}{c} \\
 &= \frac{dx}{c} - \frac{\left(bd - ce - \frac{b^2d - 2acd - bce}{\sqrt{b^2 - 4ac}}\right) \int \frac{1}{\frac{b}{2} - \frac{1}{2}\sqrt{b^2 - 4ac} + cx^2} dx}{2c} - \frac{\left(bd - ce + \frac{b^2d - 2acd - bce}{\sqrt{b^2 - 4ac}}\right) \int \frac{1}{\frac{b}{2} + \frac{1}{2}\sqrt{b^2 - 4ac} + cx^2} dx}{2c} \\
 &= \frac{dx}{c} - \frac{\left(bd - ce - \frac{b^2d - 2acd - bce}{\sqrt{b^2 - 4ac}}\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b - \sqrt{b^2 - 4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b - \sqrt{b^2 - 4ac}}} - \frac{\left(bd - ce + \frac{b^2d - 2acd - bce}{\sqrt{b^2 - 4ac}}\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b + \sqrt{b^2 - 4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b + \sqrt{b^2 - 4ac}}}
 \end{aligned}$$

**Mathematica [A]** time = 0.190567, size = 251, normalized size = 1.21

$$\frac{\left(bd\sqrt{b^2-4ac}-ce\sqrt{b^2-4ac}+2acd+b^2(-d)+bce\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(bd\sqrt{b^2-4ac}-ce\sqrt{b^2-4ac}-2acd+b^2d\right)}{\sqrt{2}c^{3/2}\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}}}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e/x^2)/(c + a/x^4 + b/x^2), x]

[Out] (d\*x)/c - ((- (b^2\*d) + 2\*a\*c\*d + b\*Sqrt[b^2 - 4\*a\*c]\*d + b\*c\*e - c\*Sqrt[b^2 - 4\*a\*c]\*e)\*ArcTan[(Sqrt[2]\*Sqrt[c]\*x)/Sqrt[b - Sqrt[b^2 - 4\*a\*c]])/(Sqrt[2]\*c^(3/2)\*Sqrt[b^2 - 4\*a\*c]\*Sqrt[b - Sqrt[b^2 - 4\*a\*c]]) - ((b^2\*d - 2\*a\*c\*d + b\*Sqrt[b^2 - 4\*a\*c]\*d - b\*c\*e - c\*Sqrt[b^2 - 4\*a\*c]\*e)\*ArcTan[(Sqrt[2]\*Sqrt[c]\*x)/Sqrt[b + Sqrt[b^2 - 4\*a\*c]])/(Sqrt[2]\*c^(3/2)\*Sqrt[b^2 - 4\*a\*c]\*Sqrt[b + Sqrt[b^2 - 4\*a\*c]])

**Maple [B]** time = 0.027, size = 560, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d+e/x^2)/(c+a/x^4+b/x^2), x)

[Out] d\*x/c-1/2/c\*2^(1/2)/((b+(-4\*a\*c+b^2)^(1/2))\*c)^(1/2)\*arctan(c\*x\*2^(1/2)/((b+(-4\*a\*c+b^2)^(1/2))\*c)^(1/2))\*b\*d+1/2\*2^(1/2)/((b+(-4\*a\*c+b^2)^(1/2))\*c)^(1/2)\*arctan(c\*x\*2^(1/2)/((b+(-4\*a\*c+b^2)^(1/2))\*c)^(1/2))\*e+1/(-4\*a\*c+b^2)^(1/2)\*2^(1/2)/((b+(-4\*a\*c+b^2)^(1/2))\*c)^(1/2)\*arctan(c\*x\*2^(1/2)/((b+(-4\*a\*c+b^2)^(1/2))\*c)^(1/2))\*a\*d-1/2/c/(-4\*a\*c+b^2)^(1/2)\*2^(1/2)/((b+(-4\*a\*c+b^2)^(1/2))\*c)^(1/2)\*arctan(c\*x\*2^(1/2)/((b+(-4\*a\*c+b^2)^(1/2))\*c)^(1/2))\*b^2\*d+1/2/(-4\*a\*c+b^2)^(1/2)\*2^(1/2)/((b+(-4\*a\*c+b^2)^(1/2))\*c)^(1/2)\*arctan(c\*x\*2^(1/2)/((b+(-4\*a\*c+b^2)^(1/2))\*c)^(1/2))\*b\*e+1/2/c\*2^(1/2)/((-b+(-4\*a\*c+b^2)^(1/2))\*c)^(1/2)\*arctanh(c\*x\*2^(1/2)/((-b+(-4\*a\*c+b^2)^(1/2))\*c)^(1/2))\*b\*d-1/2\*2^(1/2)/((-b+(-4\*a\*c+b^2)^(1/2))\*c)^(1/2)\*arctanh(c\*x\*2^(1/2)/((-b+(-4\*a\*c+b^2)^(1/2))\*c)^(1/2))\*e+1/(-4\*a\*c+b^2)^(1/2)\*2^(1/2)/((-b+(-4\*a\*c+b^2)^(1/2))\*c)^(1/2)\*arctanh(c\*x\*2^(1/2)/((-b+(-4\*a\*c+b^2)^(1/2))\*c)^(1/2))\*a\*d-1/2/c/(-4\*a\*c+b^2)^(1/2)\*2^(1/2)/((-b+(-4\*a\*c+b^2)^(1/2))\*c)^(1/2)\*arctanh(c\*x\*2^(1/2)/((-b+(-4\*a\*c+b^2)^(1/2))\*c)^(1/2))\*b^2\*d+1/2/(-4\*a\*c+b^2)^(1/2)\*2^(1/2)/((-b+(-4\*a\*c+b^2)^(1/2))\*c)^(1/2)\*arctanh(c\*x\*2^(1/2)/((-b+(-4\*a\*c+b^2)^(1/2))\*c)^(1/2))

$$(-4*a*c+b^2)^{(1/2)}*c^{(1/2)}*b*e$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{dx}{c} + \frac{-\int \frac{(bd-ce)x^2+ad}{cx^4+bx^2+a} dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e/x^2)/(c+a/x^4+b/x^2),x, algorithm="maxima")

[Out] d\*x/c + integrate(-((b\*d - c\*e)\*x^2 + a\*d)/(c\*x^4 + b\*x^2 + a), x)/c

**Fricas [B]** time = 1.90901, size = 5042, normalized size = 24.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e/x^2)/(c+a/x^4+b/x^2),x, algorithm="fricas")

[Out]  $\frac{1}{2} * (\sqrt{1/2} * c * \sqrt{-(b*c^2*e^2 + (b^3 - 3*a*b*c)*d^2 - 2*(b^2*c - 2*a*c^2)*d*e + (b^2*c^3 - 4*a*c^4)*\sqrt{-(4*b*c^3*d*e^3 - c^4*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^4 + 4*(b^3*c - a*b*c^2)*d^3*e - 2*(3*b^2*c^2 - a*c^3)*d^2*e^2})/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4) * \log(2*(3*b^2*c*d^2*e^2 - 3*b*c^2*d*e^3 + c^3*e^4 + (a*b^2 - a^2*c)*d^4 - (b^3 + a*b*c)*d^3*e)*x + \sqrt{1/2} * ((b^4 - 5*a*b^2*c + 4*a^2*c^2)*d^3 - 2*(b^3*c - 4*a*b*c^2)*d^2*e + (b^2*c^2 - 4*a*c^3)*d*e^2 - ((b^3*c^3 - 4*a*b*c^4)*d - 2*(b^2*c^4 - 4*a*c^5)*e)*\sqrt{-(4*b*c^3*d*e^3 - c^4*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^4 + 4*(b^3*c - a*b*c^2)*d^3*e - 2*(3*b^2*c^2 - a*c^3)*d^2*e^2})/(b^2*c^6 - 4*a*c^7)) * \sqrt{-(b*c^2*e^2 + (b^3 - 3*a*b*c)*d^2 - 2*(b^2*c - 2*a*c^2)*d*e + (b^2*c^3 - 4*a*c^4)*\sqrt{-(4*b*c^3*d*e^3 - c^4*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^4 + 4*(b^3*c - a*b*c^2)*d^3*e - 2*(3*b^2*c^2 - a*c^3)*d^2*e^2})/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4) - \sqrt{1/2} * c * \sqrt{-(b*c^2*e^2 + (b^3 - 3*a*b*c)*d^2 - 2*(b^2*c - 2*a*c^2)*d*e + (b^2*c^3 - 4*a*c^4)*\sqrt{-(4*b*c^3*d*e^3 - c^4*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^4 + 4*(b^3*c - a*b*c^2)*d^3*e - 2*(3*b^2*c^2 - a*c^3)*d^2*e^2})/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4) * \log(2*(3*b^2*c*d^2*e^2 - 3*b*c^2*d*e^3 + c^3*e^4 + (a*b^2 - a^2*c)*d^4 - (b^3 + a*b*c)*d^3*e)*x - \sqrt{1/2} * ((b^4 - 5*a*b^2*c + 4*a^2*c^2)*d^3 - 2*$

$$\begin{aligned}
& (b^3c - 4ab^2c^2)d^2e + (b^2c^2 - 4a^2c^3)de^2 - ((b^3c^3 - 4ab^2c^4)d - 2(b^2c^4 - 4a^2c^5)e)\sqrt{-(4b^3c^3d^3e^3 - c^4e^4 - (b^4 - 2ab^2c + a^2c^2)d^4 + 4(b^3c - ab^2c^2)d^3e - 2(3b^2c^2 - a^2c^3)d^2e^2)/(b^2c^6 - 4a^2c^7))}\sqrt{-(b^2c^2e^2 + (b^3 - 3ab^2c)d^2 - 2(b^2c - 2a^2c^2)de + (b^2c^3 - 4a^2c^4)\sqrt{-(4b^3c^3d^3e^3 - c^4e^4 - (b^4 - 2ab^2c + a^2c^2)d^4 + 4(b^3c - ab^2c^2)d^3e - 2(3b^2c^2 - a^2c^3)d^2e^2)/(b^2c^6 - 4a^2c^7))})/(b^2c^3 - 4a^2c^4))} + \sqrt{1/2}c\sqrt{-(b^2c^2e^2 + (b^3 - 3ab^2c)d^2 - 2(b^2c - 2a^2c^2)de - (b^2c^3 - 4a^2c^4)\sqrt{-(4b^3c^3d^3e^3 - c^4e^4 - (b^4 - 2ab^2c + a^2c^2)d^4 + 4(b^3c - ab^2c^2)d^3e - 2(3b^2c^2 - a^2c^3)d^2e^2)/(b^2c^6 - 4a^2c^7))})/(b^2c^3 - 4a^2c^4)}\log(2(3b^2cd^2e^2 - 3b^2c^2d^3e^3 + c^3e^4 + (ab^2 - a^2c)d^4 - (b^3 + abc)d^3e)x + \sqrt{1/2}((b^4 - 5ab^2c + 4a^2c^2)d^3 - 2(b^3c - 4ab^2c^2)d^2e + (b^2c^2 - 4a^2c^3)d^2e^2 + ((b^3c^3 - 4ab^2c^4)d - 2(b^2c^4 - 4a^2c^5)e)\sqrt{-(4b^3c^3d^3e^3 - c^4e^4 - (b^4 - 2ab^2c + a^2c^2)d^4 + 4(b^3c - ab^2c^2)d^3e - 2(3b^2c^2 - a^2c^3)d^2e^2)/(b^2c^6 - 4a^2c^7))})\sqrt{-(b^2c^2e^2 + (b^3 - 3ab^2c)d^2 - 2(b^2c - 2a^2c^2)de - (b^2c^3 - 4a^2c^4)\sqrt{-(4b^3c^3d^3e^3 - c^4e^4 - (b^4 - 2ab^2c + a^2c^2)d^4 + 4(b^3c - ab^2c^2)d^3e - 2(3b^2c^2 - a^2c^3)d^2e^2)/(b^2c^6 - 4a^2c^7))})/(b^2c^3 - 4a^2c^4))} - \sqrt{1/2}c\sqrt{-(b^2c^2e^2 + (b^3 - 3ab^2c)d^2 - 2(b^2c - 2a^2c^2)de - (b^2c^3 - 4a^2c^4)\sqrt{-(4b^3c^3d^3e^3 - c^4e^4 - (b^4 - 2ab^2c + a^2c^2)d^4 + 4(b^3c - ab^2c^2)d^3e - 2(3b^2c^2 - a^2c^3)d^2e^2)/(b^2c^6 - 4a^2c^7))})/(b^2c^3 - 4a^2c^4)}\log(2(3b^2c^2d^2e^2 - 3b^2c^2d^3e^3 + c^3e^4 + (ab^2 - a^2c)d^4 - (b^3 + abc)d^3e)x - \sqrt{1/2}((b^4 - 5ab^2c + 4a^2c^2)d^3 - 2(b^3c - 4ab^2c^2)d^2e + (b^2c^2 - 4a^2c^3)d^2e^2 + ((b^3c^3 - 4ab^2c^4)d - 2(b^2c^4 - 4a^2c^5)e)\sqrt{-(4b^3c^3d^3e^3 - c^4e^4 - (b^4 - 2ab^2c + a^2c^2)d^4 + 4(b^3c - ab^2c^2)d^3e - 2(3b^2c^2 - a^2c^3)d^2e^2)/(b^2c^6 - 4a^2c^7))})\sqrt{-(b^2c^2e^2 + (b^3 - 3ab^2c)d^2 - 2(b^2c - 2a^2c^2)de - (b^2c^3 - 4a^2c^4)\sqrt{-(4b^3c^3d^3e^3 - c^4e^4 - (b^4 - 2ab^2c + a^2c^2)d^4 + 4(b^3c - ab^2c^2)d^3e - 2(3b^2c^2 - a^2c^3)d^2e^2)/(b^2c^6 - 4a^2c^7))})/(b^2c^3 - 4a^2c^4))} + 2dx)/c
\end{aligned}$$

**Sympy [B]** time = 10.6236, size = 428, normalized size = 2.06

$$\text{RootSum}\left(t^4(256a^2c^5 - 128ab^2c^4 + 16b^4c^3) + t^2(48a^2bc^2d^2 - 64a^2c^3de - 28ab^3cd^2 + 48ab^2c^2de - 16abc^3e^2 + 4b^5d^2 - \dots\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e/x\*\*2)/(c+a/x\*\*4+b/x\*\*2), x)

```
[Out] RootSum(_t**4*(256*a**2*c**5 - 128*a*b**2*c**4 + 16*b**4*c**3) + _t**2*(48*
a**2*b*c**2*d**2 - 64*a**2*c**3*d*e - 28*a*b**3*c*d**2 + 48*a*b**2*c**2*d*e
- 16*a*b*c**3*e**2 + 4*b**5*d**2 - 8*b**4*c*d*e + 4*b**3*c**2*e**2) + a**3
*d**4 - 2*a**2*b*d**3*e + 2*a**2*c*d**2*e**2 + a*b**2*d**2*e**2 - 2*a*b*c*d
*e**3 + a*c**2*e**4, Lambda(_t, _t*log(x + (32*_t**3*a*b*c**4*d - 64*_t**3*
a*c**5*e - 8*_t**3*b**3*c**3*d + 16*_t**3*b**2*c**4*e - 4*_t*a**2*c**2*d**3
+ 8*_t*a*b**2*c*d**3 - 18*_t*a*b*c**2*d**2*e + 12*_t*a*c**3*d*e**2 - 2*_t*
b**4*d**3 + 6*_t*b**3*c*d**2*e - 6*_t*b**2*c**2*d*e**2 + 2*_t*b*c**3*e**3)/
(a**2*c*d**4 - a*b**2*d**4 + a*b*c*d**3*e + b**3*d**3*e - 3*b**2*c*d**2*e**
2 + 3*b*c**2*d*e**3 - c**3*e**4)))) + d*x/c
```

---

**Giac [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e/x^2)/(c+a/x^4+b/x^2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```



$$3.38 \quad \int \frac{d + \frac{e}{x^3}}{c + \frac{a}{x^6}} dx$$

**Optimal.** Leaf size=311

$$\frac{(\sqrt{3}\sqrt{ad} + \sqrt{ce}) \log(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{cx} + \sqrt[3]{a} + \sqrt[3]{cx^2})}{12\sqrt[3]{ac^{7/6}}} - \frac{(\sqrt{3}\sqrt{ad} - \sqrt{ce}) \log(\sqrt{3}\sqrt[6]{a}\sqrt[6]{cx} + \sqrt[3]{a} + \sqrt[3]{cx^2})}{12\sqrt[3]{ac^{7/6}}} + \frac{(\sqrt{ad} - \sqrt{3}\sqrt{ce}) \log(\sqrt{3}\sqrt[6]{a}\sqrt[6]{cx} + \sqrt[3]{a} + \sqrt[3]{cx^2})}{6\sqrt[3]{a}}$$

[Out] (d\*x)/c - (a^(1/6)\*d\*ArcTan[(c^(1/6)\*x)/a^(1/6)])/(3\*c^(7/6)) + ((Sqrt[a]\*d - Sqrt[3]\*Sqrt[c]\*e)\*ArcTan[Sqrt[3] - (2\*c^(1/6)\*x)/a^(1/6)])/(6\*a^(1/3)\*c^(7/6)) - ((Sqrt[a]\*d + Sqrt[3]\*Sqrt[c]\*e)\*ArcTan[Sqrt[3] + (2\*c^(1/6)\*x)/a^(1/6)])/(6\*a^(1/3)\*c^(7/6)) - (e\*Log[a^(1/3) + c^(1/3)\*x^2])/(6\*a^(1/3)\*c^(2/3)) + ((Sqrt[3]\*Sqrt[a]\*d + Sqrt[c]\*e)\*Log[a^(1/3) - Sqrt[3]\*a^(1/6)\*c^(1/6)\*x + c^(1/3)\*x^2])/(12\*a^(1/3)\*c^(7/6)) - ((Sqrt[3]\*Sqrt[a]\*d - Sqrt[c]\*e)\*Log[a^(1/3) + Sqrt[3]\*a^(1/6)\*c^(1/6)\*x + c^(1/3)\*x^2])/(12\*a^(1/3)\*c^(7/6))

**Rubi [A]** time = 0.289657, antiderivative size = 311, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 10, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.588$ , Rules used = {1394, 1503, 1416, 635, 203, 260, 634, 617, 204, 628}

$$\frac{(\sqrt{3}\sqrt{ad} + \sqrt{ce}) \log(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{cx} + \sqrt[3]{a} + \sqrt[3]{cx^2})}{12\sqrt[3]{ac^{7/6}}} - \frac{(\sqrt{3}\sqrt{ad} - \sqrt{ce}) \log(\sqrt{3}\sqrt[6]{a}\sqrt[6]{cx} + \sqrt[3]{a} + \sqrt[3]{cx^2})}{12\sqrt[3]{ac^{7/6}}} + \frac{(\sqrt{ad} - \sqrt{3}\sqrt{ce}) \log(\sqrt{3}\sqrt[6]{a}\sqrt[6]{cx} + \sqrt[3]{a} + \sqrt[3]{cx^2})}{6\sqrt[3]{a}}$$

Antiderivative was successfully verified.

[In] Int[(d + e/x^3)/(c + a/x^6), x]

[Out] (d\*x)/c - (a^(1/6)\*d\*ArcTan[(c^(1/6)\*x)/a^(1/6)])/(3\*c^(7/6)) + ((Sqrt[a]\*d - Sqrt[3]\*Sqrt[c]\*e)\*ArcTan[Sqrt[3] - (2\*c^(1/6)\*x)/a^(1/6)])/(6\*a^(1/3)\*c^(7/6)) - ((Sqrt[a]\*d + Sqrt[3]\*Sqrt[c]\*e)\*ArcTan[Sqrt[3] + (2\*c^(1/6)\*x)/a^(1/6)])/(6\*a^(1/3)\*c^(7/6)) - (e\*Log[a^(1/3) + c^(1/3)\*x^2])/(6\*a^(1/3)\*c^(2/3)) + ((Sqrt[3]\*Sqrt[a]\*d + Sqrt[c]\*e)\*Log[a^(1/3) - Sqrt[3]\*a^(1/6)\*c^(1/6)\*x + c^(1/3)\*x^2])/(12\*a^(1/3)\*c^(7/6)) - ((Sqrt[3]\*Sqrt[a]\*d - Sqrt[c]\*e)\*Log[a^(1/3) + Sqrt[3]\*a^(1/6)\*c^(1/6)\*x + c^(1/3)\*x^2])/(12\*a^(1/3)\*c^(7/6))

**Rule 1394**

```
Int[((a_) + (c_)*(x_)^(n2_))^(p_)*((d_) + (e_)*(x_)^(n_))^(q_), x_Symbol]
:= Int[x^(n*(2*p + q))*(e + d/x^n)^q*(c + a/x^(2*n))^p, x] /; FreeQ[{a,
c, d, e, n}, x] && EqQ[n2, 2*n] && IntegersQ[p, q] && NegQ[n]
```

### Rule 1503

```
Int[((f_)*(x_))^(m_)*((d_) + (e_)*(x_)^(n_))*((a_) + (c_)*(x_)^(n2_))^(
p_), x_Symbol] := Simp[(e*f^(n - 1)*(f*x)^(m - n + 1)*(a + c*x^(2*n))^(p +
1))/(c*(m + n*(2*p + 1) + 1)), x] - Dist[f^n/(c*(m + n*(2*p + 1) + 1)), Int
[(f*x)^(m - n)*(a + c*x^(2*n))^p*(a*e*(m - n + 1) - c*d*(m + n*(2*p + 1) +
1)*x^n), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && EqQ[n2, 2*n] && IGtQ[n,
0] && GtQ[m, n - 1] && NeQ[m + n*(2*p + 1) + 1, 0] && IntegerQ[p]
```

### Rule 1416

```
Int[((d_) + (e_)*(x_)^3)/((a_) + (c_)*(x_)^6), x_Symbol] := With[{q = Rt[
c/a, 6]}, Dist[1/(3*a*q^2), Int[(q^2*d - e*x)/(1 + q^2*x^2), x], x] + (Dist
[1/(6*a*q^2), Int[(2*q^2*d - (Sqrt[3]*q^3*d - e)*x)/(1 - Sqrt[3]*q*x + q^2*
x^2), x], x] + Dist[1/(6*a*q^2), Int[(2*q^2*d + (Sqrt[3]*q^3*d + e)*x)/(1 +
Sqrt[3]*q*x + q^2*x^2), x], x])) /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 +
a*e^2, 0] && PosQ[c/a]
```

### Rule 635

```
Int[((d_) + (e_)*(x_))/((a_) + (c_)*(x_)^2), x_Symbol] := Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && !NiceSqrtQ[-(a*c)]
```

### Rule 203

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

### Rule 260

```
Int[(x_)^(m_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] := Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

### Rule 634

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
```

```
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

### Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])]; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

### Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

### Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

### Rubi steps

$$\begin{aligned}
\int \frac{d + \frac{e}{x^3}}{c + \frac{a}{x^6}} dx &= \int \frac{x^3 (e + dx^3)}{a + cx^6} dx \\
&= \frac{dx}{c} - \frac{\int \frac{ad - cex^3}{a + cx^6} dx}{c} \\
&= \frac{dx}{c} - \frac{\int \frac{2a^{2/3} \sqrt[3]{cd} - (\sqrt{3}\sqrt{a}\sqrt{cd+ce})x}{1 - \frac{\sqrt{3}\sqrt[6]{cx}}{\sqrt[6]{a}} + \frac{\sqrt[3]{cx^2}}{\sqrt[3]{a}}} dx}{6a^{2/3}c^{4/3}} - \frac{\int \frac{2a^{2/3} \sqrt[3]{cd} + (\sqrt{3}\sqrt{a}\sqrt{cd-ce})x}{1 + \frac{\sqrt{3}\sqrt[6]{cx}}{\sqrt[6]{a}} + \frac{\sqrt[3]{cx^2}}{\sqrt[3]{a}}} dx}{6a^{2/3}c^{4/3}} - \frac{\int \frac{a^{2/3} \sqrt[3]{cd+ce}}{1 + \frac{\sqrt[3]{cx^2}}{\sqrt[3]{a}}} dx}{3a^{2/3}c^{4/3}} \\
&= \frac{dx}{c} - \frac{d \int \frac{1}{1 + \frac{\sqrt[3]{cx^2}}{\sqrt[3]{a}}} dx}{3c} - \frac{e \int \frac{x}{1 + \frac{\sqrt[3]{cx^2}}{\sqrt[3]{a}}} dx}{3a^{2/3}\sqrt[3]{c}} - \frac{(\sqrt{3}\sqrt{ad} - \sqrt{ce}) \int \frac{\frac{\sqrt{3}\sqrt[6]{c}}{\sqrt[6]{a}} + \frac{2\sqrt[3]{cx}}{\sqrt[3]{a}}}{1 + \frac{\sqrt{3}\sqrt[6]{cx}}{\sqrt[6]{a}} + \frac{\sqrt[3]{cx^2}}{\sqrt[3]{a}}} dx}{12\sqrt[3]{ac}^{7/6}} + \frac{(\sqrt{3}\sqrt{ad} + \sqrt{ce}) \int \frac{\frac{\sqrt{3}\sqrt[6]{c}}{\sqrt[6]{a}} + \frac{2\sqrt[3]{cx}}{\sqrt[3]{a}}}{1 - \frac{\sqrt{3}\sqrt[6]{cx}}{\sqrt[6]{a}} + \frac{\sqrt[3]{cx^2}}{\sqrt[3]{a}}} dx}{12\sqrt[3]{ac}^{7/6}} \\
&= \frac{dx}{c} - \frac{\sqrt[6]{ad} \tan^{-1}\left(\frac{\sqrt[6]{cx}}{\sqrt[6]{a}}\right)}{3c^{7/6}} - \frac{e \log(\sqrt[3]{a} + \sqrt[3]{cx^2})}{6\sqrt[3]{ac}^{2/3}} + \frac{(\sqrt{3}\sqrt{ad} + \sqrt{ce}) \log(\sqrt[3]{a} - \sqrt{3}\sqrt[6]{a}\sqrt[6]{cx} + \sqrt[3]{cx^2})}{12\sqrt[3]{ac}^{7/6}} - \frac{(\sqrt{3}\sqrt{ad} - \sqrt{ce}) \log(\sqrt{3}\sqrt[6]{a}\sqrt[6]{cx} + \sqrt[3]{a} + \sqrt[3]{cx^2})}{12\sqrt[3]{ac}^{7/6}} \\
&= \frac{dx}{c} - \frac{\sqrt[6]{ad} \tan^{-1}\left(\frac{\sqrt[6]{cx}}{\sqrt[6]{a}}\right)}{3c^{7/6}} + \frac{(\sqrt{ad} - \sqrt{3}\sqrt{ce}) \tan^{-1}\left(\sqrt{3} - \frac{2\sqrt[6]{cx}}{\sqrt[6]{a}}\right)}{6\sqrt[3]{ac}^{7/6}} - \frac{(\sqrt{ad} + \sqrt{3}\sqrt{ce}) \tan^{-1}\left(\sqrt{3} + \frac{2\sqrt[6]{cx}}{\sqrt[6]{a}}\right)}{6\sqrt[3]{ac}^{7/6}} - \frac{e \log(\sqrt{3}\sqrt[6]{a}\sqrt[6]{cx} + \sqrt[3]{a} + \sqrt[3]{cx^2})}{12\sqrt[3]{ac}^{7/6}}
\end{aligned}$$

**Mathematica [A]** time = 0.114622, size = 346, normalized size = 1.11

$$-\frac{(-\sqrt{3}a^{7/6}\sqrt{cd} - a^{2/3}ce) \log(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{cx} + \sqrt[3]{a} + \sqrt[3]{cx^2})}{12ac^{5/3}} - \frac{(\sqrt{3}a^{7/6}\sqrt{cd} - a^{2/3}ce) \log(\sqrt{3}\sqrt[6]{a}\sqrt[6]{cx} + \sqrt[3]{a} + \sqrt[3]{cx^2})}{12ac^{5/3}} + \frac{(\sqrt{3}a^{7/6}\sqrt{cd} + a^{2/3}ce) \log(\sqrt{3}\sqrt[6]{a}\sqrt[6]{cx} + \sqrt[3]{a} + \sqrt[3]{cx^2})}{12ac^{5/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e/x^3)/(c + a/x^6),x]

[Out] (d\*x)/c - (a^(1/6)\*d\*ArcTan[(c^(1/6)\*x)/a^(1/6)]/(3\*c^(7/6))) + ((-a^(7/6)\*Sqrt[c]\*d) + Sqrt[3]\*a^(2/3)\*c\*e)\*ArcTan[(-(Sqrt[3]\*a^(1/6)) + 2\*c^(1/6)\*x)/a^(1/6)]/(6\*a\*c^(5/3)) + ((-a^(7/6)\*Sqrt[c]\*d) - Sqrt[3]\*a^(2/3)\*c\*e)\*ArcTan[(Sqrt[3]\*a^(1/6) + 2\*c^(1/6)\*x)/a^(1/6)]/(6\*a\*c^(5/3)) - (e\*Log[a^(1/3) + c^(1/3)\*x^2])/(6\*a^(1/3)\*c^(2/3)) - ((-Sqrt[3]\*a^(7/6)\*Sqrt[c]\*d) - a^(2/3)\*c\*e)\*Log[a^(1/3) - Sqrt[3]\*a^(1/6)\*c^(1/6)\*x + c^(1/3)\*x^2]/(12\*a\*c^(5/3)) - ((Sqrt[3]\*a^(7/6)\*Sqrt[c]\*d - a^(2/3)\*c\*e)\*Log[a^(1/3) + Sqrt[3]\*a^(1/6)\*c^(1/6)\*x + c^(1/3)\*x^2]/(12\*a\*c^(5/3))

**Maple [A]** time = 0.044, size = 334, normalized size = 1.1

$$\frac{dx}{c} - \frac{\sqrt{3}d}{12a} \left(\frac{a}{c}\right)^{\frac{7}{6}} \ln\left(x^2 + \sqrt{3}\sqrt{\frac{a}{c}}x + \sqrt{\frac{a}{c}}\right) + \frac{e}{12a} \left(\frac{a}{c}\right)^{\frac{2}{3}} \ln\left(x^2 + \sqrt{3}\sqrt{\frac{a}{c}}x + \sqrt{\frac{a}{c}}\right) - \frac{d}{6c} \sqrt{\frac{a}{c}} \arctan\left(2x\frac{1}{\sqrt{\frac{a}{c}}} + \sqrt{3}\right) - \frac{\sqrt{3}}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d+e/x^3)/(c+a/x^6),x)

[Out]  $d*x/c - 1/12*(a/c)^{(7/6)}/a*\ln(x^2+3^{(1/2)}*(a/c)^{(1/6)*x+(a/c)^{(1/3)})*3^{(1/2)*d+1/12*(a/c)^{(2/3)}/a*\ln(x^2+3^{(1/2)}*(a/c)^{(1/6)*x+(a/c)^{(1/3)})*e-1/6*c*(a/c)^{(1/6)*\arctan(2*x/(a/c)^{(1/6)}+3^{(1/2)})}*d-1/6*(a/c)^{(2/3)}/a*\arctan(2*x/(a/c)^{(1/6)}+3^{(1/2)})*3^{(1/2)*e+1/12/a*\ln(x^2-3^{(1/2)}*(a/c)^{(1/6)*x+(a/c)^{(1/3)})*(a/c)^{(2/3)*e+1/12/c*\ln(x^2-3^{(1/2)}*(a/c)^{(1/6)*x+(a/c)^{(1/3)})*3^{(1/2)}*(a/c)^{(1/6)*d+1/6/a*(a/c)^{(2/3)*\arctan(2*x/(a/c)^{(1/6)}-3^{(1/2)})}*3^{(1/2)*e-1/6/c*(a/c)^{(1/6)*\arctan(2*x/(a/c)^{(1/6)}-3^{(1/2)})}*d-1/6*(a/c)^{(2/3)}/a*e*\ln(x^2+(a/c)^{(1/3)})-1/3/c*(a/c)^{(1/6)*d*\arctan(x/(a/c)^{(1/6)})}$

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e/x^3)/(c+a/x^6),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [B]** time = 2.89753, size = 6369, normalized size = 20.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e/x^3)/(c+a/x^6),x, algorithm="fricas")

[Out]  $-1/12*(4*\sqrt{3})*c*((a*c^3*\sqrt{-(a^2*d^6 - 6*a*c*d^4*e^2 + 9*c^2*d^2*e^4)/(a*c^7)}) + 3*a*d^2*e - c*e^3)/(a*c^3))^{(1/3)*\arctan(1/3*(2*(\sqrt{3})*(a^2*c^$

$$\begin{aligned}
& 6*d^2 - a*c^7*e^2)*\sqrt{-(a^2*d^6 - 6*a*c*d^4*e^2 + 9*c^2*d^2*e^4)/(a*c^7)} \\
& - 2*\sqrt{3}*(a^2*c^3*d^4*e - 3*a*c^4*d^2*e^3))*\sqrt{((a^3*d^7 - a^2*c*d^5* \\
& e^2 - 5*a*c^2*d^3*e^4 - 3*c^3*d*e^6)*x^2 + (2*a^2*c^6*d*e*\sqrt{-(a^2*d^6 - \\
& 6*a*c*d^4*e^2 + 9*c^2*d^2*e^4)/(a*c^7)} + a^3*c^2*d^5 - 4*a^2*c^3*d^3*e^2 + \\
& 3*a*c^4*d*e^4))*((a*c^3*\sqrt{-(a^2*d^6 - 6*a*c*d^4*e^2 + 9*c^2*d^2*e^4)/(a* \\
& c^7)} + 3*a*d^2*e - c*e^3)/(a*c^3))^{2/3} + ((a^2*c^5*d^2*e + a*c^6*e^3)*x* \\
& \sqrt{-(a^2*d^6 - 6*a*c*d^4*e^2 + 9*c^2*d^2*e^4)/(a*c^7)} + (a^3*c*d^6 - 2*a \\
& ^2*c^2*d^4*e^2 - 3*a*c^3*d^2*e^4)*x)*((a*c^3*\sqrt{-(a^2*d^6 - 6*a*c*d^4*e^2 \\
& + 9*c^2*d^2*e^4)/(a*c^7)} + 3*a*d^2*e - c*e^3)/(a*c^3))^{1/3})/(a^3*d^7 - \\
& a^2*c*d^5*e^2 - 5*a*c^2*d^3*e^4 - 3*c^3*d*e^6))*((a*c^3*\sqrt{-(a^2*d^6 - 6* \\
& a*c*d^4*e^2 + 9*c^2*d^2*e^4)/(a*c^7)} + 3*a*d^2*e - c*e^3)/(a*c^3))^{2/3} - \\
& 2*(\sqrt{3}*(a^2*c^6*d^2 - a*c^7*e^2)*x*\sqrt{-(a^2*d^6 - 6*a*c*d^4*e^2 + 9* \\
& c^2*d^2*e^4)/(a*c^7)} - 2*\sqrt{3}*(a^2*c^3*d^4*e - 3*a*c^4*d^2*e^3)*x)*((a \\
& c^3*\sqrt{-(a^2*d^6 - 6*a*c*d^4*e^2 + 9*c^2*d^2*e^4)/(a*c^7)} + 3*a*d^2*e - \\
& c*e^3)/(a*c^3))^{2/3} + \sqrt{3}*(a^3*d^7 - a^2*c*d^5*e^2 - 5*a*c^2*d^3*e^4 \\
& - 3*c^3*d*e^6))/(a^3*d^7 - a^2*c*d^5*e^2 - 5*a*c^2*d^3*e^4 - 3*c^3*d*e^6)) \\
& - 4*\sqrt{3}*c*(-(a*c^3*\sqrt{-(a^2*d^6 - 6*a*c*d^4*e^2 + 9*c^2*d^2*e^4)/(a*c \\
& ^7)} - 3*a*d^2*e + c*e^3)/(a*c^3))^{1/3}*\arctan(1/3*(2*(\sqrt{3}*(a^2*c^6*d^ \\
& 2 - a*c^7*e^2)*\sqrt{-(a^2*d^6 - 6*a*c*d^4*e^2 + 9*c^2*d^2*e^4)/(a*c^7)} + 2 \\
& *\sqrt{3}*(a^2*c^3*d^4*e - 3*a*c^4*d^2*e^3))*\sqrt{((a^3*d^7 - a^2*c*d^5*e^2 \\
& - 5*a*c^2*d^3*e^4 - 3*c^3*d*e^6)*x^2 - (2*a^2*c^6*d*e*\sqrt{-(a^2*d^6 - 6*a* \\
& c*d^4*e^2 + 9*c^2*d^2*e^4)/(a*c^7)} - a^3*c^2*d^5 + 4*a^2*c^3*d^3*e^2 - 3*a \\
& *c^4*d*e^4))*(-(a*c^3*\sqrt{-(a^2*d^6 - 6*a*c*d^4*e^2 + 9*c^2*d^2*e^4)/(a*c^7 \\
& )} - 3*a*d^2*e + c*e^3)/(a*c^3))^{2/3} - ((a^2*c^5*d^2*e + a*c^6*e^3)*x*\sqrt{ \\
& -(a^2*d^6 - 6*a*c*d^4*e^2 + 9*c^2*d^2*e^4)/(a*c^7)} - (a^3*c*d^6 - 2*a^2* \\
& c^2*d^4*e^2 - 3*a*c^3*d^2*e^4)*x)*(-(a*c^3*\sqrt{-(a^2*d^6 - 6*a*c*d^4*e^2 + \\
& 9*c^2*d^2*e^4)/(a*c^7)} - 3*a*d^2*e + c*e^3)/(a*c^3))^{1/3})/(a^3*d^7 - a^ \\
& 2*c*d^5*e^2 - 5*a*c^2*d^3*e^4 - 3*c^3*d*e^6))*(-(a*c^3*\sqrt{-(a^2*d^6 - 6*a \\
& *c*d^4*e^2 + 9*c^2*d^2*e^4)/(a*c^7)} - 3*a*d^2*e + c*e^3)/(a*c^3))^{2/3} - \\
& 2*(\sqrt{3}*(a^2*c^6*d^2 - a*c^7*e^2)*x*\sqrt{-(a^2*d^6 - 6*a*c*d^4*e^2 + 9*c \\
& ^2*d^2*e^4)/(a*c^7)} + 2*\sqrt{3}*(a^2*c^3*d^4*e - 3*a*c^4*d^2*e^3)*x)*(-(a \\
& c^3*\sqrt{-(a^2*d^6 - 6*a*c*d^4*e^2 + 9*c^2*d^2*e^4)/(a*c^7)} - 3*a*d^2*e + \\
& c*e^3)/(a*c^3))^{2/3} - \sqrt{3}*(a^3*d^7 - a^2*c*d^5*e^2 - 5*a*c^2*d^3*e^4 \\
& - 3*c^3*d*e^6))/(a^3*d^7 - a^2*c*d^5*e^2 - 5*a*c^2*d^3*e^4 - 3*c^3*d*e^6)) \\
& + c*((a*c^3*\sqrt{-(a^2*d^6 - 6*a*c*d^4*e^2 + 9*c^2*d^2*e^4)/(a*c^7)} + 3*a* \\
& d^2*e - c*e^3)/(a*c^3))^{1/3}*\log(-(a^3*d^7 - a^2*c*d^5*e^2 - 5*a*c^2*d^3*e \\
& ^4 - 3*c^3*d*e^6)*x^2 - (2*a^2*c^6*d*e*\sqrt{-(a^2*d^6 - 6*a*c*d^4*e^2 + 9*c \\
& ^2*d^2*e^4)/(a*c^7)} + a^3*c^2*d^5 - 4*a^2*c^3*d^3*e^2 + 3*a*c^4*d*e^4))*((a \\
& c^3*\sqrt{-(a^2*d^6 - 6*a*c*d^4*e^2 + 9*c^2*d^2*e^4)/(a*c^7)} + 3*a*d^2*e - \\
& c*e^3)/(a*c^3))^{2/3} - ((a^2*c^5*d^2*e + a*c^6*e^3)*x*\sqrt{-(a^2*d^6 - 6* \\
& a*c*d^4*e^2 + 9*c^2*d^2*e^4)/(a*c^7)} + (a^3*c*d^6 - 2*a^2*c^2*d^4*e^2 - 3* \\
& a*c^3*d^2*e^4)*x)*((a*c^3*\sqrt{-(a^2*d^6 - 6*a*c*d^4*e^2 + 9*c^2*d^2*e^4)/( \\
& a*c^7)} + 3*a*d^2*e - c*e^3)/(a*c^3))^{1/3}) + c*(-(a*c^3*\sqrt{-(a^2*d^6 - \\
& 6*a*c*d^4*e^2 + 9*c^2*d^2*e^4)/(a*c^7)} - 3*a*d^2*e + c*e^3)/(a*c^3))^{1/3} \\
& *\log(-(a^3*d^7 - a^2*c*d^5*e^2 - 5*a*c^2*d^3*e^4 - 3*c^3*d*e^6)*x^2 + (2*a^
\end{aligned}$$

$$2c^6d^2e\sqrt{-(a^2d^6 - 6ac^2d^4e^2 + 9c^2d^2e^4)/(ac^7)} - a^3c^2d^5 + 4a^2c^3d^3e^2 - 3a^2c^4d^2e^4) * (-(ac^3\sqrt{-(a^2d^6 - 6ac^2d^4e^2 + 9c^2d^2e^4)/(ac^7)} - 3ad^2e + ce^3)/(ac^3))^{2/3} + ((a^2c^5d^2e + ac^6e^3)*x\sqrt{-(a^2d^6 - 6ac^2d^4e^2 + 9c^2d^2e^4)/(ac^7)} - (a^3cd^6 - 2a^2c^2d^4e^2 - 3ac^3d^2e^4)*x) * (-(ac^3\sqrt{-(a^2d^6 - 6ac^2d^4e^2 + 9c^2d^2e^4)/(ac^7)} - 3ad^2e + ce^3)/(ac^3))^{1/3} - 2c*((ac^3\sqrt{-(a^2d^6 - 6ac^2d^4e^2 + 9c^2d^2e^4)/(ac^7)} + 3ad^2e - ce^3)/(ac^3))^{1/3} * \log(-(a^2d^5 - 2ac^2d^3e^2 - 3c^2d^2e^4)*x + (ac^5e\sqrt{-(a^2d^6 - 6ac^2d^4e^2 + 9c^2d^2e^4)/(ac^7)} + a^2cd^4 - 3ac^2d^2e^2) * ((ac^3\sqrt{-(a^2d^6 - 6ac^2d^4e^2 + 9c^2d^2e^4)/(ac^7)} + 3ad^2e - ce^3)/(ac^3))^{1/3}) - 2c * (-(ac^3\sqrt{-(a^2d^6 - 6ac^2d^4e^2 + 9c^2d^2e^4)/(ac^7)} - 3ad^2e + ce^3)/(ac^3))^{1/3} * \log(-(a^2d^5 - 2ac^2d^3e^2 - 3c^2d^2e^4)*x - (ac^5e\sqrt{-(a^2d^6 - 6ac^2d^4e^2 + 9c^2d^2e^4)/(ac^7)} - a^2cd^4 + 3ac^2d^2e^2) * (-(ac^3\sqrt{-(a^2d^6 - 6ac^2d^4e^2 + 9c^2d^2e^4)/(ac^7)} - 3ad^2e + ce^3)/(ac^3))^{1/3}) - 12d*x)/c$$

**Sympy [A]** time = 2.48061, size = 167, normalized size = 0.54

$$\text{RootSum}\left(46656t^6a^2c^7 + t^3(-1296a^2c^4d^2e + 432ac^5e^3) + a^3d^6 + 3a^2cd^4e^2 + 3ac^2d^2e^4 + c^3e^6, \left(t \mapsto t \log\left(x + \frac{-1296t^4}{\dots}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e/x\*\*3)/(c+a/x\*\*6),x)

[Out] RootSum(46656\*\_t\*\*6\*a\*\*2\*c\*\*7 + \_t\*\*3\*(-1296\*a\*\*2\*c\*\*4\*d\*\*2\*e + 432\*a\*c\*\*5\*e\*\*3) + a\*\*3\*d\*\*6 + 3\*a\*\*2\*c\*d\*\*4\*e\*\*2 + 3\*a\*c\*\*2\*d\*\*2\*e\*\*4 + c\*\*3\*e\*\*6, Lambda(\_t, \_t\*log(x + (-1296\*\_t\*\*4\*a\*c\*\*5\*e - 6\*\_t\*a\*\*2\*c\*d\*\*4 + 36\*\_t\*a\*c\*\*2\*d\*\*2\*e\*\*2 - 6\*\_t\*c\*\*3\*e\*\*4)/(a\*\*2\*d\*\*5 - 2\*a\*c\*d\*\*3\*e\*\*2 - 3\*c\*\*2\*d\*\*4))) + d\*x/c

**Giac [A]** time = 1.14669, size = 406, normalized size = 1.31

$$\frac{dx}{c} - \frac{(ac^5)^{\frac{1}{6}} d \arctan\left(\frac{x}{\left(\frac{a}{c}\right)^{\frac{1}{6}}}\right)}{3c^2} - \frac{(ac^5)^{\frac{2}{3}} |c| e \log\left(x^2 + \left(\frac{a}{c}\right)^{\frac{1}{3}}\right)}{6ac^5} - \frac{\left(\left(ac^5\right)^{\frac{1}{6}} ac^2d + \sqrt{3}\left(ac^5\right)^{\frac{2}{3}} e\right) \arctan\left(\frac{2x + \sqrt{3}\left(\frac{a}{c}\right)^{\frac{1}{6}}}{\left(\frac{a}{c}\right)^{\frac{1}{6}}}\right)}{6ac^4} - \frac{\left(ac^5\right)^{\frac{1}{6}}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e/x^3)/(c+a/x^6),x, algorithm="giac")
```

```
[Out] d*x/c - 1/3*(a*c^5)^(1/6)*d*arctan(x/(a/c)^(1/6))/c^2 - 1/6*(a*c^5)^(2/3)*a
bs(c)*e*log(x^2 + (a/c)^(1/3))/(a*c^5) - 1/6*((a*c^5)^(1/6)*a*c^2*d + sqrt(
3)*(a*c^5)^(2/3)*e)*arctan((2*x + sqrt(3)*(a/c)^(1/6))/(a/c)^(1/6))/(a*c^4)
- 1/6*((a*c^5)^(1/6)*a*c^2*d - sqrt(3)*(a*c^5)^(2/3)*e)*arctan((2*x - sqrt
(3)*(a/c)^(1/6))/(a/c)^(1/6))/(a*c^4) - 1/12*(sqrt(3)*(a*c^5)^(1/6)*a*c^2*d
- (a*c^5)^(2/3)*e)*log(x^2 + sqrt(3)*x*(a/c)^(1/6) + (a/c)^(1/3))/(a*c^4)
+ 1/12*(sqrt(3)*(a*c^5)^(1/6)*a*c^2*d + (a*c^5)^(2/3)*e)*log(x^2 - sqrt(3)*
x*(a/c)^(1/6) + (a/c)^(1/3))/(a*c^4)
```



$$3.39 \quad \int \frac{d + \frac{e}{x^3}}{c + \frac{a}{x^6} + \frac{b}{x^3}} dx$$

**Optimal.** Leaf size=716

$$\frac{\left(-\frac{-2acd+b^2d-bce}{\sqrt{b^2-4ac}} + bd - ce\right) \log\left(-\sqrt[3]{2}\sqrt[3]{cx}\sqrt[3]{b-\sqrt{b^2-4ac}} + \left(b-\sqrt{b^2-4ac}\right)^{2/3} + 2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}c^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}} + \frac{\left(-\frac{-2acd+b^2d-bce}{\sqrt{b^2-4ac}} + bd - ce\right)}{\sqrt{b^2-4ac}}$$

```
[Out] (d*x)/c + ((b*d - c*e - (b^2*d - 2*a*c*d - b*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan
[(1 - (2*2^(1/3)*c^(1/3)*x)/(b - Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(1/
3)*Sqrt[3]*c^(4/3)*(b - Sqrt[b^2 - 4*a*c])^(2/3)) + ((b*d - c*e + (b^2*d -
2*a*c*d - b*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x)/(b +
Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(1/3)*Sqrt[3]*c^(4/3)*(b + Sqrt[b^2
- 4*a*c])^(2/3)) - ((b*d - c*e - (b^2*d - 2*a*c*d - b*c*e)/Sqrt[b^2 - 4*a*c
])*Log[(b - Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x]/(3*2^(1/3)*c^(4/
3)*(b - Sqrt[b^2 - 4*a*c])^(2/3)) - ((b*d - c*e + (b^2*d - 2*a*c*d - b*c*e)
/Sqrt[b^2 - 4*a*c])*Log[(b + Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x]
/(3*2^(1/3)*c^(4/3)*(b + Sqrt[b^2 - 4*a*c])^(2/3)) + ((b*d - c*e - (b^2*d -
2*a*c*d - b*c*e)/Sqrt[b^2 - 4*a*c])*Log[(b - Sqrt[b^2 - 4*a*c])^(2/3) - 2^(
1/3)*c^(1/3)*(b - Sqrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2)]/(6*2^(
1/3)*c^(4/3)*(b - Sqrt[b^2 - 4*a*c])^(2/3)) + ((b*d - c*e + (b^2*d - 2*a*c
*d - b*c*e)/Sqrt[b^2 - 4*a*c])*Log[(b + Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*
c^(1/3)*(b + Sqrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2)]/(6*2^(1/3)*
c^(4/3)*(b + Sqrt[b^2 - 4*a*c])^(2/3))
```

**Rubi [A]** time = 1.63351, antiderivative size = 716, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 9, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$ , Rules used = {1393, 1502, 1422, 200, 31, 634, 617, 204, 628}

$$\frac{\left(-\frac{-2acd+b^2d-bce}{\sqrt{b^2-4ac}} + bd - ce\right) \log\left(-\sqrt[3]{2}\sqrt[3]{cx}\sqrt[3]{b-\sqrt{b^2-4ac}} + \left(b-\sqrt{b^2-4ac}\right)^{2/3} + 2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}c^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}} + \frac{\left(-\frac{-2acd+b^2d-bce}{\sqrt{b^2-4ac}} + bd - ce\right)}{\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

[In] Int[(d + e/x^3)/(c + a/x^6 + b/x^3),x]

[Out] (d\*x)/c + ((b\*d - c\*e - (b^2\*d - 2\*a\*c\*d - b\*c\*e)/Sqrt[b^2 - 4\*a\*c])\*ArcTan[(1 - (2\*2^(1/3)\*c^(1/3)\*x)/(b - Sqrt[b^2 - 4\*a\*c])^(1/3)]/Sqrt[3])/((2^(1/3)\*Sqrt[3]\*c^(4/3)\*(b - Sqrt[b^2 - 4\*a\*c])^(2/3)) + ((b\*d - c\*e + (b^2\*d - 2\*a\*c\*d - b\*c\*e)/Sqrt[b^2 - 4\*a\*c])\*ArcTan[(1 - (2\*2^(1/3)\*c^(1/3)\*x)/(b + Sqrt[b^2 - 4\*a\*c])^(1/3)]/Sqrt[3])/((2^(1/3)\*Sqrt[3]\*c^(4/3)\*(b + Sqrt[b^2 - 4\*a\*c])^(2/3)) - ((b\*d - c\*e - (b^2\*d - 2\*a\*c\*d - b\*c\*e)/Sqrt[b^2 - 4\*a\*c])\*Log[(b - Sqrt[b^2 - 4\*a\*c])^(1/3) + 2^(1/3)\*c^(1/3)\*x]/(3\*2^(1/3)\*c^(4/3)\*(b - Sqrt[b^2 - 4\*a\*c])^(2/3)) - ((b\*d - c\*e + (b^2\*d - 2\*a\*c\*d - b\*c\*e)/Sqrt[b^2 - 4\*a\*c])\*Log[(b + Sqrt[b^2 - 4\*a\*c])^(1/3) + 2^(1/3)\*c^(1/3)\*x]/(3\*2^(1/3)\*c^(4/3)\*(b + Sqrt[b^2 - 4\*a\*c])^(2/3)) + ((b\*d - c\*e - (b^2\*d - 2\*a\*c\*d - b\*c\*e)/Sqrt[b^2 - 4\*a\*c])\*Log[(b - Sqrt[b^2 - 4\*a\*c])^(2/3) - 2^(1/3)\*c^(1/3)\*(b - Sqrt[b^2 - 4\*a\*c])^(1/3)\*x + 2^(2/3)\*c^(2/3)\*x^2]/(6\*2^(1/3)\*c^(4/3)\*(b - Sqrt[b^2 - 4\*a\*c])^(2/3)) + ((b\*d - c\*e + (b^2\*d - 2\*a\*c\*d - b\*c\*e)/Sqrt[b^2 - 4\*a\*c])\*Log[(b + Sqrt[b^2 - 4\*a\*c])^(2/3) - 2^(1/3)\*c^(1/3)\*(b + Sqrt[b^2 - 4\*a\*c])^(1/3)\*x + 2^(2/3)\*c^(2/3)\*x^2]/(6\*2^(1/3)\*c^(4/3)\*(b + Sqrt[b^2 - 4\*a\*c])^(2/3))

### Rule 1393

Int[((a\_) + (c\_)\*(x\_)^(n2\_)) + (b\_)\*(x\_)^(n\_)]^(p\_)\*((d\_) + (e\_)\*(x\_)^(n\_))^(q\_), x\_Symbol] := Int[x^(n\*(2\*p + q))\*(e + d/x^n)^q\*(c + b/x^n + a/x^(2\*n))^p, x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2\*n] && IntegersQ[p, q] && NegQ[n]

### Rule 1502

Int[((f\_)\*(x\_)^(m\_))\*((d\_) + (e\_)\*(x\_)^(n\_))\*((a\_) + (b\_)\*(x\_)^(n\_) + (c\_)\*(x\_)^(n2\_))]^(p\_), x\_Symbol] := Simp[(e\*f^(n - 1)\*(f\*x)^(m - n + 1)\*(a + b\*x^n + c\*x^(2\*n))^(p + 1))/(c\*(m + n\*(2\*p + 1) + 1)), x] - Dist[f^n/(c\*(m + n\*(2\*p + 1) + 1)), Int[(f\*x)^(m - n)\*(a + b\*x^n + c\*x^(2\*n))^p\*Simp[a\*e\*(m - n + 1) + (b\*e\*(m + n\*p + 1) - c\*d\*(m + n\*(2\*p + 1) + 1))\*x^n, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[n2, 2\*n] && NeQ[b^2 - 4\*a\*c, 0] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n\*(2\*p + 1) + 1, 0] && IntegerQ[p]

### Rule 1422

Int[((d\_) + (e\_)\*(x\_)^(n\_))/((a\_) + (b\_)\*(x\_)^(n\_) + (c\_)\*(x\_)^(n2\_)), x\_Symbol] := With[{q = Rt[b^2 - 4\*a\*c, 2]}, Dist[e/2 + (2\*c\*d - b\*e)/(2\*q), Int[1/(b/2 - q/2 + c\*x^n), x], x] + Dist[e/2 - (2\*c\*d - b\*e)/(2\*q), Int[1/(b/2 + q/2 + c\*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2\*n] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && (PosQ[b^2 - 4\*a

\*c] || !IGtQ[n/2, 0])

### Rule 200

Int[((a\_) + (b\_)\*(x\_)^3)^(-1), x\_Symbol] := Dist[1/(3\*Rt[a, 3]^2), Int[1/(Rt[a, 3] + Rt[b, 3]\*x), x], x] + Dist[1/(3\*Rt[a, 3]^2), Int[(2\*Rt[a, 3] - Rt[b, 3]\*x)/(Rt[a, 3]^2 - Rt[a, 3]\*Rt[b, 3]\*x + Rt[b, 3]^2\*x^2), x], x] /; FreeQ[{a, b}, x]

### Rule 31

Int[((a\_) + (b\_)\*(x\_)^(-1), x\_Symbol] := Simp[Log[RemoveContent[a + b\*x, x]]/b, x] /; FreeQ[{a, b}, x]

### Rule 634

Int[((d\_) + (e\_)\*(x\_))/((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2), x\_Symbol] := Dist[(2\*c\*d - b\*e)/(2\*c), Int[1/(a + b\*x + c\*x^2), x], x] + Dist[e/(2\*c), Int[(b + 2\*c\*x)/(a + b\*x + c\*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2\*c\*d - b\*e, 0] && NeQ[b^2 - 4\*a\*c, 0] && !NiceSqrtQ[b^2 - 4\*a\*c]

### Rule 617

Int[((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(-1), x\_Symbol] := With[{q = 1 - 4\*Simplify[(a\*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2\*c\*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4\*a\*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

### Rule 204

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

### Rule 628

Int[((d\_) + (e\_)\*(x\_))/((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2), x\_Symbol] := Simp[(d\*Log[RemoveContent[a + b\*x + c\*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

### Rubi steps

$$\begin{aligned}
\int \frac{d + \frac{e}{x^3}}{c + \frac{a}{x^6} + \frac{b}{x^3}} dx &= \int \frac{x^3(e + dx^3)}{a + bx^3 + cx^6} dx \\
&= \frac{dx}{c} - \frac{\int \frac{ad + (bd - ce)x^3}{a + bx^3 + cx^6} dx}{c} \\
&= \frac{dx}{c} - \frac{\left(bd - ce - \frac{b^2d - 2acd - bce}{\sqrt{b^2 - 4ac}}\right) \int \frac{1}{\frac{b}{2} - \frac{1}{2}\sqrt{b^2 - 4ac} + cx^3} dx}{2c} - \frac{\left(bd - ce + \frac{b^2d - 2acd - bce}{\sqrt{b^2 - 4ac}}\right) \int \frac{1}{\frac{b}{2} + \frac{1}{2}\sqrt{b^2 - 4ac} + cx^3} dx}{2c} \\
&= \frac{dx}{c} - \frac{\left(bd - ce - \frac{b^2d - 2acd - bce}{\sqrt{b^2 - 4ac}}\right) \int \frac{1}{\frac{\sqrt[3]{b - \sqrt{b^2 - 4ac}}}{\sqrt[3]{2}} + \sqrt[3]{cx}} dx}{3\sqrt[3]{2c} \left(b - \sqrt{b^2 - 4ac}\right)^{2/3}} - \frac{\left(bd - ce - \frac{b^2d - 2acd - bce}{\sqrt{b^2 - 4ac}}\right) \int \frac{2^{2/3} \sqrt[3]{b - \sqrt{b^2 - 4ac}} - \sqrt[3]{c}}{\left(b - \sqrt{b^2 - 4ac}\right)^{2/3} - \sqrt[3]{c} \sqrt[3]{b - \sqrt{b^2 - 4ac}}} dx}{3\sqrt[3]{2c} \left(b - \sqrt{b^2 - 4ac}\right)^{2/3}} \\
&= \frac{dx}{c} - \frac{\left(bd - ce - \frac{b^2d - 2acd - bce}{\sqrt{b^2 - 4ac}}\right) \log\left(\sqrt[3]{b - \sqrt{b^2 - 4ac}} + \sqrt[3]{2} \sqrt[3]{cx}\right)}{3\sqrt[3]{2c^{4/3}} \left(b - \sqrt{b^2 - 4ac}\right)^{2/3}} - \frac{\left(bd - ce + \frac{b^2d - 2acd - bce}{\sqrt{b^2 - 4ac}}\right) \log\left(\sqrt[3]{b + \sqrt{b^2 - 4ac}} + \sqrt[3]{c}\right)}{3\sqrt[3]{2c^{4/3}} \left(b + \sqrt{b^2 - 4ac}\right)^{2/3}} \\
&= \frac{dx}{c} - \frac{\left(bd - ce - \frac{b^2d - 2acd - bce}{\sqrt{b^2 - 4ac}}\right) \log\left(\sqrt[3]{b - \sqrt{b^2 - 4ac}} + \sqrt[3]{2} \sqrt[3]{cx}\right)}{3\sqrt[3]{2c^{4/3}} \left(b - \sqrt{b^2 - 4ac}\right)^{2/3}} - \frac{\left(bd - ce + \frac{b^2d - 2acd - bce}{\sqrt{b^2 - 4ac}}\right) \log\left(\sqrt[3]{b + \sqrt{b^2 - 4ac}} + \sqrt[3]{c}\right)}{3\sqrt[3]{2c^{4/3}} \left(b + \sqrt{b^2 - 4ac}\right)^{2/3}} \\
&= \frac{dx}{c} + \frac{\left(bd - ce - \frac{b^2d - 2acd - bce}{\sqrt{b^2 - 4ac}}\right) \tan^{-1}\left(\frac{1 - \frac{2\sqrt[3]{2}\sqrt[3]{cx}}{\sqrt[3]{b - \sqrt{b^2 - 4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt[3]{3c^{4/3}} \left(b - \sqrt{b^2 - 4ac}\right)^{2/3}} + \frac{\left(bd - ce + \frac{b^2d - 2acd - bce}{\sqrt{b^2 - 4ac}}\right) \tan^{-1}\left(\frac{1 - \frac{2\sqrt[3]{2}\sqrt[3]{cx}}{\sqrt[3]{b + \sqrt{b^2 - 4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt[3]{3c^{4/3}} \left(b + \sqrt{b^2 - 4ac}\right)^{2/3}}
\end{aligned}$$

**Mathematica [C]** time = 0.0558162, size = 88, normalized size = 0.12

$$\frac{dx}{c} - \frac{\text{RootSum}\left[\#1^3b + \#1^6c + a\&, \frac{\#1^3bd \log(x-\#1) - \#1^3ce \log(x-\#1) + ad \log(x-\#1)}{\#1^2b + 2\#1^5c} \&\right]}{3c}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e/x^3)/(c + a/x^6 + b/x^3), x]

[Out]  $(d*x)/c - \text{RootSum}[a + b*x^3 + c*x^6 \& , (a*d*\text{Log}[x - \#1] + b*d*\text{Log}[x - \#1] * \#1^3 - c*e*\text{Log}[x - \#1] * \#1^3) / (b*\#1^2 + 2*c*\#1^5) \& ] / (3*c)$

**Maple [C]** time = 0.013, size = 67, normalized size = 0.1

$$\frac{dx}{c} + \frac{1}{3c} \sum_{_R=\text{RootOf}(_Z^6+_Z^3b+a)} \frac{((-bd+ce)_R^3-ad)\ln(x-_R)}{2_R^5c+_R^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d+e/x^3)/(c+a/x^6+b/x^3),x)`

[Out]  $d*x/c + 1/3/c*\text{sum}(((b*d+c*e)*_R^3-a*d)/(2*_R^5*c+_R^2*b)*\ln(x-_R),_R=\text{RootOf}(_Z^6*c+_Z^3*b+a))$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{dx}{c} + \frac{-\int \frac{(bd-ce)x^3+ad}{cx^6+bx^3+a} dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d+e/x^3)/(c+a/x^6+b/x^3),x, algorithm="maxima")`

[Out]  $d*x/c + \text{integrate}(-((b*d - c*e)*x^3 + a*d)/(c*x^6 + b*x^3 + a), x)/c$

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d+e/x^3)/(c+a/x^6+b/x^3),x, algorithm="fricas")`

[Out] Timed out

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e/x**3)/(c+a/x**6+b/x**3),x)
```

```
[Out] Timed out
```

---

**Giac [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e/x^3)/(c+a/x^6+b/x^3),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

$$3.40 \quad \int \frac{d + \frac{e}{x^4}}{c + \frac{a}{x^8}} dx$$

**Optimal.** Leaf size=753

$$\frac{(\sqrt{a}(d - \sqrt{2}d) + \sqrt{ce}) \log\left(-\sqrt{2 - \sqrt{2}} \sqrt[8]{a} \sqrt[8]{cx} + \sqrt[4]{a} + \sqrt[4]{cx^2}\right)}{8\sqrt{2}(2 - \sqrt{2})a^{3/8}c^{9/8}} + \frac{(\sqrt{a}(d - \sqrt{2}d) + \sqrt{ce}) \log\left(\sqrt{2 - \sqrt{2}} \sqrt[8]{a} \sqrt[8]{cx} + \sqrt[4]{a} + \sqrt[4]{cx^2}\right)}{8\sqrt{2}(2 - \sqrt{2})a^{3/8}c^{9/8}}$$

```
[Out] (d*x)/c + (Sqrt[2 - Sqrt[2]]*((1 + Sqrt[2])*Sqrt[a]*d + Sqrt[c]*e)*ArcTan[(Sqrt[2 - Sqrt[2]]*a^(1/8) - 2*c^(1/8)*x)/(Sqrt[2 + Sqrt[2]]*a^(1/8))])/(8*a^(3/8)*c^(9/8)) - (Sqrt[2 + Sqrt[2]]*(Sqrt[a]*(d - Sqrt[2]*d) + Sqrt[c]*e)*ArcTan[(Sqrt[2 + Sqrt[2]]*a^(1/8) - 2*c^(1/8)*x)/(Sqrt[2 - Sqrt[2]]*a^(1/8))])/(8*a^(3/8)*c^(9/8)) - (Sqrt[2 - Sqrt[2]]*((1 + Sqrt[2])*Sqrt[a]*d + Sqrt[c]*e)*ArcTan[(Sqrt[2 - Sqrt[2]]*a^(1/8) + 2*c^(1/8)*x)/(Sqrt[2 + Sqrt[2]]*a^(1/8))])/(8*a^(3/8)*c^(9/8)) + (Sqrt[2 + Sqrt[2]]*(Sqrt[a]*(d - Sqrt[2]*d) + Sqrt[c]*e)*ArcTan[(Sqrt[2 + Sqrt[2]]*a^(1/8) + 2*c^(1/8)*x)/(Sqrt[2 - Sqrt[2]]*a^(1/8))])/(8*a^(3/8)*c^(9/8)) - ((Sqrt[a]*(d - Sqrt[2]*d) + Sqrt[c]*e)*Log[a^(1/4) - Sqrt[2 - Sqrt[2]]*a^(1/8)*c^(1/8)*x + c^(1/4)*x^2])/(8*Sqrt[2*(2 - Sqrt[2])]*a^(3/8)*c^(9/8)) + ((Sqrt[a]*(d - Sqrt[2]*d) + Sqrt[c]*e)*Log[a^(1/4) + Sqrt[2 - Sqrt[2]]*a^(1/8)*c^(1/8)*x + c^(1/4)*x^2])/(8*Sqrt[2*(2 - Sqrt[2])]*a^(3/8)*c^(9/8)) + (((1 + Sqrt[2])*Sqrt[a]*d + Sqrt[c]*e)*Log[a^(1/4) - Sqrt[2 + Sqrt[2]]*a^(1/8)*c^(1/8)*x + c^(1/4)*x^2])/(8*Sqrt[2*(2 + Sqrt[2])]*a^(3/8)*c^(9/8)) - (((1 + Sqrt[2])*Sqrt[a]*d + Sqrt[c]*e)*Log[a^(1/4) + Sqrt[2 + Sqrt[2]]*a^(1/8)*c^(1/8)*x + c^(1/4)*x^2])/(8*Sqrt[2*(2 + Sqrt[2])]*a^(3/8)*c^(9/8))
```

**Rubi [A]** time = 1.43632, antiderivative size = 753, normalized size of antiderivative = 1., number of steps used = 21, number of rules used = 8, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.471$ , Rules used = {1394, 1503, 1415, 1169, 634, 618, 204, 628}

$$\frac{(\sqrt{a}(d - \sqrt{2}d) + \sqrt{ce}) \log\left(-\sqrt{2 - \sqrt{2}} \sqrt[8]{a} \sqrt[8]{cx} + \sqrt[4]{a} + \sqrt[4]{cx^2}\right)}{8\sqrt{2}(2 - \sqrt{2})a^{3/8}c^{9/8}} + \frac{(\sqrt{a}(d - \sqrt{2}d) + \sqrt{ce}) \log\left(\sqrt{2 - \sqrt{2}} \sqrt[8]{a} \sqrt[8]{cx} + \sqrt[4]{a} + \sqrt[4]{cx^2}\right)}{8\sqrt{2}(2 - \sqrt{2})a^{3/8}c^{9/8}}$$

Antiderivative was successfully verified.

[In] Int[(d + e/x^4)/(c + a/x^8), x]

```
[Out] (d*x)/c + (Sqrt[2 - Sqrt[2]]*((1 + Sqrt[2])*Sqrt[a]*d + Sqrt[c]*e)*ArcTan[(Sqrt[2 - Sqrt[2]]*a^(1/8) - 2*c^(1/8)*x)/(Sqrt[2 + Sqrt[2]]*a^(1/8))])/(8*a^(3/8)*c^(9/8)) - (Sqrt[2 + Sqrt[2]]*(Sqrt[a]*(d - Sqrt[2]*d) + Sqrt[c]*e)*ArcTan[(Sqrt[2 + Sqrt[2]]*a^(1/8) - 2*c^(1/8)*x)/(Sqrt[2 - Sqrt[2]]*a^(1/8))])/(8*a^(3/8)*c^(9/8)) - (Sqrt[2 - Sqrt[2]]*((1 + Sqrt[2])*Sqrt[a]*d + Sqrt[c]*e)*ArcTan[(Sqrt[2 - Sqrt[2]]*a^(1/8) + 2*c^(1/8)*x)/(Sqrt[2 + Sqrt[2]]*a^(1/8))])/(8*a^(3/8)*c^(9/8)) + (Sqrt[2 + Sqrt[2]]*(Sqrt[a]*(d - Sqrt[2]*d) + Sqrt[c]*e)*ArcTan[(Sqrt[2 + Sqrt[2]]*a^(1/8) + 2*c^(1/8)*x)/(Sqrt[2 - Sqrt[2]]*a^(1/8))])/(8*a^(3/8)*c^(9/8)) - ((Sqrt[a]*(d - Sqrt[2]*d) + Sqrt[c]*e)*Log[a^(1/4) - Sqrt[2 - Sqrt[2]]*a^(1/8)*c^(1/8)*x + c^(1/4)*x^2])/(8*Sqrt[2*(2 - Sqrt[2])]*a^(3/8)*c^(9/8)) + ((Sqrt[a]*(d - Sqrt[2]*d) + Sqrt[c]*e)*Log[a^(1/4) + Sqrt[2 - Sqrt[2]]*a^(1/8)*c^(1/8)*x + c^(1/4)*x^2])/(8*Sqrt[2*(2 - Sqrt[2])]*a^(3/8)*c^(9/8)) + (((1 + Sqrt[2])*Sqrt[a]*d + Sqrt[c]*e)*Log[a^(1/4) - Sqrt[2 + Sqrt[2]]*a^(1/8)*c^(1/8)*x + c^(1/4)*x^2])/(8*Sqrt[2*(2 + Sqrt[2])]*a^(3/8)*c^(9/8)) - (((1 + Sqrt[2])*Sqrt[a]*d + Sqrt[c]*e)*Log[a^(1/4) + Sqrt[2 + Sqrt[2]]*a^(1/8)*c^(1/8)*x + c^(1/4)*x^2])/(8*Sqrt[2*(2 + Sqrt[2])]*a^(3/8)*c^(9/8))
```

### Rule 1394

```
Int[((a_) + (c_)*(x_)^(n2_))^(p_)*((d_) + (e_)*(x_)^(n_))^(q_), x_Symbol]
:> Int[x^(n*(2*p + q))*(e + d/x^n)^q*(c + a/x^(2*n))^p, x] /; FreeQ[{a, c, d, e, n}, x] && EqQ[n2, 2*n] && IntegersQ[p, q] && NegQ[n]
```

### Rule 1503

```
Int[((f_)*(x_)^(m_))*((d_) + (e_)*(x_)^(n_))*((a_) + (c_)*(x_)^(n2_))^(p_), x_Symbol]
:> Simp[(e*f^(n - 1)*(f*x)^(m - n + 1)*(a + c*x^(2*n))^(p + 1))/(c*(m + n*(2*p + 1) + 1)), x] - Dist[f^n/(c*(m + n*(2*p + 1) + 1)), Int[(f*x)^(m - n)*(a + c*x^(2*n))^p*(a*e*(m - n + 1) - c*d*(m + n*(2*p + 1) + 1)*x^n), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && EqQ[n2, 2*n] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*(2*p + 1) + 1, 0] && IntegerQ[p]
```

### Rule 1415

```
Int[((d_) + (e_)*(x_)^(n_))/((a_) + (c_)*(x_)^(n2_)), x_Symbol]
:> With[{q = Rt[a/c, 4]}, Dist[1/(2*Sqrt[2]*c*q^3), Int[(Sqrt[2]*d*q - (d - e*q^2)*x^(n/2))/(q^2 - Sqrt[2]*q*x^(n/2) + x^n), x], x] + Dist[1/(2*Sqrt[2]*c*q^3), Int[(Sqrt[2]*d*q + (d - e*q^2)*x^(n/2))/(q^2 + Sqrt[2]*q*x^(n/2) + x^n), x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[n2, 2*n] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && IGtQ[n/2, 0] && PosQ[a*c]
```

### Rule 1169

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (b_)*(x_)^2 + (c_)*(x_)^4), x_Symbol] :
```



```
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
(d - e*q)*x)/(q + r*x + x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

### Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

### Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

### Rule 204

```
Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

### Rule 628

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

### Rubi steps

$$\begin{aligned}
\int \frac{d + \frac{e}{x^4}}{c + \frac{a}{x^8}} dx &= \int \frac{x^4 (e + dx^4)}{a + cx^8} dx \\
&= \frac{dx}{c} - \frac{\int \frac{ad - cex^4}{a + cx^8} dx}{c} \\
&= \frac{dx}{c} - \frac{\int \frac{\frac{\sqrt{2}a^{5/4}d}{\sqrt[4]{c}} + (-ad - \sqrt{a}\sqrt{ce})x^2}{\frac{\sqrt{a}}{\sqrt{c}} - \frac{\sqrt{2}\sqrt[4]{a}x^2}{\sqrt[4]{c}} + x^4} dx}{2\sqrt{2}a^{3/4}c^{5/4}} - \frac{\int \frac{\frac{\sqrt{2}a^{5/4}d}{\sqrt[4]{c}} + (ad + \sqrt{a}\sqrt{ce})x^2}{\frac{\sqrt{a}}{\sqrt{c}} + \frac{\sqrt{2}\sqrt[4]{a}x^2}{\sqrt[4]{c}} + x^4} dx}{2\sqrt{2}a^{3/4}c^{5/4}} \\
&= \frac{dx}{c} - \frac{\int \frac{\frac{\sqrt{2(2-\sqrt{2})}a^{11/8}d}{c^{3/8}} - \left(\frac{\sqrt{2}a^{5/4}d}{\sqrt[4]{c}} - \frac{\sqrt[4]{a}(ad + \sqrt{a}\sqrt{ce})}{\sqrt[4]{c}}\right)x}{\frac{\sqrt[4]{a}}{\sqrt[4]{c}} - \frac{\sqrt{2-\sqrt{2}}\sqrt[8]{a}x}{\sqrt[8]{c}} + x^2} dx}{4\sqrt{2(2-\sqrt{2})}a^{9/8}c^{7/8}} - \frac{\int \frac{\frac{\sqrt{2(2+\sqrt{2})}a^{11/8}d}{c^{3/8}} + \left(\frac{\sqrt{2}a^{5/4}d}{\sqrt[4]{c}} - \frac{\sqrt[4]{a}(ad + \sqrt{a}\sqrt{ce})}{\sqrt[4]{c}}\right)x}{\frac{\sqrt[4]{a}}{\sqrt[4]{c}} + \frac{\sqrt{2-\sqrt{2}}\sqrt[8]{a}x}{\sqrt[8]{c}} + x^2} dx}{4\sqrt{2(2-\sqrt{2})}a^{9/8}c^{7/8}} - \frac{\int \frac{\frac{\sqrt{2(2+\sqrt{2})}a^{11/8}d}{c^{3/8}} - \left(\frac{\sqrt{2}a^{5/4}d}{\sqrt[4]{c}} - \frac{\sqrt[4]{a}(ad + \sqrt{a}\sqrt{ce})}{\sqrt[4]{c}}\right)x}{\frac{\sqrt[4]{a}}{\sqrt[4]{c}} - \frac{\sqrt{2-\sqrt{2}}\sqrt[8]{a}x}{\sqrt[8]{c}} + x^2} dx}{4\sqrt{2(2-\sqrt{2})}a^{9/8}c^{7/8}} \\
&= \frac{dx}{c} - \frac{((1 + \sqrt{2})\sqrt{ad} + \sqrt{ce}) \int \frac{1}{\frac{\sqrt[4]{a}}{\sqrt[4]{c}} - \frac{\sqrt{2-\sqrt{2}}\sqrt[8]{a}x}{\sqrt[8]{c}} + x^2} dx}{8\sqrt{2}\sqrt[4]{ac}^{5/4}} - \frac{((1 + \sqrt{2})\sqrt{ad} + \sqrt{ce}) \int \frac{1}{\frac{\sqrt[4]{a}}{\sqrt[4]{c}} + \frac{\sqrt{2-\sqrt{2}}\sqrt[8]{a}x}{\sqrt[8]{c}} + x^2} dx}{8\sqrt{2}\sqrt[4]{ac}^{5/4}} + \frac{((1 - \sqrt{2})\sqrt{ad} + \sqrt{ce}) \log\left(\frac{\sqrt[4]{a} - \sqrt{2-\sqrt{2}}\sqrt[8]{a}\sqrt[8]{cx} + \sqrt[4]{cx^2}}{8\sqrt{2(2-\sqrt{2})}a^{3/8}c^{9/8}}\right)}{8\sqrt{2(2-\sqrt{2})}a^{3/8}c^{9/8}} + \frac{((1 - \sqrt{2})\sqrt{ad} + \sqrt{ce}) \log\left(\frac{\sqrt[4]{a} + \sqrt{2-\sqrt{2}}\sqrt[8]{a}\sqrt[8]{cx} + \sqrt[4]{cx^2}}{8\sqrt{2(2-\sqrt{2})}a^{3/8}c^{9/8}}\right)}{8\sqrt{2(2-\sqrt{2})}a^{3/8}c^{9/8}} \\
&= \frac{dx}{c} + \frac{((1 + \sqrt{2})\sqrt{ad} + \sqrt{ce}) \tan^{-1}\left(\frac{\sqrt{2-\sqrt{2}}\sqrt[8]{a}-2\sqrt[8]{cx}}{\sqrt{2+\sqrt{2}}\sqrt[8]{a}}\right)}{4\sqrt{2(2+\sqrt{2})}a^{3/8}c^{9/8}} - \frac{\sqrt{2+\sqrt{2}}((1 - \sqrt{2})\sqrt{ad} + \sqrt{ce}) \tan^{-1}\left(\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{a}-2\sqrt[8]{cx}}{\sqrt{2-\sqrt{2}}\sqrt[8]{a}}\right)}{8a^{3/8}c^{9/8}}
\end{aligned}$$

**Mathematica [A]** time = 0.917837, size = 551, normalized size = 0.73

$$\log\left(2\sqrt[8]{a}\sqrt[8]{cx}\sin\left(\frac{\pi}{8}\right) + \sqrt[4]{a} + \sqrt[4]{cx^2}\right)\left(a^{5/8}ce\cos\left(\frac{\pi}{8}\right) - a^{9/8}\sqrt{cd}\sin\left(\frac{\pi}{8}\right)\right) + \log\left(-2\sqrt[8]{a}\sqrt[8]{cx}\sin\left(\frac{\pi}{8}\right) + \sqrt[4]{a} + \sqrt[4]{cx^2}\right)\left(a^{9/8}\sqrt{cd}\sin\left(\frac{\pi}{8}\right) - a^{5/8}ce\cos\left(\frac{\pi}{8}\right)\right)$$

Antiderivative was successfully verified.

[In] Integrate[(d + e/x^4)/(c + a/x^8), x]

```
[Out] (8*a*c^(5/8)*d*x + 2*ArcTan[Cot[Pi/8] + (c^(1/8)*x*Csc[Pi/8])/a^(1/8)]*(a^(5/8)*c*e*cos[Pi/8] - a^(9/8)*Sqrt[c]*d*sin[Pi/8]) + Log[a^(1/4) + c^(1/4)*x^2 + 2*a^(1/8)*c^(1/8)*x*sin[Pi/8]]*(a^(5/8)*c*e*cos[Pi/8] - a^(9/8)*Sqrt[c]*d*sin[Pi/8]) + 2*ArcTan[Cot[Pi/8] - (c^(1/8)*x*Csc[Pi/8])/a^(1/8)]*(-(a^(5/8)*c*e*cos[Pi/8]) + a^(9/8)*Sqrt[c]*d*sin[Pi/8]) + Log[a^(1/4) + c^(1/4)*x^2 - 2*a^(1/8)*c^(1/8)*x*sin[Pi/8]]*(-(a^(5/8)*c*e*cos[Pi/8]) + a^(9/8)*Sqrt[c]*d*sin[Pi/8]) - 2*ArcTan[(c^(1/8)*x*Sec[Pi/8])/a^(1/8) - Tan[Pi/8]]*(a^(9/8)*Sqrt[c]*d*cos[Pi/8] + a^(5/8)*c*e*sin[Pi/8]) - 2*ArcTan[(c^(1/8)*x*Sec[Pi/8])/a^(1/8) + Tan[Pi/8]]*(a^(9/8)*Sqrt[c]*d*cos[Pi/8] + a^(5/8)*c*e*sin[Pi/8]) + Log[a^(1/4) + c^(1/4)*x^2 - 2*a^(1/8)*c^(1/8)*x*cos[Pi/8]]*(a^(9/8)*Sqrt[c]*d*cos[Pi/8] + a^(5/8)*c*e*sin[Pi/8]) - Log[a^(1/4) + c^(1/4)*x^2 + 2*a^(1/8)*c^(1/8)*x*cos[Pi/8]]*(a^(9/8)*Sqrt[c]*d*cos[Pi/8] + a^(5/8)*c*e*sin[Pi/8]))/(8*a*c^(13/8))
```

**Maple [C]** time = 0.003, size = 45, normalized size = 0.1

$$\frac{dx}{c} + \frac{1}{8c^2} \sum_{_R=\text{RootOf}(_Z^8c+a)} \frac{(_R^4ce - ad) \ln(x - _R)}{-_R^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+e/x^4)/(c+a/x^8),x)
```

```
[Out] d*x/c+1/8/c^2*sum((_R^4*c*e-a*d)/_R^7*ln(x-_R),_R=RootOf(_Z^8*c+a))
```

**Maxima [F]** time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e/x^4)/(c+a/x^8),x, algorithm="maxima")
```

```
[Out] d*x/c + integrate((c*e*x^4 - a*d)/(c*x^8 + a), x)/c
```

**Fricas [B]** time = 5.35588, size = 6888, normalized size = 9.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e/x^4)/(c+a/x^8),x, algorithm="fricas")

[Out] 
$$-1/8*(4*c*(-(a*c^4*\sqrt{-(a^4*d^8 - 12*a^3*c*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a*c^3*d^2*e^6 + c^4*e^8)}/(a^3*c^9)) - 4*a*d^3*e + 4*c*d*e^3)/(a*c^4))^{1/4}*\arctan(-((3*a^4*c^4*d^6*e - 19*a^3*c^5*d^4*e^3 + 9*a^2*c^6*d^2*e^5 - a*c^7*e^7 + (a^4*c^8*d^3 - 3*a^3*c^9*d*e^2)*\sqrt{-(a^4*d^8 - 12*a^3*c*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a*c^3*d^2*e^6 + c^4*e^8)}/(a^3*c^9)))*\sqrt{((a^4*d^8 - 4*a^3*c*d^6*e^2 - 10*a^2*c^2*d^4*e^4 - 4*a*c^3*d^2*e^6 + c^4*e^8)*x^2 - (2*a^3*c^7*d*e*\sqrt{-(a^4*d^8 - 12*a^3*c*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a*c^3*d^2*e^6 + c^4*e^8)}/(a^3*c^9)) - a^4*c^2*d^6 + 7*a^3*c^3*d^4*e^2 - 7*a^2*c^4*d^2*e^4 + a*c^5*e^6)*\sqrt{-(a*c^4*\sqrt{-(a^4*d^8 - 12*a^3*c*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a*c^3*d^2*e^6 + c^4*e^8)}/(a^3*c^9)) - 4*a*d^3*e + 4*c*d*e^3)/(a*c^4)))/(a^4*d^8 - 4*a^3*c*d^6*e^2 - 10*a^2*c^2*d^4*e^4 - 4*a*c^3*d^2*e^6 + c^4*e^8)*\sqrt{-(a*c^4*\sqrt{-(a^4*d^8 - 12*a^3*c*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a*c^3*d^2*e^6 + c^4*e^8)}/(a^3*c^9)) - 4*a*d^3*e + 4*c*d*e^3)/(a*c^4)) - ((a^4*c^8*d^3 - 3*a^3*c^9*d*e^2)*x*\sqrt{-(a^4*d^8 - 12*a^3*c*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a*c^3*d^2*e^6 + c^4*e^8)}/(a^3*c^9)) + (3*a^4*c^4*d^6*e - 19*a^3*c^5*d^4*e^3 + 9*a^2*c^6*d^2*e^5 - a*c^7*e^7)*x*\sqrt{-(a*c^4*\sqrt{-(a^4*d^8 - 12*a^3*c*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a*c^3*d^2*e^6 + c^4*e^8)}/(a^3*c^9)) - 4*a*d^3*e + 4*c*d*e^3)/(a*c^4)))*(-a*c^4*\sqrt{-(a^4*d^8 - 12*a^3*c*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a*c^3*d^2*e^6 + c^4*e^8)}/(a^3*c^9)) - 4*a*d^3*e + 4*c*d*e^3)/(a*c^4))^{1/4}/(a^5*d^{10} - 3*a^4*c*d^8*e^2 - 14*a^3*c^2*d^6*e^4 - 14*a^2*c^3*d^4*e^6 - 3*a*c^4*d^2*e^8 + c^5*e^{10})) - 4*c*((a*c^4*\sqrt{-(a^4*d^8 - 12*a^3*c*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a*c^3*d^2*e^6 + c^4*e^8)}/(a^3*c^9)) + 4*a*d^3*e - 4*c*d*e^3)/(a*c^4))^{1/4}*\arctan(((3*a^4*c^4*d^6*e - 19*a^3*c^5*d^4*e^3 + 9*a^2*c^6*d^2*e^5 - a*c^7*e^7 - (a^4*c^8*d^3 - 3*a^3*c^9*d*e^2)*\sqrt{-(a^4*d^8 - 12*a^3*c*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a*c^3*d^2*e^6 + c^4*e^8)}/(a^3*c^9)))*\sqrt{((a^4*d^8 - 4*a^3*c*d^6*e^2 - 10*a^2*c^2*d^4*e^4 - 4*a*c^3*d^2*e^6 + c^4*e^8)*x^2 + (2*a^3*c^7*d*e*\sqrt{-(a^4*d^8 - 12*a^3*c*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a*c^3*d^2*e^6 + c^4*e^8)}/(a^3*c^9)) + a^4*c^2*d^6 - 7*a^3*c^3*d^4*e^2 + 7*a^2*c^4*d^2*e^4 - a*c^5*e^6)*\sqrt{((a*c^4*\sqrt{-(a^4*d^8 - 12*a^3*c*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a*c^3*d^2*e^6 + c^4*e^8)}/(a^3*c^9)) + 4*a*d^3*e - 4*c*d*e^3)/(a*c^4)))/(a^4*d^8 - 4*a^3*c*d^6*e^2 - 10*a^2*c^2*d^4*e^4 - 4*a*c^3*d^2*e^6 + c^4*e^8))*((a*c^4*\sqrt{-(a^4*d^8 - 12*a^3*c*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a*c^3*d^2*e^6 + c^4*e^8)}/(a^3*c^9)) + 4*a*d^3*e - 4*c*d*e^3)/(a*c^4))^{3/4} + ((a^4*c^8*d^3 - 3*a^3*c^9*d*e^2)*x*\sqrt{-(a^4*d^8 - 12*a^3*c*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a*c^3*d^2*e^6 + c^4*e^8)}/(a^3*c^9)) - (3*a^4*c^4*d^6*e - 19*a^3*c^5*d^4*e^3 + 9*a^2*c^6*d^2*e^5 - a*c^7*e^7)*x)*((a*c^4*\sqrt{-(a^4*d^8 - 12*a^3*c*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a*c^3*d^2*e^6 + c^4*e^8)}/(a^3*c^9)) + 4*a*d^3*e - 4*c*d*e^3)/(a*c^4))^{3/4})/(a^5*d^{10} - 3*a^4*c*d^8*e^2 - 14*a^3*c^2*d^6*e^4 - 14*a^2*c^3*d^4*e^6 - 3*a*c^4*d^2*e^8 + c^5*e^{10}))$$

$$\begin{aligned}
&^3d^4e^6 - 3ac^4d^2e^8 + c^5e^{10}) + c((ac^4\sqrt{-(a^4d^8 - 12a^3cd^6e^2 + 38a^2c^2d^4e^4 - 12a^3cd^2e^6 + c^4e^8)/(a^3c^9)}) \\
&+ 4ad^3e - 4cd^2e^3)/(ac^4))^{1/4} \log((a^3d^6 - 5a^2cd^4e^2 - 5ac^2d^2e^4 + c^3e^6)x + (a^2c^6\sqrt{-(a^4d^8 - 12a^3cd^6e^2 + 38a^2c^2d^4e^4 - 12a^3cd^2e^6 + c^4e^8)/(a^3c^9)} + a^3cd^5 - 6a^2c^2d^3e^2 + ac^3d^2e^4) \\
&((ac^4\sqrt{-(a^4d^8 - 12a^3cd^6e^2 + 38a^2c^2d^4e^4 - 12a^3cd^2e^6 + c^4e^8)/(a^3c^9)}) + 4ad^3e - 4cd^2e^3)/(ac^4))^{1/4} - c((ac^4\sqrt{-(a^4d^8 - 12a^3cd^6e^2 + 38a^2c^2d^4e^4 - 12a^3cd^2e^6 + c^4e^8)/(a^3c^9)} + 4ad^3e - 4cd^2e^3)/(ac^4))^{1/4} \log((a^3d^6 - 5a^2cd^4e^2 - 5ac^2d^2e^4 + c^3e^6)x - (a^2c^6\sqrt{-(a^4d^8 - 12a^3cd^6e^2 + 38a^2c^2d^4e^4 - 12a^3cd^2e^6 + c^4e^8)/(a^3c^9)} + a^3cd^5 - 6a^2c^2d^3e^2 + ac^3d^2e^4) \\
&((ac^4\sqrt{-(a^4d^8 - 12a^3cd^6e^2 + 38a^2c^2d^4e^4 - 12a^3cd^2e^6 + c^4e^8)/(a^3c^9)}) + 4ad^3e - 4cd^2e^3)/(ac^4))^{1/4} - c(-(ac^4\sqrt{-(a^4d^8 - 12a^3cd^6e^2 + 38a^2c^2d^4e^4 - 12a^3cd^2e^6 + c^4e^8)/(a^3c^9)} - 4ad^3e + 4cd^2e^3)/(ac^4))^{1/4} \log((a^3d^6 - 5a^2cd^4e^2 - 5ac^2d^2e^4 + c^3e^6)x + (a^2c^6\sqrt{-(a^4d^8 - 12a^3cd^6e^2 + 38a^2c^2d^4e^4 - 12a^3cd^2e^6 + c^4e^8)/(a^3c^9)} - a^3cd^5 + 6a^2c^2d^3e^2 - ac^3d^2e^4) \\
&(-(ac^4\sqrt{-(a^4d^8 - 12a^3cd^6e^2 + 38a^2c^2d^4e^4 - 12a^3cd^2e^6 + c^4e^8)/(a^3c^9)} - 4ad^3e + 4cd^2e^3)/(ac^4))^{1/4} + c(-(ac^4\sqrt{-(a^4d^8 - 12a^3cd^6e^2 + 38a^2c^2d^4e^4 - 12a^3cd^2e^6 + c^4e^8)/(a^3c^9)} - 4ad^3e + 4cd^2e^3)/(ac^4))^{1/4} \log((a^3d^6 - 5a^2cd^4e^2 - 5ac^2d^2e^4 + c^3e^6)x - (a^2c^6\sqrt{-(a^4d^8 - 12a^3cd^6e^2 + 38a^2c^2d^4e^4 - 12a^3cd^2e^6 + c^4e^8)/(a^3c^9)} - a^3cd^5 + 6a^2c^2d^3e^2 - ac^3d^2e^4) \\
&(-(ac^4\sqrt{-(a^4d^8 - 12a^3cd^6e^2 + 38a^2c^2d^4e^4 - 12a^3cd^2e^6 + c^4e^8)/(a^3c^9)} - 4ad^3e + 4cd^2e^3)/(ac^4))^{1/4} - 8dx)/c
\end{aligned}$$

**Sympy [A]** time = 18.9418, size = 204, normalized size = 0.27

$$\text{RootSum}\left(16777216t^8a^3c^9 + t^4(-32768a^3c^5d^3e + 32768a^2c^6de^3) + a^4d^8 + 4a^3cd^6e^2 + 6a^2c^2d^4e^4 + 4ac^3d^2e^6 + c^4e^8, (t\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e/x\*\*4)/(c+a/x\*\*8),x)

[Out] RootSum(16777216\*\_t\*\*8\*a\*\*3\*c\*\*9 + \_t\*\*4\*(-32768\*a\*\*3\*c\*\*5\*d\*\*3\*e + 32768\*a\*\*2\*c\*\*6\*d\*e\*\*3) + a\*\*4\*d\*\*8 + 4\*a\*\*3\*c\*d\*\*6\*e\*\*2 + 6\*a\*\*2\*c\*\*2\*d\*\*4\*e\*\*4 + 4\*a\*c\*\*3\*d\*\*2\*e\*\*6 + c\*\*4\*e\*\*8, Lambda(\_t, \_t\*log(x + (-32768\*\_t\*\*5\*a\*\*2\*c\*\*6\*e - 8\*\_t\*a\*\*3\*c\*d\*\*5 + 80\*\_t\*a\*\*2\*c\*\*2\*d\*\*3\*e\*\*2 - 40\*\_t\*a\*c\*\*3\*d\*e\*\*4)

$$\frac{d}{dx} \left( \frac{1}{c} \left( \frac{d^3 x^6 - 5a^2 c d^4 e^2 - 5a c^2 d^2 e^4 + c^3 e^6}{c} \right) \right) + \frac{d^2 x}{c}$$

**Giac [A]** time = 1.22948, size = 873, normalized size = 1.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e/x^4)/(c+a/x^8),x, algorithm="giac")

[Out] 
$$\begin{aligned} & \frac{d^2 x}{c} - \frac{1}{8} \left( \frac{c \sqrt{-\sqrt{2} + 2} \left(\frac{a}{c}\right)^{5/8} e + a d \sqrt{\sqrt{2} + 2} \left(\frac{a}{c}\right)^{1/8}}{a^2 c} \arctan\left(\frac{2x + \sqrt{-\sqrt{2} + 2} \left(\frac{a}{c}\right)^{1/8}}{\sqrt{\sqrt{2} + 2} \left(\frac{a}{c}\right)^{1/8}}\right) \right. \\ & - \frac{1}{8} \left( \frac{c \sqrt{-\sqrt{2} + 2} \left(\frac{a}{c}\right)^{5/8} e + a d \sqrt{\sqrt{2} + 2} \left(\frac{a}{c}\right)^{1/8}}{a^2 c} \arctan\left(\frac{2x - \sqrt{-\sqrt{2} + 2} \left(\frac{a}{c}\right)^{1/8}}{\sqrt{\sqrt{2} + 2} \left(\frac{a}{c}\right)^{1/8}}\right) \right. \\ & + \frac{1}{8} \left( \frac{c \sqrt{\sqrt{2} + 2} \left(\frac{a}{c}\right)^{5/8} e - a d \sqrt{-\sqrt{2} + 2} \left(\frac{a}{c}\right)^{1/8}}{a^2 c} \arctan\left(\frac{2x + \sqrt{\sqrt{2} + 2} \left(\frac{a}{c}\right)^{1/8}}{\sqrt{-\sqrt{2} + 2} \left(\frac{a}{c}\right)^{1/8}}\right) \right. \\ & + \frac{1}{8} \left( \frac{c \sqrt{\sqrt{2} + 2} \left(\frac{a}{c}\right)^{5/8} e - a d \sqrt{-\sqrt{2} + 2} \left(\frac{a}{c}\right)^{1/8}}{a^2 c} \arctan\left(\frac{2x - \sqrt{\sqrt{2} + 2} \left(\frac{a}{c}\right)^{1/8}}{\sqrt{-\sqrt{2} + 2} \left(\frac{a}{c}\right)^{1/8}}\right) \right) \\ & - \frac{1}{16} \left( \frac{c \sqrt{-\sqrt{2} + 2} \left(\frac{a}{c}\right)^{5/8} e + a d \sqrt{\sqrt{2} + 2} \left(\frac{a}{c}\right)^{1/8}}{a^2 c} \log\left(x^2 + x \sqrt{\sqrt{2} + 2} \left(\frac{a}{c}\right)^{1/8} + \left(\frac{a}{c}\right)^{1/4}\right) \right. \\ & + \frac{1}{16} \left( \frac{c \sqrt{-\sqrt{2} + 2} \left(\frac{a}{c}\right)^{5/8} e + a d \sqrt{\sqrt{2} + 2} \left(\frac{a}{c}\right)^{1/8}}{a^2 c} \log\left(x^2 - x \sqrt{\sqrt{2} + 2} \left(\frac{a}{c}\right)^{1/8} + \left(\frac{a}{c}\right)^{1/4}\right) \right) \\ & - \frac{1}{16} \left( \frac{c \sqrt{\sqrt{2} + 2} \left(\frac{a}{c}\right)^{5/8} e - a d \sqrt{-\sqrt{2} + 2} \left(\frac{a}{c}\right)^{1/8}}{a^2 c} \log\left(x^2 + x \sqrt{-\sqrt{2} + 2} \left(\frac{a}{c}\right)^{1/8} + \left(\frac{a}{c}\right)^{1/4}\right) \right. \\ & - \frac{1}{16} \left( \frac{c \sqrt{\sqrt{2} + 2} \left(\frac{a}{c}\right)^{5/8} e - a d \sqrt{-\sqrt{2} + 2} \left(\frac{a}{c}\right)^{1/8}}{a^2 c} \log\left(x^2 - x \sqrt{-\sqrt{2} + 2} \left(\frac{a}{c}\right)^{1/8} + \left(\frac{a}{c}\right)^{1/4}\right) \right) \end{aligned}$$

$$3.41 \quad \int \frac{d + \frac{e}{x^4}}{c + \frac{a}{x^8} + \frac{b}{x^4}} dx$$

**Optimal.** Leaf size=433

$$\frac{\left(\frac{-2acd+b^2d-bce}{\sqrt{b^2-4ac}} + bd - ce\right) \tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{cx}}{\sqrt{-\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}c^{5/4}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}} + \frac{\left(\frac{-2acd+b^2d-bce}{\sqrt{b^2-4ac}} + bd - ce\right) \tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{cx}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}c^{5/4}\left(\sqrt{b^2-4ac}-b\right)^{3/4}} + \frac{\left(\frac{-2acd+b^2d-bce}{\sqrt{b^2-4ac}} + bd - ce\right) \tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{cx}}{\sqrt{b^2-4ac}}\right)}{2\sqrt[4]{2}c^{5/4}\left(-\sqrt{b^2-4ac}\right)^{3/4}}$$

[Out] (d\*x)/c + ((b\*d - c\*e + (b^2\*d - 2\*a\*c\*d - b\*c\*e)/Sqrt[b^2 - 4\*a\*c])\*ArcTan[(2^(1/4)\*c^(1/4)\*x)/(-b - Sqrt[b^2 - 4\*a\*c])^(1/4)]/(2\*2^(1/4)\*c^(5/4)\*(-b - Sqrt[b^2 - 4\*a\*c])^(3/4)) + ((b\*d - c\*e - (b^2\*d - 2\*a\*c\*d - b\*c\*e)/Sqrt[b^2 - 4\*a\*c])\*ArcTan[(2^(1/4)\*c^(1/4)\*x)/(-b + Sqrt[b^2 - 4\*a\*c])^(1/4)]/(2\*2^(1/4)\*c^(5/4)\*(-b + Sqrt[b^2 - 4\*a\*c])^(3/4)) + ((b\*d - c\*e + (b^2\*d - 2\*a\*c\*d - b\*c\*e)/Sqrt[b^2 - 4\*a\*c])\*ArcTanh[(2^(1/4)\*c^(1/4)\*x)/(-b - Sqrt[b^2 - 4\*a\*c])^(1/4)]/(2\*2^(1/4)\*c^(5/4)\*(-b - Sqrt[b^2 - 4\*a\*c])^(3/4)) + ((b\*d - c\*e - (b^2\*d - 2\*a\*c\*d - b\*c\*e)/Sqrt[b^2 - 4\*a\*c])\*ArcTanh[(2^(1/4)\*c^(1/4)\*x)/(-b + Sqrt[b^2 - 4\*a\*c])^(1/4)]/(2\*2^(1/4)\*c^(5/4)\*(-b + Sqrt[b^2 - 4\*a\*c])^(3/4))

**Rubi [A]** time = 0.98864, antiderivative size = 433, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$ , Rules used = {1393, 1502, 1422, 212, 208, 205}

$$\frac{\left(\frac{-2acd+b^2d-bce}{\sqrt{b^2-4ac}} + bd - ce\right) \tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{cx}}{\sqrt{-\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}c^{5/4}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}} + \frac{\left(\frac{-2acd+b^2d-bce}{\sqrt{b^2-4ac}} + bd - ce\right) \tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{cx}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}c^{5/4}\left(\sqrt{b^2-4ac}-b\right)^{3/4}} + \frac{\left(\frac{-2acd+b^2d-bce}{\sqrt{b^2-4ac}} + bd - ce\right) \tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{cx}}{\sqrt{b^2-4ac}}\right)}{2\sqrt[4]{2}c^{5/4}\left(-\sqrt{b^2-4ac}\right)^{3/4}}$$

Antiderivative was successfully verified.

[In] Int[(d + e/x^4)/(c + a/x^8 + b/x^4), x]

[Out] (d\*x)/c + ((b\*d - c\*e + (b^2\*d - 2\*a\*c\*d - b\*c\*e)/Sqrt[b^2 - 4\*a\*c])\*ArcTan[(2^(1/4)\*c^(1/4)\*x)/(-b - Sqrt[b^2 - 4\*a\*c])^(1/4)]/(2\*2^(1/4)\*c^(5/4)\*(-b - Sqrt[b^2 - 4\*a\*c])^(3/4)) + ((b\*d - c\*e - (b^2\*d - 2\*a\*c\*d - b\*c\*e)/Sqrt[b^2 - 4\*a\*c])\*ArcTan[(2^(1/4)\*c^(1/4)\*x)/(-b + Sqrt[b^2 - 4\*a\*c])^(1/4)]/(2\*2^(1/4)\*c^(5/4)\*(-b + Sqrt[b^2 - 4\*a\*c])^(3/4)) + ((b\*d - c\*e + (b^2\*d - 2\*a\*c\*d - b\*c\*e)/Sqrt[b^2 - 4\*a\*c])\*ArcTanh[(2^(1/4)\*c^(1/4)\*x)/(-b - Sqrt[b^2 - 4\*a\*c])^(1/4)]/(2\*2^(1/4)\*c^(5/4)\*(-b - Sqrt[b^2 - 4\*a\*c])^(3/4)) + ((b\*d - c\*e - (b^2\*d - 2\*a\*c\*d - b\*c\*e)/Sqrt[b^2 - 4\*a\*c])\*ArcTanh[(2^(1/4)\*c^(1/4)\*x)/(-b + Sqrt[b^2 - 4\*a\*c])^(1/4)]/(2\*2^(1/4)\*c^(5/4)\*(-b + Sqrt[b^2 - 4\*a\*c])^(3/4))

$$\frac{t[b^2 - 4ac]^{1/4}}{(2^2)^{1/4} c^{5/4} (-b - \sqrt{b^2 - 4ac})^{3/4}} + \frac{((bd - ce - (b^2d - 2acd - bce))/\sqrt{b^2 - 4ac}) \operatorname{ArcTanh}[(2^{1/4} c^{1/4} x)/(-b + \sqrt{b^2 - 4ac})^{1/4}]}{(2^2)^{1/4} c^{5/4} (-b + \sqrt{b^2 - 4ac})^{3/4}}$$

### Rule 1393

$$\operatorname{Int}[(a + (c \cdot x)^{n_2}) + (b \cdot x)^{n_1}]^{p_1} ((d + (e \cdot x)^{n_1})^{q_1}, x_{\text{Symbol}}] \rightarrow \operatorname{Int}[x^{n(2p+q)} (e + d/x^n)^q (c + b/x^n + a/x^{2n})^p, x] /; \operatorname{FreeQ}\{a, b, c, d, e, n\}, x\} \&\& \operatorname{EqQ}[n^2, 2n] \&\& \operatorname{IntegersQ}[p, q] \&\& \operatorname{NegQ}[n]$$

### Rule 1502

$$\operatorname{Int}[(f \cdot x)^{m_1} ((d + (e \cdot x)^{n_1}) ((a + (b \cdot x)^{n_1}) + (c \cdot x)^{n_2}))^{p_1}, x_{\text{Symbol}}] \rightarrow \operatorname{Simp}[(e f^{n-1} (f x)^{m-n+1} (a + b x^n + c x^{2n})^{p+1}) / (c(m+n(2p+1)+1)), x] - \operatorname{Dist}[f^n / (c(m+n(2p+1)+1)), \operatorname{Int}[(f x)^{m-n} (a + b x^n + c x^{2n})^p \operatorname{Simp}[a e(m-n+1) + (b e(m+n p+1) - c d(m+n(2p+1)+1)) x^n, x], x], x] /; \operatorname{FreeQ}\{a, b, c, d, e, f, p\}, x\} \&\& \operatorname{EqQ}[n^2, 2n] \&\& \operatorname{NeQ}[b^2 - 4ac, 0] \&\& \operatorname{IGtQ}[n, 0] \&\& \operatorname{GtQ}[m, n-1] \&\& \operatorname{NeQ}[m+n(2p+1)+1, 0] \&\& \operatorname{IntegerQ}[p]$$

### Rule 1422

$$\operatorname{Int}[(d + (e \cdot x)^{n_1}) / ((a + (b \cdot x)^{n_1}) + (c \cdot x)^{n_2}), x_{\text{Symbol}}] \rightarrow \operatorname{With}\{q = \operatorname{Rt}[b^2 - 4ac, 2]\}, \operatorname{Dist}[e/2 + (2cd - be)/(2q), \operatorname{Int}[1/(b/2 - q/2 + c x^n), x], x] + \operatorname{Dist}[e/2 - (2cd - be)/(2q), \operatorname{Int}[1/(b/2 + q/2 + c x^n), x], x]] /; \operatorname{FreeQ}\{a, b, c, d, e, n\}, x\} \&\& \operatorname{EqQ}[n^2, 2n] \&\& \operatorname{NeQ}[b^2 - 4ac, 0] \&\& \operatorname{NeQ}[c d^2 - b d e + a e^2, 0] \&\& (\operatorname{PosQ}[b^2 - 4ac] \mid \mid \operatorname{!IGtQ}[n/2, 0])$$

### Rule 212

$$\operatorname{Int}[(a + (b \cdot x)^4)^{-1}, x_{\text{Symbol}}] \rightarrow \operatorname{With}\{r = \operatorname{Numerator}[\operatorname{Rt}[-(a/b), 2]], s = \operatorname{Denominator}[\operatorname{Rt}[-(a/b), 2]]\}, \operatorname{Dist}[r/(2a), \operatorname{Int}[1/(r - s x^2), x], x] + \operatorname{Dist}[r/(2a), \operatorname{Int}[1/(r + s x^2), x], x]] /; \operatorname{FreeQ}\{a, b\}, x\} \&\& \operatorname{!GtQ}[a/b, 0]$$

### Rule 208

$$\operatorname{Int}[(a + (b \cdot x)^2)^{-1}, x_{\text{Symbol}}] \rightarrow \operatorname{Simp}[(\operatorname{Rt}[-(a/b), 2] \operatorname{ArcTanh}[x/\operatorname{Rt}[-(a/b), 2]])/a, x] /; \operatorname{FreeQ}\{a, b\}, x\} \&\& \operatorname{NegQ}[a/b]$$



Rule 205

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rubi steps

$$\begin{aligned}
 \int \frac{d + \frac{e}{x^4}}{c + \frac{a}{x^8} + \frac{b}{x^4}} dx &= \int \frac{x^4(e + dx^4)}{a + bx^4 + cx^8} dx \\
 &= \frac{dx}{c} - \frac{\int \frac{ad+(bd-ce)x^4}{a+bx^4+cx^8} dx}{c} \\
 &= \frac{dx}{c} - \frac{\left(bd - ce - \frac{b^2d-2acd-bce}{\sqrt{b^2-4ac}}\right) \int \frac{1}{\frac{b}{2} - \frac{1}{2}\sqrt{b^2-4ac} + cx^4} dx}{2c} - \frac{\left(bd - ce + \frac{b^2d-2acd-bce}{\sqrt{b^2-4ac}}\right) \int \frac{1}{\frac{b}{2} + \frac{1}{2}\sqrt{b^2-4ac} + cx^4} dx}{2c} \\
 &= \frac{dx}{c} + \frac{\left(bd - ce - \frac{b^2d-2acd-bce}{\sqrt{b^2-4ac}}\right) \int \frac{1}{\sqrt{-b+\sqrt{b^2-4ac}} - \sqrt{2}\sqrt{cx^2}} dx}{2c\sqrt{-b+\sqrt{b^2-4ac}}} + \frac{\left(bd - ce - \frac{b^2d-2acd-bce}{\sqrt{b^2-4ac}}\right) \int \frac{1}{\sqrt{-b+\sqrt{b^2-4ac}} + \sqrt{2}\sqrt{cx^2}} dx}{2c\sqrt{-b+\sqrt{b^2-4ac}}} \\
 &= \frac{dx}{c} + \frac{\left(bd - ce + \frac{b^2d-2acd-bce}{\sqrt{b^2-4ac}}\right) \tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{cx}}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}c^{5/4}\left(-b-\sqrt{b^2-4ac}\right)^{3/4}} + \frac{\left(bd - ce - \frac{b^2d-2acd-bce}{\sqrt{b^2-4ac}}\right) \tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{cx}}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}c^{5/4}\left(-b+\sqrt{b^2-4ac}\right)^{3/4}} + \dots
 \end{aligned}$$

**Mathematica [C]** time = 0.0780775, size = 88, normalized size = 0.2

$$\frac{dx}{c} - \frac{\text{RootSum}\left[\#1^4b + \#1^8c + a\&, \frac{\#1^4bd \log(x-\#1) - \#1^4ce \log(x-\#1) + ad \log(x-\#1)}{\#1^3b + 2\#1^7c} \& \right]}{4c}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e/x^4)/(c + a/x^8 + b/x^4), x]

[Out] (d\*x)/c - RootSum[a + b\*#1^4 + c\*#1^8 & , (a\*d\*Log[x - #1] + b\*d\*Log[x - #1]\*#1^4 - c\*e\*Log[x - #1]\*#1^4)/(b\*#1^3 + 2\*c\*#1^7) & ]/(4\*c)

**Maple [C]** time = 0.005, size = 67, normalized size = 0.2

$$\frac{dx}{c} + \frac{1}{4c} \sum_{_R=\text{RootOf}(_Z^8c+_Z^4b+a)} \frac{((-bd+ce)_R^4-ad)\ln(x-_R)}{2_R^7c+_R^3b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d+e/x^4)/(c+a/x^8+b/x^4),x)

[Out] d\*x/c+1/4/c\*sum(((b\*d+c\*e)\*\_R^4-a\*d)/(2\*\_R^7\*c+\_R^3\*b)\*ln(x-\_R),\_R=RootOf(\_Z^8\*c+\_Z^4\*b+a))

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{dx}{c} + \frac{-\int \frac{(bd-ce)x^4+ad}{cx^8+bx^4+a} dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e/x^4)/(c+a/x^8+b/x^4),x, algorithm="maxima")

[Out] d\*x/c + integrate(-((b\*d - c\*e)\*x^4 + a\*d)/(c\*x^8 + b\*x^4 + a), x)/c

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e/x^4)/(c+a/x^8+b/x^4),x, algorithm="fricas")

[Out] Timed out

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e/x**4)/(c+a/x**8+b/x**4),x)
```

```
[Out] Timed out
```

---

**Giac [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e/x^4)/(c+a/x^8+b/x^4),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

$$3.42 \quad \int \frac{(d+ex^n)^3}{a+cx^{2n}} dx$$

**Optimal.** Leaf size=141

$$\frac{ex^{n+1} (3cd^2 - ae^2) {}_2F_1\left(1, \frac{n+1}{2n}; \frac{1}{2}\left(3 + \frac{1}{n}\right); -\frac{cx^{2n}}{a}\right)}{ac(n+1)} + \frac{dx (cd^2 - 3ae^2) {}_2F_1\left(1, \frac{1}{2n}; \frac{1}{2}\left(2 + \frac{1}{n}\right); -\frac{cx^{2n}}{a}\right)}{ac} + \frac{3de^2x}{c} + \frac{e^3x^{n+1}}{c(n+1)}$$

[Out] (3\*d\*e^2\*x)/c + (e^3\*x^(1+n))/(c\*(1+n)) + (d\*(c\*d^2 - 3\*a\*e^2)\*x\*Hypergeometric2F1[1, 1/(2\*n), (2+n^(-1))/2, -((c\*x^(2\*n))/a)]/(a\*c) + (e\*(3\*c\*d^2 - a\*e^2)\*x^(1+n)\*Hypergeometric2F1[1, (1+n)/(2\*n), (3+n^(-1))/2, -((c\*x^(2\*n))/a)]/(a\*c\*(1+n)))

**Rubi [A]** time = 0.145232, antiderivative size = 141, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.19$ , Rules used = {1425, 1418, 245, 364}

$$\frac{ex^{n+1} (3cd^2 - ae^2) {}_2F_1\left(1, \frac{n+1}{2n}; \frac{1}{2}\left(3 + \frac{1}{n}\right); -\frac{cx^{2n}}{a}\right)}{ac(n+1)} + \frac{dx (cd^2 - 3ae^2) {}_2F_1\left(1, \frac{1}{2n}; \frac{1}{2}\left(2 + \frac{1}{n}\right); -\frac{cx^{2n}}{a}\right)}{ac} + \frac{3de^2x}{c} + \frac{e^3x^{n+1}}{c(n+1)}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x^n)^3/(a + c\*x^(2\*n)), x]

[Out] (3\*d\*e^2\*x)/c + (e^3\*x^(1+n))/(c\*(1+n)) + (d\*(c\*d^2 - 3\*a\*e^2)\*x\*Hypergeometric2F1[1, 1/(2\*n), (2+n^(-1))/2, -((c\*x^(2\*n))/a)]/(a\*c) + (e\*(3\*c\*d^2 - a\*e^2)\*x^(1+n)\*Hypergeometric2F1[1, (1+n)/(2\*n), (3+n^(-1))/2, -((c\*x^(2\*n))/a)]/(a\*c\*(1+n)))

### Rule 1425

Int[((d\_) + (e\_.)\*(x\_)^(n\_))^(q\_)/((a\_) + (c\_.)\*(x\_)^(n2\_)), x\_Symbol] := Int[ExpandIntegrand[(d + e\*x^n)^q/(a + c\*x^(2\*n)), x], x] /; FreeQ[{a, c, d, e, n}, x] && EqQ[n2, 2\*n] && NeQ[c\*d^2 + a\*e^2, 0] && IntegerQ[q]

### Rule 1418

Int[((d\_) + (e\_.)\*(x\_)^(n\_))/((a\_) + (c\_.)\*(x\_)^(n2\_)), x\_Symbol] := Dist[d, Int[1/(a + c\*x^(2\*n)), x], x] + Dist[e, Int[x^n/(a + c\*x^(2\*n)), x], x] /; FreeQ[{a, c, d, e, n}, x] && EqQ[n2, 2\*n] && NeQ[c\*d^2 + a\*e^2, 0] && (Po

sQ[a\*c] || !IntegerQ[n])

### Rule 245

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] :> Simp[a^p\*x\*Hypergeometric2F1[-p, 1/n, 1/n + 1, -((b\*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] || GtQ[a, 0])

### Rule 364

Int[((c\_.)\*(x\_))^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] :> Simp[(a^p\*(c\*x)^(m + 1)\*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b\*x^n)/a)])/((c\*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])

### Rubi steps

$$\begin{aligned} \int \frac{(d + ex^n)^3}{a + cx^{2n}} dx &= \int \left( \frac{3de^2}{c} + \frac{e^3 x^n}{c} + \frac{cd^3 - 3ade^2 + (3cd^2e - ae^3)x^n}{c(a + cx^{2n})} \right) dx \\ &= \frac{3de^2 x}{c} + \frac{e^3 x^{1+n}}{c(1+n)} + \frac{\int \frac{cd^3 - 3ade^2 + (3cd^2e - ae^3)x^n}{a + cx^{2n}} dx}{c} \\ &= \frac{3de^2 x}{c} + \frac{e^3 x^{1+n}}{c(1+n)} + \frac{(d(cd^2 - 3ae^2)) \int \frac{1}{a + cx^{2n}} dx}{c} + \frac{(e(3cd^2 - ae^2)) \int \frac{x^n}{a + cx^{2n}} dx}{c} \\ &= \frac{3de^2 x}{c} + \frac{e^3 x^{1+n}}{c(1+n)} + \frac{d(cd^2 - 3ae^2) x {}_2F_1\left(1, \frac{1}{2n}; \frac{1}{2}\left(2 + \frac{1}{n}\right); -\frac{cx^{2n}}{a}\right)}{ac} + \frac{e(3cd^2 - ae^2) x^{1+n} {}_2F_1\left(1, \frac{1+n}{2n}; \frac{1}{2}\right)}{ac(1+n)} \end{aligned}$$

**Mathematica [A]** time = 0.247389, size = 127, normalized size = 0.9

$$\frac{x \left( d(n+1)(cd^2 - 3ae^2) {}_2F_1\left(1, \frac{1}{2n}; \frac{1}{2}\left(2 + \frac{1}{n}\right); -\frac{cx^{2n}}{a}\right) + e \left( x^n (3cd^2 - ae^2) {}_2F_1\left(1, \frac{n+1}{2n}; \frac{1}{2}\left(3 + \frac{1}{n}\right); -\frac{cx^{2n}}{a}\right) + ae(3d(n+1)) \right) \right)}{ac(n+1)}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e\*x^n)^3/(a + c\*x^(2\*n)), x]

[Out] (x\*(d\*(c\*d^2 - 3\*a\*e^2)\*(1 + n)\*Hypergeometric2F1[1, 1/(2\*n), (2 + n^(-1))/2, -((c\*x^(2\*n))/a)] + e\*(a\*e\*(3\*d\*(1 + n) + e\*x^n) + (3\*c\*d^2 - a\*e^2)\*x^n

\*Hypergeometric2F1[1, (1 + n)/(2\*n), (3 + n^(-1))/2, -((c\*x^(2\*n))/a)])))/(a\*c\*(1 + n))

**Maple [F]** time = 0.078, size = 0, normalized size = 0.

$$\int \frac{(d + ex^n)^3}{a + cx^{2n}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d+e\*x^n)^3/(a+c\*x^(2\*n)),x)

[Out] int((d+e\*x^n)^3/(a+c\*x^(2\*n)),x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{3de^2(n+1)x + e^3xx^n}{c(n+1)} - \int -\frac{cd^3 - 3ade^2 + (3cd^2e - ae^3)x^n}{c^2x^{2n} + ac} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e\*x^n)^3/(a+c\*x^(2\*n)),x, algorithm="maxima")

[Out] (3\*d\*e^2\*(n + 1)\*x + e^3\*x\*x^n)/(c\*(n + 1)) - integrate(-(c\*d^3 - 3\*a\*d\*e^2 + (3\*c\*d^2\*e - a\*e^3)\*x^n)/(c^2\*x^(2\*n) + a\*c), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{e^3x^{3n} + 3de^2x^{2n} + 3d^2ex^n + d^3}{cx^{2n} + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e\*x^n)^3/(a+c\*x^(2\*n)),x, algorithm="fricas")

[Out] integral((e^3\*x^(3\*n) + 3\*d\*e^2\*x^(2\*n) + 3\*d^2\*e\*x^n + d^3)/(c\*x^(2\*n) + a), x)

---

**Sympy [C]** time = 10.3546, size = 337, normalized size = 2.39

$$\frac{3de^2x\Phi\left(\frac{ax^{-2n}e^{i\pi}}{c}, 1, \frac{e^{i\pi}}{2n}\right)\Gamma\left(\frac{1}{2n}\right)}{4cn^2\Gamma\left(1 + \frac{1}{2n}\right)} + \frac{d^3x\Phi\left(\frac{cx^{2n}e^{i\pi}}{a}, 1, \frac{1}{2n}\right)\Gamma\left(\frac{1}{2n}\right)}{4an^2\Gamma\left(1 + \frac{1}{2n}\right)} + \frac{3d^2exx^n\Phi\left(\frac{cx^{2n}e^{i\pi}}{a}, 1, \frac{1}{2} + \frac{1}{2n}\right)\Gamma\left(\frac{1}{2} + \frac{1}{2n}\right)}{4an\Gamma\left(\frac{3}{2} + \frac{1}{2n}\right)} + \frac{3d^2exx^n\Phi\left(\frac{cx^{2n}e^{i\pi}}{a}, 1, \frac{1}{2} + \frac{1}{2n}\right)\Gamma\left(\frac{1}{2} + \frac{1}{2n}\right)}{4an\Gamma\left(\frac{3}{2} + \frac{1}{2n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e\*x\*\*n)\*\*3/(a+c\*x\*\*(2\*n)),x)

[Out]  $-3*d*e^{2n}x*lerchphi(a*x^{-(2*n)}*exp\_polar(I*pi)/c, 1, exp\_polar(I*pi)/(2*n)) * gamma(1/(2*n))/(4*c*n^{2n}*gamma(1 + 1/(2*n))) + d^{3n}x*lerchphi(c*x^{(2*n)}*exp\_polar(I*pi)/a, 1, 1/(2*n)) * gamma(1/(2*n))/(4*a*n^{2n}*gamma(1 + 1/(2*n))) + 3*d^{2n}e*x*x^n*lerchphi(c*x^{(2*n)}*exp\_polar(I*pi)/a, 1, 1/2 + 1/(2*n)) * gamma(1/2 + 1/(2*n))/(4*a*n*gamma(3/2 + 1/(2*n))) + 3*d^{2n}e*x*x^n*lerchphi(c*x^{(2*n)}*exp\_polar(I*pi)/a, 1, 1/2 + 1/(2*n)) * gamma(1/2 + 1/(2*n))/(4*a*n*gamma(3/2 + 1/(2*n))) + 3*e^{3n}x*x^{3n}*lerchphi(c*x^{(2*n)}*exp\_polar(I*pi)/a, 1, 3/2 + 1/(2*n)) * gamma(3/2 + 1/(2*n))/(4*a*n*gamma(5/2 + 1/(2*n))) + e^{3n}x*x^{3n}*lerchphi(c*x^{(2*n)}*exp\_polar(I*pi)/a, 1, 3/2 + 1/(2*n)) * gamma(3/2 + 1/(2*n))/(4*a*n^{2n}*gamma(5/2 + 1/(2*n)))$

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^n + d)^3}{cx^{2n} + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e\*x^n)^3/(a+c\*x^(2\*n)),x, algorithm="giac")

[Out] integrate((e\*x^n + d)^3/(c\*x^(2\*n) + a), x)

### 3.43 $\int \frac{(d+ex^n)^2}{a+cx^{2n}} dx$

**Optimal.** Leaf size=107

$$\frac{x(cd^2 - ae^2) {}_2F_1\left(1, \frac{1}{2n}; \frac{1}{2}\left(2 + \frac{1}{n}\right); -\frac{cx^{2n}}{a}\right)}{ac} + \frac{2dex^{n+1} {}_2F_1\left(1, \frac{n+1}{2n}; \frac{1}{2}\left(3 + \frac{1}{n}\right); -\frac{cx^{2n}}{a}\right)}{a(n+1)} + \frac{e^2x}{c}$$

[Out] (e^2\*x)/c + ((c\*d^2 - a\*e^2)\*x\*Hypergeometric2F1[1, 1/(2\*n), (2 + n^(-1))/2, -((c\*x^(2\*n))/a)])/(a\*c) + (2\*d\*e\*x^(1 + n)\*Hypergeometric2F1[1, (1 + n)/(2\*n), (3 + n^(-1))/2, -((c\*x^(2\*n))/a)])/(a\*(1 + n))

**Rubi [A]** time = 0.097461, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.19$ , Rules used = {1425, 1418, 245, 364}

$$\frac{x(cd^2 - ae^2) {}_2F_1\left(1, \frac{1}{2n}; \frac{1}{2}\left(2 + \frac{1}{n}\right); -\frac{cx^{2n}}{a}\right)}{ac} + \frac{2dex^{n+1} {}_2F_1\left(1, \frac{n+1}{2n}; \frac{1}{2}\left(3 + \frac{1}{n}\right); -\frac{cx^{2n}}{a}\right)}{a(n+1)} + \frac{e^2x}{c}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x^n)^2/(a + c\*x^(2\*n)), x]

[Out] (e^2\*x)/c + ((c\*d^2 - a\*e^2)\*x\*Hypergeometric2F1[1, 1/(2\*n), (2 + n^(-1))/2, -((c\*x^(2\*n))/a)])/(a\*c) + (2\*d\*e\*x^(1 + n)\*Hypergeometric2F1[1, (1 + n)/(2\*n), (3 + n^(-1))/2, -((c\*x^(2\*n))/a)])/(a\*(1 + n))

#### Rule 1425

Int[((d\_) + (e\_.)\*(x\_)^(n\_))^(q\_)/((a\_) + (c\_.)\*(x\_)^(n2\_)), x\_Symbol] :> Int[ExpandIntegrand[(d + e\*x^n)^q/(a + c\*x^(2\*n)), x], x] /; FreeQ[{a, c, d, e, n}, x] && EqQ[n2, 2\*n] && NeQ[c\*d^2 + a\*e^2, 0] && IntegerQ[q]

#### Rule 1418

Int[((d\_) + (e\_.)\*(x\_)^(n\_))/((a\_) + (c\_.)\*(x\_)^(n2\_)), x\_Symbol] :> Dist[d, Int[1/(a + c\*x^(2\*n)), x], x] + Dist[e, Int[x^n/(a + c\*x^(2\*n)), x], x] /; FreeQ[{a, c, d, e, n}, x] && EqQ[n2, 2\*n] && NeQ[c\*d^2 + a\*e^2, 0] && (PosQ[a\*c] || !IntegerQ[n])

#### Rule 245



```
Int[((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Simp[a^p*x*Hypergeometric2F1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] || GtQ[a, 0])
```

### Rule 364

```
Int[((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Simp[(a^p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a)])/((c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])
```

### Rubi steps

$$\begin{aligned} \int \frac{(d + ex^n)^2}{a + cx^{2n}} dx &= \int \left( \frac{e^2}{c} + \frac{cd^2 - ae^2 + 2cdex^n}{c(a + cx^{2n})} \right) dx \\ &= \frac{e^2 x}{c} + \frac{\int \frac{cd^2 - ae^2 + 2cdex^n}{a + cx^{2n}} dx}{c} \\ &= \frac{e^2 x}{c} + (2de) \int \frac{x^n}{a + cx^{2n}} dx + \frac{(cd^2 - ae^2) \int \frac{1}{a + cx^{2n}} dx}{c} \\ &= \frac{e^2 x}{c} + \frac{(cd^2 - ae^2) x {}_2F_1\left(1, \frac{1}{2n}; \frac{1}{2}\left(2 + \frac{1}{n}\right); -\frac{cx^{2n}}{a}\right)}{ac} + \frac{2dex^{1+n} {}_2F_1\left(1, \frac{1+n}{2n}; \frac{1}{2}\left(3 + \frac{1}{n}\right); -\frac{cx^{2n}}{a}\right)}{a(1+n)} \end{aligned}$$

**Mathematica [A]** time = 0.150999, size = 107, normalized size = 1.

$$\frac{x(cd^2 - ae^2) {}_2F_1\left(1, \frac{1}{2n}; \frac{1}{2}\left(2 + \frac{1}{n}\right); -\frac{cx^{2n}}{a}\right)}{ac} + \frac{2dex^{n+1} {}_2F_1\left(1, \frac{n+1}{2n}; \frac{1}{2}\left(3 + \frac{1}{n}\right); -\frac{cx^{2n}}{a}\right)}{a(n+1)} + \frac{e^2 x}{c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^n)^2/(a + c*x^(2*n)), x]
```

```
[Out] (e^2*x)/c + ((c*d^2 - a*e^2)*x*Hypergeometric2F1[1, 1/(2*n), (2 + n^(-1))/2, -((c*x^(2*n))/a)])/(a*c) + (2*d*e*x^(1 + n)*Hypergeometric2F1[1, (1 + n)/(2*n), (3 + n^(-1))/2, -((c*x^(2*n))/a)])/(a*(1 + n))
```

**Maple [F]** time = 0.062, size = 0, normalized size = 0.

$$\int \frac{(d + ex^n)^2}{a + cx^{2n}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d+e\*x^n)^2/(a+c\*x^(2\*n)),x)

[Out] int((d+e\*x^n)^2/(a+c\*x^(2\*n)),x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{e^2x}{c} + \int \frac{2cdex^n + cd^2 - ae^2}{c^2x^{2n} + ac} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e\*x^n)^2/(a+c\*x^(2\*n)),x, algorithm="maxima")

[Out] e^2\*x/c + integrate((2\*c\*d\*e\*x^n + c\*d^2 - a\*e^2)/(c^2\*x^(2\*n) + a\*c), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{e^2x^{2n} + 2dex^n + d^2}{cx^{2n} + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e\*x^n)^2/(a+c\*x^(2\*n)),x, algorithm="fricas")

[Out] integral((e^2\*x^(2\*n) + 2\*d\*e\*x^n + d^2)/(c\*x^(2\*n) + a), x)

**Sympy [C]** time = 7.57686, size = 207, normalized size = 1.93

$$-\frac{e^2x\Phi\left(\frac{ax^{-2n}e^{i\pi}}{c}, 1, \frac{e^{i\pi}}{2n}\right)\Gamma\left(\frac{1}{2n}\right)}{4cn^2\Gamma\left(1 + \frac{1}{2n}\right)} + \frac{d^2x\Phi\left(\frac{cx^{2n}e^{i\pi}}{a}, 1, \frac{1}{2n}\right)\Gamma\left(\frac{1}{2n}\right)}{4an^2\Gamma\left(1 + \frac{1}{2n}\right)} + \frac{dexx^n\Phi\left(\frac{cx^{2n}e^{i\pi}}{a}, 1, \frac{1}{2} + \frac{1}{2n}\right)\Gamma\left(\frac{1}{2} + \frac{1}{2n}\right)}{2an\Gamma\left(\frac{3}{2} + \frac{1}{2n}\right)} + \frac{dexx^n\Phi\left(\frac{cx^{2n}e^{i\pi}}{a}, 1, \frac{1}{2} + \frac{1}{2n}\right)\Gamma\left(\frac{1}{2} + \frac{1}{2n}\right)}{2an^2\Gamma\left(\frac{3}{2} + \frac{1}{2n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e\*x\*\*n)\*\*2/(a+c\*x\*\*(2\*n)),x)

[Out] -e\*\*2\*x\*lerchphi(a\*x\*\*(-2\*n)\*exp\_polar(I\*pi)/c, 1, exp\_polar(I\*pi)/(2\*n))\*gamma(1/(2\*n))/(4\*c\*n\*\*2\*gamma(1 + 1/(2\*n))) + d\*\*2\*x\*lerchphi(c\*x\*\*(2\*n)\*exp\_polar(I\*pi)/a, 1, 1/(2\*n))\*gamma(1/(2\*n))/(4\*a\*n\*\*2\*gamma(1 + 1/(2\*n))) + d\*e\*x\*x\*\*n\*lerchphi(c\*x\*\*(2\*n)\*exp\_polar(I\*pi)/a, 1, 1/2 + 1/(2\*n))\*gamma(1/2 + 1/(2\*n))/(2\*a\*n\*gamma(3/2 + 1/(2\*n))) + d\*e\*x\*x\*\*n\*lerchphi(c\*x\*\*(2\*n)\*exp\_polar(I\*pi)/a, 1, 1/2 + 1/(2\*n))\*gamma(1/2 + 1/(2\*n))/(2\*a\*n\*\*2\*gamma(3/2 + 1/(2\*n)))

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^n + d)^2}{cx^{2n} + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e\*x^n)^2/(a+c\*x^(2\*n)),x, algorithm="giac")

[Out] integrate((e\*x^n + d)^2/(c\*x^(2\*n) + a), x)

$$3.44 \quad \int \frac{d+ex^n}{a+cx^{2n}} dx$$

**Optimal.** Leaf size=83

$$\frac{dx {}_2F_1\left(1, \frac{1}{2n}; \frac{1}{2}\left(2 + \frac{1}{n}\right); -\frac{cx^{2n}}{a}\right)}{a} + \frac{ex^{n+1} {}_2F_1\left(1, \frac{n+1}{2n}; \frac{1}{2}\left(3 + \frac{1}{n}\right); -\frac{cx^{2n}}{a}\right)}{a(n+1)}$$

[Out] (d\*x\*Hypergeometric2F1[1, 1/(2\*n), (2 + n^(-1))/2, -((c\*x^(2\*n))/a)])/a + (e\*x^(1 + n)\*Hypergeometric2F1[1, (1 + n)/(2\*n), (3 + n^(-1))/2, -((c\*x^(2\*n))/a)])/a)/(a\*(1 + n))

**Rubi [A]** time = 0.0274489, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$ , Rules used = {1418, 245, 364}

$$\frac{dx {}_2F_1\left(1, \frac{1}{2n}; \frac{1}{2}\left(2 + \frac{1}{n}\right); -\frac{cx^{2n}}{a}\right)}{a} + \frac{ex^{n+1} {}_2F_1\left(1, \frac{n+1}{2n}; \frac{1}{2}\left(3 + \frac{1}{n}\right); -\frac{cx^{2n}}{a}\right)}{a(n+1)}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x^n)/(a + c\*x^(2\*n)),x]

[Out] (d\*x\*Hypergeometric2F1[1, 1/(2\*n), (2 + n^(-1))/2, -((c\*x^(2\*n))/a)])/a + (e\*x^(1 + n)\*Hypergeometric2F1[1, (1 + n)/(2\*n), (3 + n^(-1))/2, -((c\*x^(2\*n))/a)])/a)/(a\*(1 + n))

#### Rule 1418

Int[((d\_) + (e\_.)\*(x\_)^(n\_))/((a\_) + (c\_.)\*(x\_)^(n2\_)), x\_Symbol] :> Dist[d, Int[1/(a + c\*x^(2\*n)), x], x] + Dist[e, Int[x^n/(a + c\*x^(2\*n)), x], x] / ; FreeQ[{a, c, d, e, n}, x] && EqQ[n2, 2\*n] && NeQ[c\*d^2 + a\*e^2, 0] && (PosQ[a\*c] || !IntegerQ[n])

#### Rule 245

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] :> Simp[a^p\*x\*Hypergeometric2F1[-p, 1/n, 1/n + 1, -((b\*x^n)/a)], x] / ; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] || GtQ[a, 0])

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/((c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{d + ex^n}{a + cx^{2n}} dx = d \int \frac{1}{a + cx^{2n}} dx + e \int \frac{x^n}{a + cx^{2n}} dx$$

$$= \frac{dx {}_2F_1\left(1, \frac{1}{2n}; \frac{1}{2}\left(2 + \frac{1}{n}\right); -\frac{cx^{2n}}{a}\right)}{a} + \frac{ex^{1+n} {}_2F_1\left(1, \frac{1+n}{2n}; \frac{1}{2}\left(3 + \frac{1}{n}\right); -\frac{cx^{2n}}{a}\right)}{a(1+n)}$$

**Mathematica [A]** time = 0.0385568, size = 83, normalized size = 1.

$$\frac{dx {}_2F_1\left(1, \frac{1}{2n}; \frac{1}{2}\left(2 + \frac{1}{n}\right); -\frac{cx^{2n}}{a}\right)}{a} + \frac{ex^{n+1} {}_2F_1\left(1, \frac{n+1}{2n}; \frac{1}{2}\left(3 + \frac{1}{n}\right); -\frac{cx^{2n}}{a}\right)}{a(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^n)/(a + c*x^(2*n)),x]
```

```
[Out] (d*x*Hypergeometric2F1[1, 1/(2*n), (2 + n^(-1))/2, -((c*x^(2*n))/a)])/a + (
e*x^(1 + n)*Hypergeometric2F1[1, (1 + n)/(2*n), (3 + n^(-1))/2, -((c*x^(2*n)
)/a)])/((a*(1 + n))
```

**Maple [F]** time = 0.045, size = 0, normalized size = 0.

$$\int \frac{d + ex^n}{a + cx^{2n}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+e*x^n)/(a+c*x^(2*n)),x)
```

```
[Out] int((d+e*x^n)/(a+c*x^(2*n)),x)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{ex^n + d}{cx^{2n} + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e\*x^n)/(a+c\*x^(2\*n)),x, algorithm="maxima")

[Out] integrate((e\*x^n + d)/(c\*x^(2\*n) + a), x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{ex^n + d}{cx^{2n} + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e\*x^n)/(a+c\*x^(2\*n)),x, algorithm="fricas")

[Out] integral((e\*x^n + d)/(c\*x^(2\*n) + a), x)

---

**Sympy [C]** time = 5.79687, size = 153, normalized size = 1.84

$$\frac{dx\Phi\left(\frac{cx^{2n}e^{i\pi}}{a}, 1, \frac{1}{2n}\right)\Gamma\left(\frac{1}{2n}\right)}{4an^2\Gamma\left(1 + \frac{1}{2n}\right)} + \frac{exx^n\Phi\left(\frac{cx^{2n}e^{i\pi}}{a}, 1, \frac{1}{2} + \frac{1}{2n}\right)\Gamma\left(\frac{1}{2} + \frac{1}{2n}\right)}{4an\Gamma\left(\frac{3}{2} + \frac{1}{2n}\right)} + \frac{exx^n\Phi\left(\frac{cx^{2n}e^{i\pi}}{a}, 1, \frac{1}{2} + \frac{1}{2n}\right)\Gamma\left(\frac{1}{2} + \frac{1}{2n}\right)}{4an^2\Gamma\left(\frac{3}{2} + \frac{1}{2n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e\*x\*\*n)/(a+c\*x\*\*(2\*n)),x)

[Out] d\*x\*lerchphi(c\*x\*\*(2\*n)\*exp\_polar(I\*pi)/a, 1, 1/(2\*n))\*gamma(1/(2\*n))/(4\*a\*n\*\*2\*gamma(1 + 1/(2\*n))) + e\*x\*x\*\*n\*lerchphi(c\*x\*\*(2\*n)\*exp\_polar(I\*pi)/a, 1, 1/2 + 1/(2\*n))\*gamma(1/2 + 1/(2\*n))/(4\*a\*n\*gamma(3/2 + 1/(2\*n))) + e\*x\*x\*\*n\*lerchphi(c\*x\*\*(2\*n)\*exp\_polar(I\*pi)/a, 1, 1/2 + 1/(2\*n))\*gamma(1/2 + 1/(2\*n))/(4\*a\*n\*\*2\*gamma(3/2 + 1/(2\*n)))

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{ex^n + d}{cx^{2n} + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e\*x^n)/(a+c\*x^(2\*n)),x, algorithm="giac")

[Out] integrate((e\*x^n + d)/(c\*x^(2\*n) + a), x)

$$3.45 \quad \int \frac{1}{(d+ex^n)(a+cx^{2n})} dx$$

**Optimal.** Leaf size=152

$$-\frac{cex^{n+1} {}_2F_1\left(1, \frac{n+1}{2n}; \frac{1}{2}\left(3 + \frac{1}{n}\right); -\frac{cx^{2n}}{a}\right)}{a(n+1)(ae^2 + cd^2)} + \frac{cdx {}_2F_1\left(1, \frac{1}{2n}; \frac{1}{2}\left(2 + \frac{1}{n}\right); -\frac{cx^{2n}}{a}\right)}{a(ae^2 + cd^2)} + \frac{e^2x {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{ex^n}{d}\right)}{d(ae^2 + cd^2)}$$

[Out] (c\*d\*x\*Hypergeometric2F1[1, 1/(2\*n), (2 + n^(-1))/2, -((c\*x^(2\*n))/a)])/(a\*(c\*d^2 + a\*e^2)) + (e^2\*x\*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((e\*x^n)/d)])/(d\*(c\*d^2 + a\*e^2)) - (c\*e\*x^(1 + n)\*Hypergeometric2F1[1, (1 + n)/(2\*n), (3 + n^(-1))/2, -((c\*x^(2\*n))/a)])/(a\*(c\*d^2 + a\*e^2)\*(1 + n))

**Rubi [A]** time = 0.110592, antiderivative size = 152, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.19$ , Rules used = {1425, 245, 1418, 364}

$$-\frac{cex^{n+1} {}_2F_1\left(1, \frac{n+1}{2n}; \frac{1}{2}\left(3 + \frac{1}{n}\right); -\frac{cx^{2n}}{a}\right)}{a(n+1)(ae^2 + cd^2)} + \frac{cdx {}_2F_1\left(1, \frac{1}{2n}; \frac{1}{2}\left(2 + \frac{1}{n}\right); -\frac{cx^{2n}}{a}\right)}{a(ae^2 + cd^2)} + \frac{e^2x {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{ex^n}{d}\right)}{d(ae^2 + cd^2)}$$

Antiderivative was successfully verified.

[In] Int[1/((d + e\*x^n)\*(a + c\*x^(2\*n))),x]

[Out] (c\*d\*x\*Hypergeometric2F1[1, 1/(2\*n), (2 + n^(-1))/2, -((c\*x^(2\*n))/a)])/(a\*(c\*d^2 + a\*e^2)) + (e^2\*x\*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((e\*x^n)/d)])/(d\*(c\*d^2 + a\*e^2)) - (c\*e\*x^(1 + n)\*Hypergeometric2F1[1, (1 + n)/(2\*n), (3 + n^(-1))/2, -((c\*x^(2\*n))/a)])/(a\*(c\*d^2 + a\*e^2)\*(1 + n))

#### Rule 1425

Int[((d\_) + (e\_.)\*(x\_)^(n\_))^(q\_)/((a\_) + (c\_.)\*(x\_)^(n2\_)), x\_Symbol] :> Int[ExpandIntegrand[(d + e\*x^n)^q/(a + c\*x^(2\*n)), x], x] /; FreeQ[{a, c, d, e, n}, x] && EqQ[n2, 2\*n] && NeQ[c\*d^2 + a\*e^2, 0] && IntegerQ[q]

#### Rule 245

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] :> Simp[a^p\*x\*Hypergeometric2F1[-p, 1/n, 1/n + 1, -((b\*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||



GtQ[a, 0])

### Rule 1418

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (c_.)*(x_)^(n2_)), x_Symbol] := Dist[d
, Int[1/(a + c*x^(2*n)), x], x] + Dist[e, Int[x^n/(a + c*x^(2*n)), x], x] /
; FreeQ[{a, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[c*d^2 + a*e^2, 0] && (Po
sQ[a*c] || !IntegerQ[n])
```

### Rule 364

```
Int[((c_.)*(x_)^(m_.))*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/((c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

### Rubi steps

$$\begin{aligned} \int \frac{1}{(d + ex^n)(a + cx^{2n})} dx &= \int \left( \frac{e^2}{(cd^2 + ae^2)(d + ex^n)} - \frac{c(-d + ex^n)}{(cd^2 + ae^2)(a + cx^{2n})} \right) dx \\ &= -\frac{c \int \frac{-d+ex^n}{a+cx^{2n}} dx}{cd^2 + ae^2} + \frac{e^2 \int \frac{1}{d+ex^n} dx}{cd^2 + ae^2} \\ &= \frac{e^2 x {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{ex^n}{d}\right)}{d(cd^2 + ae^2)} + \frac{(cd) \int \frac{1}{a+cx^{2n}} dx}{cd^2 + ae^2} - \frac{(ce) \int \frac{x^n}{a+cx^{2n}} dx}{cd^2 + ae^2} \\ &= \frac{cdx {}_2F_1\left(1, \frac{1}{2n}; \frac{1}{2}\left(2 + \frac{1}{n}\right); -\frac{cx^{2n}}{a}\right)}{a(cd^2 + ae^2)} + \frac{e^2 x {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{ex^n}{d}\right)}{d(cd^2 + ae^2)} - \frac{cex^{1+n} {}_2F_1\left(1, \frac{1+n}{2n}; \frac{1}{2}\left(3 + \frac{1}{n}\right); -\frac{cx^{2n}}{a}\right)}{a(cd^2 + ae^2)(1 + ex^n)} \end{aligned}$$

**Mathematica [A]** time = 0.143869, size = 131, normalized size = 0.86

$$\frac{x \left( cd^2(n+1) {}_2F_1\left(1, \frac{1}{2n}; \frac{1}{2}\left(2 + \frac{1}{n}\right); -\frac{cx^{2n}}{a}\right) + e \left( ae(n+1) {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{ex^n}{d}\right) - cdx^n {}_2F_1\left(1, \frac{n+1}{2n}; \frac{1}{2}\left(3 + \frac{1}{n}\right); -\frac{cx^{2n}}{a}\right) \right) \right)}{ad(n+1)(ae^2 + cd^2)}$$

Antiderivative was successfully verified.

[In] Integrate[1/((d + e\*x^n)\*(a + c\*x^(2\*n))),x]

[Out] (x\*(c\*d^2\*(1 + n)\*Hypergeometric2F1[1, 1/(2\*n), (2 + n^(-1))/2, -((c\*x^(2\*n))/a)] + e\*(a\*e\*(1 + n)\*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((e\*x^n)/

d)] - c\*d\*x^n\*Hypergeometric2F1[1, (1 + n)/(2\*n), (3 + n^(-1))/2, -((c\*x^(2\*n))/a)])))/(a\*d\*(c\*d^2 + a\*e^2)\*(1 + n))

**Maple [F]** time = 0.079, size = 0, normalized size = 0.

$$\int \frac{1}{(d + ex^n)(a + cx^{2n})} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(d+e\*x^n)/(a+c\*x^(2\*n)),x)

[Out] int(1/(d+e\*x^n)/(a+c\*x^(2\*n)),x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(cx^{2n} + a)(ex^n + d)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d+e\*x^n)/(a+c\*x^(2\*n)),x, algorithm="maxima")

[Out] integrate(1/((c\*x^(2\*n) + a)\*(e\*x^n + d)), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{aex^n + ad + (cex^n + cd)x^{2n}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d+e\*x^n)/(a+c\*x^(2\*n)),x, algorithm="fricas")

[Out] integral(1/(a\*e\*x^n + a\*d + (c\*e\*x^n + c\*d)\*x^(2\*n)), x)

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a + cx^{2n})(d + ex^n)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d+e\*x\*\*n)/(a+c\*x\*\*(2\*n)),x)

[Out] Integral(1/((a + c\*x\*\*(2\*n))\*(d + e\*x\*\*n)), x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(cx^{2n} + a)(ex^n + d)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d+e\*x^n)/(a+c\*x^(2\*n)),x, algorithm="giac")

[Out] integrate(1/((c\*x^(2\*n) + a)\*(e\*x^n + d)), x)

$$3.46 \quad \int \frac{1}{(d+ex^n)^2(a+cx^{2n})} dx$$

**Optimal.** Leaf size=205

$$\frac{2c^2dex^{n+1} {}_2F_1\left(1, \frac{n+1}{2n}; \frac{1}{2}\left(3 + \frac{1}{n}\right); -\frac{cx^{2n}}{a}\right)}{a(n+1)(ae^2 + cd^2)^2} + \frac{cx(cd^2 - ae^2) {}_2F_1\left(1, \frac{1}{2n}; \frac{1}{2}\left(2 + \frac{1}{n}\right); -\frac{cx^{2n}}{a}\right)}{a(ae^2 + cd^2)^2} + \frac{2ce^2x {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{ex^n}{d}\right)}{(ae^2 + cd^2)^2} +$$

[Out] (c\*(c\*d^2 - a\*e^2)\*x\*Hypergeometric2F1[1, 1/(2\*n), (2 + n^(-1))/2, -((c\*x^(2\*n))/a)]/(a\*(c\*d^2 + a\*e^2)^2) + (2\*c\*e^2\*x\*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((e\*x^n)/d)]/(c\*d^2 + a\*e^2)^2 - (2\*c^2\*d\*e\*x^(1 + n)\*Hypergeometric2F1[1, (1 + n)/(2\*n), (3 + n^(-1))/2, -((c\*x^(2\*n))/a)]/(a\*(c\*d^2 + a\*e^2)^2\*(1 + n)) + (e^2\*x\*Hypergeometric2F1[2, n^(-1), 1 + n^(-1), -((e\*x^n)/d)]/(d^2\*(c\*d^2 + a\*e^2))

**Rubi [A]** time = 0.173819, antiderivative size = 205, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.19$ , Rules used = {1425, 245, 1418, 364}

$$\frac{2c^2dex^{n+1} {}_2F_1\left(1, \frac{n+1}{2n}; \frac{1}{2}\left(3 + \frac{1}{n}\right); -\frac{cx^{2n}}{a}\right)}{a(n+1)(ae^2 + cd^2)^2} + \frac{cx(cd^2 - ae^2) {}_2F_1\left(1, \frac{1}{2n}; \frac{1}{2}\left(2 + \frac{1}{n}\right); -\frac{cx^{2n}}{a}\right)}{a(ae^2 + cd^2)^2} + \frac{2ce^2x {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{ex^n}{d}\right)}{(ae^2 + cd^2)^2} +$$

Antiderivative was successfully verified.

[In] Int[1/((d + e\*x^n)^2\*(a + c\*x^(2\*n))),x]

[Out] (c\*(c\*d^2 - a\*e^2)\*x\*Hypergeometric2F1[1, 1/(2\*n), (2 + n^(-1))/2, -((c\*x^(2\*n))/a)]/(a\*(c\*d^2 + a\*e^2)^2) + (2\*c\*e^2\*x\*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((e\*x^n)/d)]/(c\*d^2 + a\*e^2)^2 - (2\*c^2\*d\*e\*x^(1 + n)\*Hypergeometric2F1[1, (1 + n)/(2\*n), (3 + n^(-1))/2, -((c\*x^(2\*n))/a)]/(a\*(c\*d^2 + a\*e^2)^2\*(1 + n)) + (e^2\*x\*Hypergeometric2F1[2, n^(-1), 1 + n^(-1), -((e\*x^n)/d)]/(d^2\*(c\*d^2 + a\*e^2))

**Rule 1425**

Int[((d\_) + (e\_.)\*(x\_)^(n\_))^(q\_)/((a\_) + (c\_.)\*(x\_)^(n2\_)), x\_Symbol] :> Int[ExpandIntegrand[(d + e\*x^n)^q/(a + c\*x^(2\*n)), x], x] /; FreeQ[{a, c, d, e, n}, x] && EqQ[n, 2\*n] && NeQ[c\*d^2 + a\*e^2, 0] && IntegerQ[q]

Rule 245

Int[((a\_) + (b\_)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[a^p\*x\*Hypergeometric2F1[-p, 1/n, 1/n + 1, -((b\*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] || GtQ[a, 0])

Rule 1418

Int[((d\_) + (e\_)\*(x\_)^(n\_))/((a\_) + (c\_)\*(x\_)^(n2\_)), x\_Symbol] := Dist[d, Int[1/(a + c\*x^(2\*n)), x], x] + Dist[e, Int[x^n/(a + c\*x^(2\*n)), x], x] /; FreeQ[{a, c, d, e, n}, x] && EqQ[n2, 2\*n] && NeQ[c\*d^2 + a\*e^2, 0] && (PosQ[a\*c] || !IntegerQ[n])

Rule 364

Int[((c\_)\*(x\_)^(m\_))\*((a\_) + (b\_)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[(a^p\*(c\*x)^(m + 1)\*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b\*x^n)/a)])/((c\*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])

Rubi steps

$$\begin{aligned} \int \frac{1}{(d + ex^n)^2 (a + cx^{2n})} dx &= \int \left( \frac{e^2}{(cd^2 + ae^2)(d + ex^n)^2} + \frac{2cde^2}{(cd^2 + ae^2)^2 (d + ex^n)} - \frac{c(-cd^2 + ae^2 + 2cdex^n)}{(cd^2 + ae^2)^2 (a + cx^{2n})} \right) dx \\ &= -\frac{c \int \frac{-cd^2 + ae^2 + 2cdex^n}{a + cx^{2n}} dx}{(cd^2 + ae^2)^2} + \frac{(2cde^2) \int \frac{1}{d + ex^n} dx}{(cd^2 + ae^2)^2} + \frac{e^2 \int \frac{1}{(d + ex^n)^2} dx}{cd^2 + ae^2} \\ &= \frac{2ce^2 x {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{ex^n}{d}\right)}{(cd^2 + ae^2)^2} + \frac{e^2 x {}_2F_1\left(2, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{ex^n}{d}\right)}{d^2 (cd^2 + ae^2)} - \frac{(2c^2 de) \int \frac{x^n}{a + cx^{2n}} dx}{(cd^2 + ae^2)^2} + \frac{c(cd^2 - ae^2)}{a(cd^2 + ae^2)^2} \\ &= \frac{c(cd^2 - ae^2) x {}_2F_1\left(1, \frac{1}{2n}; \frac{1}{2}\left(2 + \frac{1}{n}\right); -\frac{cx^{2n}}{a}\right)}{a(cd^2 + ae^2)^2} + \frac{2ce^2 x {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{ex^n}{d}\right)}{(cd^2 + ae^2)^2} - \frac{2c^2 dex^{1+n} {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{ex^n}{d}\right)}{a(cd^2 + ae^2)^2} \end{aligned}$$

**Mathematica [A]** time = 0.266714, size = 186, normalized size = 0.91

$$\frac{x \left( e \left( -2c^2 d^3 x^n {}_2F_1\left(1, \frac{n+1}{2n}; \frac{1}{2}\left(3 + \frac{1}{n}\right); -\frac{cx^{2n}}{a}\right) + ae(n+1)(ae^2 + cd^2) {}_2F_1\left(2, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{ex^n}{d}\right) + 2acd^2 e(n+1) {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{ex^n}{d}\right) \right)}{a(n+1)(ade^2 + cd^3)^2}$$

Antiderivative was successfully verified.

[In] Integrate[1/((d + e\*x^n)^2\*(a + c\*x^(2\*n))),x]

[Out] (x\*(c\*d^2\*(c\*d^2 - a\*e^2)\*(1 + n)\*Hypergeometric2F1[1, 1/(2\*n), (2 + n^(-1))/2, -((c\*x^(2\*n))/a)] + e\*(2\*a\*c\*d^2\*e\*(1 + n)\*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((e\*x^n)/d)] - 2\*c^2\*d^3\*x^n\*Hypergeometric2F1[1, (1 + n)/(2\*n), (3 + n^(-1))/2, -((c\*x^(2\*n))/a)] + a\*e\*(c\*d^2 + a\*e^2)\*(1 + n)\*Hypergeometric2F1[2, n^(-1), 1 + n^(-1), -((e\*x^n)/d)]))/((a\*(c\*d^3 + a\*d\*e^2)^2\*(1 + n))

**Maple [F]** time = 0.141, size = 0, normalized size = 0.

$$\int \frac{1}{(d + ex^n)^2 (a + cx^{2n})} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(d+e\*x^n)^2/(a+c\*x^(2\*n)),x)

[Out] int(1/(d+e\*x^n)^2/(a+c\*x^(2\*n)),x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{e^2 x}{cd^4 n + ad^2 e^2 n + (cd^3 en + ade^3 n)x^n} + (cd^2 e^2 (3n - 1) + ae^4 (n - 1)) \int \frac{1}{c^2 d^6 n + 2acd^4 e^2 n + a^2 d^2 e^4 n + (c^2 d^5 en + 2acd^3 e^3 n)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d+e\*x^n)^2/(a+c\*x^(2\*n)),x, algorithm="maxima")

[Out] e^2\*x/(c\*d^4\*n + a\*d^2\*e^2\*n + (c\*d^3\*e\*n + a\*d\*e^3\*n)\*x^n) + (c\*d^2\*e^2\*(3\*n - 1) + a\*e^4\*(n - 1))\*integrate(1/(c^2\*d^6\*n + 2\*a\*c\*d^4\*e^2\*n + a^2\*d^2\*e^4\*n + (c^2\*d^5\*e\*n + 2\*a\*c\*d^3\*e^3\*n)\*x^n), x) - integrate((2\*c^2\*d\*e\*x^n - c^2\*d^2 + a\*c\*e^2)/(a\*c^2\*d^4 + 2\*a^2\*c\*d^2\*e^2 + a^3\*e^4 + (c^3\*d^4 + 2\*a\*c^2\*d^2\*e^2 + a^2\*c\*e^4)\*x^(2\*n)), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{ae^2x^{2n} + 2adex^n + ad^2 + (ce^2x^{2n} + 2cdex^n + cd^2)x^{2n}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(d+e*x^n)^2/(a+c*x^(2*n)),x, algorithm="fricas")`

[Out] `integral(1/(a*e^2*x^(2*n) + 2*a*d*e*x^n + a*d^2 + (c*e^2*x^(2*n) + 2*c*d*e*x^n + c*d^2)*x^(2*n)), x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(d+e*x**n)**2/(a+c*x**(2*n)),x)`

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(cx^{2n} + a)(ex^n + d)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(d+e*x^n)^2/(a+c*x^(2*n)),x, algorithm="giac")`

[Out] `integrate(1/((c*x^(2*n) + a)*(e*x^n + d)^2), x)`

$$3.47 \quad \int \frac{d+ex^n}{a-cx^{2n}} dx$$

**Optimal.** Leaf size=81

$$\frac{dx {}_2F_1\left(1, \frac{1}{2n}; \frac{1}{2}\left(2 + \frac{1}{n}\right); \frac{cx^{2n}}{a}\right)}{a} + \frac{ex^{n+1} {}_2F_1\left(1, \frac{n+1}{2n}; \frac{1}{2}\left(3 + \frac{1}{n}\right); \frac{cx^{2n}}{a}\right)}{a(n+1)}$$

[Out] (d\*x\*Hypergeometric2F1[1, 1/(2\*n), (2 + n^(-1))/2, (c\*x^(2\*n))/a])/a + (e\*x^(1 + n)\*Hypergeometric2F1[1, (1 + n)/(2\*n), (3 + n^(-1))/2, (c\*x^(2\*n))/a])/(a\*(1 + n))

**Rubi [A]** time = 0.0283905, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.15$ , Rules used = {1418, 245, 364}

$$\frac{dx {}_2F_1\left(1, \frac{1}{2n}; \frac{1}{2}\left(2 + \frac{1}{n}\right); \frac{cx^{2n}}{a}\right)}{a} + \frac{ex^{n+1} {}_2F_1\left(1, \frac{n+1}{2n}; \frac{1}{2}\left(3 + \frac{1}{n}\right); \frac{cx^{2n}}{a}\right)}{a(n+1)}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x^n)/(a - c\*x^(2\*n)),x]

[Out] (d\*x\*Hypergeometric2F1[1, 1/(2\*n), (2 + n^(-1))/2, (c\*x^(2\*n))/a])/a + (e\*x^(1 + n)\*Hypergeometric2F1[1, (1 + n)/(2\*n), (3 + n^(-1))/2, (c\*x^(2\*n))/a])/(a\*(1 + n))

### Rule 1418

Int[((d\_) + (e\_.)\*(x\_)^(n\_))/((a\_) + (c\_.)\*(x\_)^(n2\_)), x\_Symbol] := Dist[d, Int[1/(a + c\*x^(2\*n)), x], x] + Dist[e, Int[x^n/(a + c\*x^(2\*n)), x], x] / ; FreeQ[{a, c, d, e, n}, x] && EqQ[n2, 2\*n] && NeQ[c\*d^2 + a\*e^2, 0] && (PosQ[a\*c] || !IntegerQ[n])

### Rule 245

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[a^p\*x\*Hypergeometric2F1[-p, 1/n, 1/n + 1, -(b\*x^n)/a], x] / ; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] || GtQ[a, 0])



Rule 364

Int[((c\_.)\*(x\_))^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] :> Simp[(a^p\*(c\*x)^(m + 1)\*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b\*x^n)/a)])/((c\*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])

Rubi steps

$$\int \frac{d + ex^n}{a - cx^{2n}} dx = d \int \frac{1}{a - cx^{2n}} dx + e \int \frac{x^n}{a - cx^{2n}} dx$$

$$= \frac{dx {}_2F_1\left(1, \frac{1}{2n}; \frac{1}{2}\left(2 + \frac{1}{n}\right); \frac{cx^{2n}}{a}\right)}{a} + \frac{ex^{1+n} {}_2F_1\left(1, \frac{1+n}{2n}; \frac{1}{2}\left(3 + \frac{1}{n}\right); \frac{cx^{2n}}{a}\right)}{a(1+n)}$$

**Mathematica [A]** time = 0.060855, size = 81, normalized size = 1.

$$\frac{dx {}_2F_1\left(1, \frac{1}{2n}; \frac{1}{2}\left(2 + \frac{1}{n}\right); \frac{cx^{2n}}{a}\right)}{a} + \frac{ex^{n+1} {}_2F_1\left(1, \frac{n+1}{2n}; \frac{1}{2}\left(3 + \frac{1}{n}\right); \frac{cx^{2n}}{a}\right)}{a(n+1)}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e\*x^n)/(a - c\*x^(2\*n)),x]

[Out] (d\*x\*Hypergeometric2F1[1, 1/(2\*n), (2 + n^(-1))/2, (c\*x^(2\*n))/a])/a + (e\*x^(1 + n)\*Hypergeometric2F1[1, (1 + n)/(2\*n), (3 + n^(-1))/2, (c\*x^(2\*n))/a])/((a\*(1 + n))

**Maple [F]** time = 0.043, size = 0, normalized size = 0.

$$\int \frac{d + ex^n}{a - cx^{2n}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d+e\*x^n)/(a-c\*x^(2\*n)),x)

[Out] int((d+e\*x^n)/(a-c\*x^(2\*n)),x)

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$-\int \frac{ex^n + d}{cx^{2n} - a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e\*x^n)/(a-c\*x^(2\*n)),x, algorithm="maxima")

[Out] -integrate((e\*x^n + d)/(c\*x^(2\*n) - a), x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{ex^n + d}{cx^{2n} - a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e\*x^n)/(a-c\*x^(2\*n)),x, algorithm="fricas")

[Out] integral(-(e\*x^n + d)/(c\*x^(2\*n) - a), x)

---

**Sympy [C]** time = 6.49664, size = 158, normalized size = 1.95

$$\frac{dx\Phi\left(\frac{cx^{2n}e^{2i\pi}}{a}, 1, \frac{1}{2n}\right)\Gamma\left(\frac{1}{2n}\right)}{4an^2\Gamma\left(1 + \frac{1}{2n}\right)} + \frac{exx^n\Phi\left(\frac{cx^{2n}e^{2i\pi}}{a}, 1, \frac{1}{2} + \frac{1}{2n}\right)\Gamma\left(\frac{1}{2} + \frac{1}{2n}\right)}{4an\Gamma\left(\frac{3}{2} + \frac{1}{2n}\right)} + \frac{exx^n\Phi\left(\frac{cx^{2n}e^{2i\pi}}{a}, 1, \frac{1}{2} + \frac{1}{2n}\right)\Gamma\left(\frac{1}{2} + \frac{1}{2n}\right)}{4an^2\Gamma\left(\frac{3}{2} + \frac{1}{2n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e\*x\*\*n)/(a-c\*x\*\*(2\*n)),x)

[Out] d\*x\*lerchphi(c\*x\*\*(2\*n)\*exp\_polar(2\*I\*pi)/a, 1, 1/(2\*n))\*gamma(1/(2\*n))/(4\*a\*n\*\*2\*gamma(1 + 1/(2\*n))) + e\*x\*x\*\*n\*lerchphi(c\*x\*\*(2\*n)\*exp\_polar(2\*I\*pi)/a, 1, 1/2 + 1/(2\*n))\*gamma(1/2 + 1/(2\*n))/(4\*a\*n\*gamma(3/2 + 1/(2\*n))) + e\*x\*x\*\*n\*lerchphi(c\*x\*\*(2\*n)\*exp\_polar(2\*I\*pi)/a, 1, 1/2 + 1/(2\*n))\*gamma(1/2 + 1/(2\*n))/(4\*a\*n\*\*2\*gamma(3/2 + 1/(2\*n)))

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int -\frac{ex^n + d}{cx^{2n} - a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e\*x^n)/(a-c\*x^(2\*n)),x, algorithm="giac")

[Out] integrate(-(e\*x^n + d)/(c\*x^(2\*n) - a), x)

$$3.48 \quad \int \frac{(d+ex^n)^3}{(a+cx^{2n})^2} dx$$

**Optimal.** Leaf size=288

$$\frac{e(1-n)x^{n+1}(3cd^2 - ae^2) {}_2F_1\left(1, \frac{n+1}{2n}; \frac{1}{2}\left(3 + \frac{1}{n}\right); -\frac{cx^{2n}}{a}\right)}{2a^2cn(n+1)} - \frac{d(1-2n)x(cd^2 - 3ae^2) {}_2F_1\left(1, \frac{1}{2n}; \frac{1}{2}\left(2 + \frac{1}{n}\right); -\frac{cx^{2n}}{a}\right)}{2a^2cn} + \frac{x(ex^{2n})}{2a^2cn}$$

[Out] (x\*(d\*(c\*d^2 - 3\*a\*e^2) + e\*(3\*c\*d^2 - a\*e^2)\*x^n))/(2\*a\*c\*n\*(a + c\*x^(2\*n))) + (3\*d\*e^2\*x\*Hypergeometric2F1[1, 1/(2\*n), (2 + n^(-1))/2, -((c\*x^(2\*n))/a)])/a - (d\*(c\*d^2 - 3\*a\*e^2)\*(1 - 2\*n)\*x\*Hypergeometric2F1[1, 1/(2\*n), (2 + n^(-1))/2, -((c\*x^(2\*n))/a)])/2\*a^2\*c\*n + (e^3\*x^(1 + n)\*Hypergeometric2F1[1, (1 + n)/(2\*n), (3 + n^(-1))/2, -((c\*x^(2\*n))/a)])/a - (e\*(3\*c\*d^2 - a\*e^2)\*(1 - n)\*x^(1 + n)\*Hypergeometric2F1[1, (1 + n)/(2\*n), (3 + n^(-1))/2, -((c\*x^(2\*n))/a)])/2\*a^2\*c\*n\*(1 + n)

**Rubi [A]** time = 0.252169, antiderivative size = 288, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$ , Rules used = {1437, 1431, 1418, 245, 364}

$$\frac{e(1-n)x^{n+1}(3cd^2 - ae^2) {}_2F_1\left(1, \frac{n+1}{2n}; \frac{1}{2}\left(3 + \frac{1}{n}\right); -\frac{cx^{2n}}{a}\right)}{2a^2cn(n+1)} - \frac{d(1-2n)x(cd^2 - 3ae^2) {}_2F_1\left(1, \frac{1}{2n}; \frac{1}{2}\left(2 + \frac{1}{n}\right); -\frac{cx^{2n}}{a}\right)}{2a^2cn} + \frac{x(ex^{2n})}{2a^2cn}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x^n)^3/(a + c\*x^(2\*n))^2, x]

[Out] (x\*(d\*(c\*d^2 - 3\*a\*e^2) + e\*(3\*c\*d^2 - a\*e^2)\*x^n))/(2\*a\*c\*n\*(a + c\*x^(2\*n))) + (3\*d\*e^2\*x\*Hypergeometric2F1[1, 1/(2\*n), (2 + n^(-1))/2, -((c\*x^(2\*n))/a)])/a - (d\*(c\*d^2 - 3\*a\*e^2)\*(1 - 2\*n)\*x\*Hypergeometric2F1[1, 1/(2\*n), (2 + n^(-1))/2, -((c\*x^(2\*n))/a)])/2\*a^2\*c\*n + (e^3\*x^(1 + n)\*Hypergeometric2F1[1, (1 + n)/(2\*n), (3 + n^(-1))/2, -((c\*x^(2\*n))/a)])/a - (e\*(3\*c\*d^2 - a\*e^2)\*(1 - n)\*x^(1 + n)\*Hypergeometric2F1[1, (1 + n)/(2\*n), (3 + n^(-1))/2, -((c\*x^(2\*n))/a)])/2\*a^2\*c\*n\*(1 + n)

**Rule 1437**

Int[((d\_) + (e\_.)\*(x\_)^(n\_))^(q\_)\*((a\_) + (c\_.)\*(x\_)^(n2\_))^(p\_), x\_Symbol]
 :> Int[ExpandIntegrand[(d + e\*x^n)^q\*(a + c\*x^(2\*n))^p, x], x] /; FreeQ[{a

```
, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && NeQ[c*d^2 + a*e^2, 0] && ((Integ
ersQ[p, q] && !IntegerQ[n]) || IGtQ[p, 0] || (IGtQ[q, 0] && !IntegerQ[n])
)
```

### Rule 1431

```
Int[((d_) + (e_.)*(x_)^(n_))*((a_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol] := -
Simp[(x*(d + e*x^n)*(a + c*x^(2*n))^(p + 1))/(2*a*n*(p + 1)), x] + Dist[1/(
2*a*n*(p + 1)), Int[(d*(2*n*p + 2*n + 1) + e*(2*n*p + 3*n + 1)*x^n)*(a + c*
x^(2*n))^(p + 1), x], x] /; FreeQ[{a, c, d, e, n}, x] && EqQ[n2, 2*n] && ILt
tQ[p, -1]
```

### Rule 1418

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (c_.)*(x_)^(n2_)), x_Symbol] := Dist[d
, Int[1/(a + c*x^(2*n)), x], x] + Dist[e, Int[x^n/(a + c*x^(2*n)), x], x] /
; FreeQ[{a, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[c*d^2 + a*e^2, 0] && (Po
sQ[a*c] || !IntegerQ[n])
```

### Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

### Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)]]/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

### Rubi steps

$$\begin{aligned}
\int \frac{(d + ex^n)^3}{(a + cx^{2n})^2} dx &= \int \left( \frac{cd^3 - 3ade^2 + (3cd^2e - ae^3)x^n}{c(a + cx^{2n})^2} + \frac{e^2(3d + ex^n)}{c(a + cx^{2n})} \right) dx \\
&= \frac{\int \frac{cd^3 - 3ade^2 + (3cd^2e - ae^3)x^n}{(a + cx^{2n})^2} dx}{c} + \frac{e^2 \int \frac{3d + ex^n}{a + cx^{2n}} dx}{c} \\
&= \frac{x(d(cd^2 - 3ae^2) + e(3cd^2 - ae^2)x^n)}{2acn(a + cx^{2n})} + \frac{(3de^2) \int \frac{1}{a + cx^{2n}} dx}{c} + \frac{e^3 \int \frac{x^n}{a + cx^{2n}} dx}{c} - \frac{\int \frac{(cd^3 - 3ade^2)(1 - 2n) + (3cd^2e - ae^3)x^n}{a + cx^{2n}} dx}{2acn} \\
&= \frac{x(d(cd^2 - 3ae^2) + e(3cd^2 - ae^2)x^n)}{2acn(a + cx^{2n})} + \frac{3de^2 x {}_2F_1\left(1, \frac{1}{2n}; \frac{1}{2}\left(2 + \frac{1}{n}\right); -\frac{cx^{2n}}{a}\right)}{ac} + \frac{e^3 x^{1+n} {}_2F_1\left(1, \frac{1+n}{2n}; \frac{1}{2}\left(3 + \frac{1}{n}\right); -\frac{cx^{2n}}{a}\right)}{ac(1+n)} \\
&= \frac{x(d(cd^2 - 3ae^2) + e(3cd^2 - ae^2)x^n)}{2acn(a + cx^{2n})} + \frac{3de^2 x {}_2F_1\left(1, \frac{1}{2n}; \frac{1}{2}\left(2 + \frac{1}{n}\right); -\frac{cx^{2n}}{a}\right)}{ac} - \frac{d(cd^2 - 3ae^2)(1 - 2n)}{2acn}
\end{aligned}$$

**Mathematica [A]** time = 0.282452, size = 188, normalized size = 0.65

$$\frac{x \left( \frac{ex^n (3cd^2 - ae^2) {}_2F_1\left(2, \frac{n+1}{2n}; \frac{1}{2}\left(3 + \frac{1}{n}\right); -\frac{cx^{2n}}{a}\right)}{n+1} + d(cd^2 - 3ae^2) {}_2F_1\left(2, \frac{1}{2n}; \frac{1}{2}\left(2 + \frac{1}{n}\right); -\frac{cx^{2n}}{a}\right) + 3ade^2 {}_2F_1\left(1, \frac{1}{2n}; \frac{1}{2}\left(2 + \frac{1}{n}\right); -\frac{cx^{2n}}{a}\right) \right)}{a^2 c}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e\*x^n)^3/(a + c\*x^(2\*n))^2,x]

[Out] (x\*(3\*a\*d\*e^2\*Hypergeometric2F1[1, 1/(2\*n), (2 + n^(-1))/2, -((c\*x^(2\*n))/a)]) + (a\*e^3\*x^n\*Hypergeometric2F1[1, (1 + n)/(2\*n), (3 + n^(-1))/2, -((c\*x^(2\*n))/a)]))/(1 + n) + d\*(c\*d^2 - 3\*a\*e^2)\*Hypergeometric2F1[2, 1/(2\*n), (2 + n^(-1))/2, -((c\*x^(2\*n))/a)] + (e\*(3\*c\*d^2 - a\*e^2)\*x^n\*Hypergeometric2F1[2, (1 + n)/(2\*n), (3 + n^(-1))/2, -((c\*x^(2\*n))/a)]))/(1 + n))/(a^2\*c)

**Maple [F]** time = 0.072, size = 0, normalized size = 0.

$$\int \frac{(d + ex^n)^3}{(a + cx^{2n})^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d+e*x^n)^3/(a+c*x^(2*n))^2,x)`

[Out] `int((d+e*x^n)^3/(a+c*x^(2*n))^2,x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{(3cd^2e - ae^3)xx^n + (cd^3 - 3ade^2)x}{2(ac^2nx^{2n} + a^2cn)} + \int \frac{cd^3(2n-1) + 3ade^2 + (ae^3(n+1) + 3cd^2e(n-1))x^n}{2(ac^2nx^{2n} + a^2cn)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d+e*x^n)^3/(a+c*x^(2*n))^2,x, algorithm="maxima")`

[Out] `1/2*((3*c*d^2*e - a*e^3)*x*x^n + (c*d^3 - 3*a*d*e^2)*x)/(a*c^2*n*x^(2*n) + a^2*c*n) + integrate(1/2*(c*d^3*(2*n - 1) + 3*a*d*e^2 + (a*e^3*(n + 1) + 3*c*d^2*e*(n - 1))*x^n)/(a*c^2*n*x^(2*n) + a^2*c*n), x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{e^3x^{3n} + 3de^2x^{2n} + 3d^2ex^n + d^3}{c^2x^{4n} + 2acx^{2n} + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d+e*x^n)^3/(a+c*x^(2*n))^2,x, algorithm="fricas")`

[Out] `integral((e^3*x^(3*n) + 3*d*e^2*x^(2*n) + 3*d^2*e*x^n + d^3)/(c^2*x^(4*n) + 2*a*c*x^(2*n) + a^2), x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x**n)**3/(a+c*x**(2*n))**2,x)
```

```
[Out] Timed out
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^n + d)^3}{(cx^{2n} + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x^n)^3/(a+c*x^(2*n))^2,x, algorithm="giac")
```

```
[Out] integrate((e*x^n + d)^3/(c*x^(2*n) + a)^2, x)
```



$$3.49 \quad \int \frac{(d+ex^n)^2}{(a+cx^{2n})^2} dx$$

**Optimal.** Leaf size=203

$$\frac{(1-2n)x(cd^2 - ae^2) {}_2F_1\left(1, \frac{1}{2n}; \frac{1}{2}\left(2 + \frac{1}{n}\right); -\frac{cx^{2n}}{a}\right)}{2a^2cn} - \frac{de(1-n)x^{n+1} {}_2F_1\left(1, \frac{n+1}{2n}; \frac{1}{2}\left(3 + \frac{1}{n}\right); -\frac{cx^{2n}}{a}\right)}{a^2n(n+1)} + \frac{x(-ae^2 + cd^2 + 2c)}{2acn(a + cx^{2n})}$$

[Out] (x\*(c\*d^2 - a\*e^2 + 2\*c\*d\*e\*x^n))/(2\*a\*c\*n\*(a + c\*x^(2\*n))) + (e^2\*x\*Hypergeometric2F1[1, 1/(2\*n), (2 + n^(-1))/2, -((c\*x^(2\*n))/a)]/(a\*c) - ((c\*d^2 - a\*e^2)\*(1 - 2\*n)\*x\*Hypergeometric2F1[1, 1/(2\*n), (2 + n^(-1))/2, -((c\*x^(2\*n))/a)])/(2\*a^2\*c\*n) - (d\*e\*(1 - n)\*x^(1 + n)\*Hypergeometric2F1[1, (1 + n)/(2\*n), (3 + n^(-1))/2, -((c\*x^(2\*n))/a)]/(a^2\*n\*(1 + n)))

**Rubi [A]** time = 0.167352, antiderivative size = 203, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$ , Rules used = {1437, 1431, 1418, 245, 364}

$$\frac{(1-2n)x(cd^2 - ae^2) {}_2F_1\left(1, \frac{1}{2n}; \frac{1}{2}\left(2 + \frac{1}{n}\right); -\frac{cx^{2n}}{a}\right)}{2a^2cn} - \frac{de(1-n)x^{n+1} {}_2F_1\left(1, \frac{n+1}{2n}; \frac{1}{2}\left(3 + \frac{1}{n}\right); -\frac{cx^{2n}}{a}\right)}{a^2n(n+1)} + \frac{x(-ae^2 + cd^2 + 2c)}{2acn(a + cx^{2n})}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x^n)^2/(a + c\*x^(2\*n))^2,x]

[Out] (x\*(c\*d^2 - a\*e^2 + 2\*c\*d\*e\*x^n))/(2\*a\*c\*n\*(a + c\*x^(2\*n))) + (e^2\*x\*Hypergeometric2F1[1, 1/(2\*n), (2 + n^(-1))/2, -((c\*x^(2\*n))/a)]/(a\*c) - ((c\*d^2 - a\*e^2)\*(1 - 2\*n)\*x\*Hypergeometric2F1[1, 1/(2\*n), (2 + n^(-1))/2, -((c\*x^(2\*n))/a)])/(2\*a^2\*c\*n) - (d\*e\*(1 - n)\*x^(1 + n)\*Hypergeometric2F1[1, (1 + n)/(2\*n), (3 + n^(-1))/2, -((c\*x^(2\*n))/a)]/(a^2\*n\*(1 + n)))

**Rule 1437**

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol]
  :> Int[ExpandIntegrand[(d + e*x^n)^q*(a + c*x^(2*n))^p, x], x] /; FreeQ[{a
, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && NeQ[c*d^2 + a*e^2, 0] && ((Integ
ersQ[p, q] && !IntegerQ[n]) || IGtQ[p, 0] || (IGtQ[q, 0] && !IntegerQ[n])
)
```

Rule 1431

```
Int[((d_) + (e_.)*(x_)^(n_))*((a_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol] := -
Simp[(x*(d + e*x^n)*(a + c*x^(2*n))^(p + 1))/(2*a*n*(p + 1)), x] + Dist[1/(
2*a*n*(p + 1)), Int[(d*(2*n*p + 2*n + 1) + e*(2*n*p + 3*n + 1)*x^n)*(a + c*
x^(2*n))^(p + 1), x], x] /; FreeQ[{a, c, d, e, n}, x] && EqQ[n2, 2*n] && IL
tQ[p, -1]
```

Rule 1418

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (c_.)*(x_)^(n2_)), x_Symbol] := Dist[d
, Int[1/(a + c*x^(2*n)), x], x] + Dist[e, Int[x^n/(a + c*x^(2*n)), x], x] /
; FreeQ[{a, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[c*d^2 + a*e^2, 0] && (Po
sQ[a*c] || !IntegerQ[n])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -(b*x^n)/a], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_)^(m_.))*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -(b*x^n)/a
])/ (c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(d + ex^n)^2}{(a + cx^{2n})^2} dx &= \int \left( \frac{cd^2 - ae^2 + 2cdex^n}{c(a + cx^{2n})^2} + \frac{e^2}{c(a + cx^{2n})} \right) dx \\
&= \frac{\int \frac{cd^2 - ae^2 + 2cdex^n}{(a + cx^{2n})^2} dx}{c} + \frac{e^2 \int \frac{1}{a + cx^{2n}} dx}{c} \\
&= \frac{x(cd^2 - ae^2 + 2cdex^n)}{2acn(a + cx^{2n})} + \frac{e^2 x {}_2F_1\left(1, \frac{1}{2n}; \frac{1}{2}\left(2 + \frac{1}{n}\right); -\frac{cx^{2n}}{a}\right)}{ac} - \frac{\int \frac{(cd^2 - ae^2)(1 - 2n) + 2cde(1 - n)x^n}{a + cx^{2n}} dx}{2acn} \\
&= \frac{x(cd^2 - ae^2 + 2cdex^n)}{2acn(a + cx^{2n})} + \frac{e^2 x {}_2F_1\left(1, \frac{1}{2n}; \frac{1}{2}\left(2 + \frac{1}{n}\right); -\frac{cx^{2n}}{a}\right)}{ac} - \frac{((cd^2 - ae^2)(1 - 2n)) \int \frac{1}{a + cx^{2n}} dx}{2acn} - (d + ex^n) \int \frac{1}{a + cx^{2n}} dx \\
&= \frac{x(cd^2 - ae^2 + 2cdex^n)}{2acn(a + cx^{2n})} + \frac{e^2 x {}_2F_1\left(1, \frac{1}{2n}; \frac{1}{2}\left(2 + \frac{1}{n}\right); -\frac{cx^{2n}}{a}\right)}{ac} - \frac{(cd^2 - ae^2)(1 - 2n)x {}_2F_1\left(1, \frac{1}{2n}; \frac{1}{2}\left(2 + \frac{1}{n}\right); -\frac{cx^{2n}}{a}\right)}{2a^2cn}
\end{aligned}$$

**Mathematica [A]** time = 0.160188, size = 136, normalized size = 0.67

$$\frac{x(n+1)(cd^2 - ae^2) {}_2F_1\left(2, \frac{1}{2n}; \frac{1}{2}\left(2 + \frac{1}{n}\right); -\frac{cx^{2n}}{a}\right) + 2cdex^n {}_2F_1\left(2, \frac{n+1}{2n}; \frac{1}{2}\left(3 + \frac{1}{n}\right); -\frac{cx^{2n}}{a}\right) + ae^2(n+1) {}_2F_1\left(1, \frac{1}{2n}; \frac{1}{2}\left(2 + \frac{1}{n}\right); -\frac{cx^{2n}}{a}\right)}{a^2c(n+1)}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e\*x^n)^2/(a + c\*x^(2\*n))^2,x]

[Out] (x\*(a\*e^2\*(1 + n)\*Hypergeometric2F1[1, 1/(2\*n), (2 + n^(-1))/2, -((c\*x^(2\*n)))/a]) + (c\*d^2 - a\*e^2)\*(1 + n)\*Hypergeometric2F1[2, 1/(2\*n), (2 + n^(-1))/2, -((c\*x^(2\*n)))/a]) + 2\*c\*d\*e\*x^n\*Hypergeometric2F1[2, (1 + n)/(2\*n), (3 + n^(-1))/2, -((c\*x^(2\*n)))/a]))/(a^2\*c\*(1 + n))

**Maple [F]** time = 0.069, size = 0, normalized size = 0.

$$\int \frac{(d + ex^n)^2}{(a + cx^{2n})^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d+e*x^n)^2/(a+c*x^(2*n))^2,x)`

[Out] `int((d+e*x^n)^2/(a+c*x^(2*n))^2,x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{2cdexx^n + (cd^2 - ae^2)x}{2(ac^2nx^{2n} + a^2cn)} + \int \frac{2cde(n-1)x^n + cd^2(2n-1) + ae^2}{2(ac^2nx^{2n} + a^2cn)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d+e*x^n)^2/(a+c*x^(2*n))^2,x, algorithm="maxima")`

[Out] `1/2*(2*c*d*e*x*x^n + (c*d^2 - a*e^2)*x)/(a*c^2*n*x^(2*n) + a^2*c*n) + integrate(1/2*(2*c*d*e*(n - 1)*x^n + c*d^2*(2*n - 1) + a*e^2)/(a*c^2*n*x^(2*n) + a^2*c*n), x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{e^2x^{2n} + 2dex^n + d^2}{c^2x^{4n} + 2acx^{2n} + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d+e*x^n)^2/(a+c*x^(2*n))^2,x, algorithm="fricas")`

[Out] `integral((e^2*x^(2*n) + 2*d*e*x^n + d^2)/(c^2*x^(4*n) + 2*a*c*x^(2*n) + a^2), x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d+e*x**n)**2/(a+c*x**(2*n))**2,x)`

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^n + d)^2}{(cx^{2n} + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e\*x^n)^2/(a+c\*x^(2\*n))^2,x, algorithm="giac")

[Out] integrate((e\*x^n + d)^2/(c\*x^(2\*n) + a)^2, x)

$$3.50 \quad \int \frac{d+ex^n}{(a+cx^{2n})^2} dx$$

**Optimal.** Leaf size=134

$$-\frac{d(1-2n)x {}_2F_1\left(1, \frac{1}{2n}; \frac{1}{2}\left(2 + \frac{1}{n}\right); -\frac{cx^{2n}}{a}\right)}{2a^2n} - \frac{e(1-n)x^{n+1} {}_2F_1\left(1, \frac{n+1}{2n}; \frac{1}{2}\left(3 + \frac{1}{n}\right); -\frac{cx^{2n}}{a}\right)}{2a^2n(n+1)} + \frac{x(d+ex^n)}{2an(a+cx^{2n})}$$

[Out] (x\*(d + e\*x^n))/(2\*a\*n\*(a + c\*x^(2\*n))) - (d\*(1 - 2\*n)\*x\*Hypergeometric2F1[1, 1/(2\*n), (2 + n^(-1))/2, -((c\*x^(2\*n))/a)])/(2\*a^2\*n) - (e\*(1 - n)\*x^(1 + n)\*Hypergeometric2F1[1, (1 + n)/(2\*n), (3 + n^(-1))/2, -((c\*x^(2\*n))/a)])/(2\*a^2\*n\*(1 + n))

**Rubi [A]** time = 0.0543761, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.21$ , Rules used = {1431, 1418, 245, 364}

$$-\frac{d(1-2n)x {}_2F_1\left(1, \frac{1}{2n}; \frac{1}{2}\left(2 + \frac{1}{n}\right); -\frac{cx^{2n}}{a}\right)}{2a^2n} - \frac{e(1-n)x^{n+1} {}_2F_1\left(1, \frac{n+1}{2n}; \frac{1}{2}\left(3 + \frac{1}{n}\right); -\frac{cx^{2n}}{a}\right)}{2a^2n(n+1)} + \frac{x(d+ex^n)}{2an(a+cx^{2n})}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x^n)/(a + c\*x^(2\*n))^2,x]

[Out] (x\*(d + e\*x^n))/(2\*a\*n\*(a + c\*x^(2\*n))) - (d\*(1 - 2\*n)\*x\*Hypergeometric2F1[1, 1/(2\*n), (2 + n^(-1))/2, -((c\*x^(2\*n))/a)])/(2\*a^2\*n) - (e\*(1 - n)\*x^(1 + n)\*Hypergeometric2F1[1, (1 + n)/(2\*n), (3 + n^(-1))/2, -((c\*x^(2\*n))/a)])/(2\*a^2\*n\*(1 + n))

### Rule 1431

Int[((d\_) + (e\_.)\*(x\_)^(n\_))\*((a\_) + (c\_.)\*(x\_)^(n2\_))^(p\_), x\_Symbol] :> -Simp[(x\*(d + e\*x^n)\*(a + c\*x^(2\*n))^(p + 1))/(2\*a\*n\*(p + 1)), x] + Dist[1/(2\*a\*n\*(p + 1)), Int[(d\*(2\*n\*p + 2\*n + 1) + e\*(2\*n\*p + 3\*n + 1)\*x^n)\*(a + c\*x^(2\*n))^(p + 1), x], x] /; FreeQ[{a, c, d, e, n}, x] && EqQ[n2, 2\*n] && ILtQ[p, -1]

### Rule 1418

```
Int[((d_) + (e_)*(x_)^(n_))/((a_) + (c_)*(x_)^(n2_)), x_Symbol] := Dist[d
, Int[1/(a + c*x^(2*n)), x], x] + Dist[e, Int[x^n/(a + c*x^(2*n)), x], x] /
; FreeQ[{a, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[c*d^2 + a*e^2, 0] && (Po
sQ[a*c] || !IntegerQ[n])
```

### Rule 245

```
Int[((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

### Rule 364

```
Int[((c_)*(x_)^(m_))*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/((c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

### Rubi steps

$$\begin{aligned} \int \frac{d + ex^n}{(a + cx^{2n})^2} dx &= \frac{x(d + ex^n)}{2an(a + cx^{2n})} - \frac{\int \frac{d(1-2n) + e(1-n)x^n}{a + cx^{2n}} dx}{2an} \\ &= \frac{x(d + ex^n)}{2an(a + cx^{2n})} - \frac{(d(1-2n)) \int \frac{1}{a + cx^{2n}} dx}{2an} - \frac{(e(1-n)) \int \frac{x^n}{a + cx^{2n}} dx}{2an} \\ &= \frac{x(d + ex^n)}{2an(a + cx^{2n})} - \frac{d(1-2n)x {}_2F_1\left(1, \frac{1}{2n}; \frac{1}{2}\left(2 + \frac{1}{n}\right); -\frac{cx^{2n}}{a}\right)}{2a^2n} - \frac{e(1-n)x^{1+n} {}_2F_1\left(1, \frac{1+n}{2n}; \frac{1}{2}\left(3 + \frac{1}{n}\right); -\frac{cx^{2n}}{a}\right)}{2a^2n(1+n)} \end{aligned}$$

**Mathematica [A]** time = 0.0423645, size = 83, normalized size = 0.62

$$\frac{dx {}_2F_1\left(2, \frac{1}{2n}; \frac{1}{2}\left(2 + \frac{1}{n}\right); -\frac{cx^{2n}}{a}\right)}{a^2} + \frac{ex^{n+1} {}_2F_1\left(2, \frac{n+1}{2n}; \frac{1}{2}\left(3 + \frac{1}{n}\right); -\frac{cx^{2n}}{a}\right)}{a^2(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^n)/(a + c*x^(2*n))^2, x]
```

```
[Out] (d*x*Hypergeometric2F1[2, 1/(2*n), (2 + n^(-1))/2, -((c*x^(2*n))/a)]/a^2 +
(e*x^(1 + n)*Hypergeometric2F1[2, (1 + n)/(2*n), (3 + n^(-1))/2, -((c*x^(2
*n))/a)])/(a^2*(1 + n))
```

**Maple [F]** time = 0.066, size = 0, normalized size = 0.

$$\int \frac{d + ex^n}{(a + cx^{2n})^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+e*x^n)/(a+c*x^(2*n))^2,x)
```

```
[Out] int((d+e*x^n)/(a+c*x^(2*n))^2,x)
```

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{ex^n + dx}{2(acnx^{2n} + a^2n)} + \int \frac{e(n-1)x^n + d(2n-1)}{2(acnx^{2n} + a^2n)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x^n)/(a+c*x^(2*n))^2,x, algorithm="maxima")
```

```
[Out] 1/2*(e*x*x^n + d*x)/(a*c*n*x^(2*n) + a^2*n) + integrate(1/2*(e*(n - 1)*x^n
+ d*(2*n - 1))/(a*c*n*x^(2*n) + a^2*n), x)
```

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{ex^n + d}{c^2x^{4n} + 2acx^{2n} + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x^n)/(a+c*x^(2*n))^2,x, algorithm="fricas")
```



[Out] `integral((e*x^n + d)/(c^2*x^(4*n) + 2*a*c*x^(2*n) + a^2), x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d+e*x**n)/(a+c*x**(2*n))**2,x)`

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{ex^n + d}{(cx^{2n} + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d+e*x^n)/(a+c*x^(2*n))^2,x, algorithm="giac")`

[Out] `integrate((e*x^n + d)/(c*x^(2*n) + a)^2, x)`

$$3.51 \quad \int \frac{1}{(d+ex^n)(a+cx^{2n})^2} dx$$

**Optimal.** Leaf size=333

$$\frac{ce(1-n)x^{n+1} {}_2F_1\left(1, \frac{n+1}{2n}; \frac{1}{2}\left(3 + \frac{1}{n}\right); -\frac{cx^{2n}}{a}\right)}{2a^2n(n+1)(ae^2 + cd^2)} - \frac{cd(1-2n)x {}_2F_1\left(1, \frac{1}{2n}; \frac{1}{2}\left(2 + \frac{1}{n}\right); -\frac{cx^{2n}}{a}\right)}{2a^2n(ae^2 + cd^2)} - \frac{ce^3x^{n+1} {}_2F_1\left(1, \frac{n+1}{2n}; \frac{1}{2}\left(3 + \frac{1}{n}\right); -\frac{cx^{2n}}{a}\right)}{a(n+1)(ae^2 + cd^2)^2}$$

[Out] (c\*x\*(d - e\*x^n))/(2\*a\*(c\*d^2 + a\*e^2)\*n\*(a + c\*x^(2\*n))) + (c\*d\*e^2\*x\*Hypergeometric2F1[1, 1/(2\*n), (2 + n^(-1))/2, -((c\*x^(2\*n))/a)]/(a\*(c\*d^2 + a\*e^2)^2) - (c\*d\*(1 - 2\*n)\*x\*Hypergeometric2F1[1, 1/(2\*n), (2 + n^(-1))/2, -((c\*x^(2\*n))/a)]/(2\*a^2\*(c\*d^2 + a\*e^2)\*n) + (e^4\*x\*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((e\*x^n)/d)]/(d\*(c\*d^2 + a\*e^2)^2) - (c\*e^3\*x^(1 + n)\*Hypergeometric2F1[1, (1 + n)/(2\*n), (3 + n^(-1))/2, -((c\*x^(2\*n))/a)]/(a\*(c\*d^2 + a\*e^2)^2\*(1 + n)) + (c\*e\*(1 - n)\*x^(1 + n)\*Hypergeometric2F1[1, (1 + n)/(2\*n), (3 + n^(-1))/2, -((c\*x^(2\*n))/a)]/(2\*a^2\*(c\*d^2 + a\*e^2)\*n\*(1 + n)))

**Rubi [A]** time = 0.224394, antiderivative size = 333, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 5, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$ , Rules used = {1437, 245, 1431, 1418, 364}

$$\frac{ce(1-n)x^{n+1} {}_2F_1\left(1, \frac{n+1}{2n}; \frac{1}{2}\left(3 + \frac{1}{n}\right); -\frac{cx^{2n}}{a}\right)}{2a^2n(n+1)(ae^2 + cd^2)} - \frac{cd(1-2n)x {}_2F_1\left(1, \frac{1}{2n}; \frac{1}{2}\left(2 + \frac{1}{n}\right); -\frac{cx^{2n}}{a}\right)}{2a^2n(ae^2 + cd^2)} - \frac{ce^3x^{n+1} {}_2F_1\left(1, \frac{n+1}{2n}; \frac{1}{2}\left(3 + \frac{1}{n}\right); -\frac{cx^{2n}}{a}\right)}{a(n+1)(ae^2 + cd^2)^2}$$

Antiderivative was successfully verified.

[In] Int[1/((d + e\*x^n)\*(a + c\*x^(2\*n))^2), x]

[Out] (c\*x\*(d - e\*x^n))/(2\*a\*(c\*d^2 + a\*e^2)\*n\*(a + c\*x^(2\*n))) + (c\*d\*e^2\*x\*Hypergeometric2F1[1, 1/(2\*n), (2 + n^(-1))/2, -((c\*x^(2\*n))/a)]/(a\*(c\*d^2 + a\*e^2)^2) - (c\*d\*(1 - 2\*n)\*x\*Hypergeometric2F1[1, 1/(2\*n), (2 + n^(-1))/2, -((c\*x^(2\*n))/a)]/(2\*a^2\*(c\*d^2 + a\*e^2)\*n) + (e^4\*x\*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((e\*x^n)/d)]/(d\*(c\*d^2 + a\*e^2)^2) - (c\*e^3\*x^(1 + n)\*Hypergeometric2F1[1, (1 + n)/(2\*n), (3 + n^(-1))/2, -((c\*x^(2\*n))/a)]/(a\*(c\*d^2 + a\*e^2)^2\*(1 + n)) + (c\*e\*(1 - n)\*x^(1 + n)\*Hypergeometric2F1[1, (1 + n)/(2\*n), (3 + n^(-1))/2, -((c\*x^(2\*n))/a)]/(2\*a^2\*(c\*d^2 + a\*e^2)\*n\*(1 + n)))

Rule 1437

```
Int[((d_) + (e_)*(x_)^(n_))^(q_)*((a_) + (c_)*(x_)^(n2_))^(p_), x_Symbol]
  :> Int[ExpandIntegrand[(d + e*x^n)^q*(a + c*x^(2*n))^p, x], x] /; FreeQ[{a
, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && NeQ[c*d^2 + a*e^2, 0] && ((Integ
ersQ[p, q] && !IntegerQ[n]) || IGtQ[p, 0] || (IGtQ[q, 0] && !IntegerQ[n])
)
```

Rule 245

```
Int[((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 1431

```
Int[((d_) + (e_)*(x_)^(n_))*((a_) + (c_)*(x_)^(n2_))^(p_), x_Symbol] :> -
Simp[(x*(d + e*x^n)*(a + c*x^(2*n))^(p + 1))/(2*a*n*(p + 1)), x] + Dist[1/(
2*a*n*(p + 1)), Int[(d*(2*n*p + 2*n + 1) + e*(2*n*p + 3*n + 1)*x^n)*(a + c*
x^(2*n))^(p + 1), x], x] /; FreeQ[{a, c, d, e, n}, x] && EqQ[n2, 2*n] && IL
tQ[p, -1]
```

Rule 1418

```
Int[((d_) + (e_)*(x_)^(n_))/((a_) + (c_)*(x_)^(n2_)), x_Symbol] :> Dist[d
, Int[1/(a + c*x^(2*n)), x], x] + Dist[e, Int[x^n/(a + c*x^(2*n)), x], x] /
; FreeQ[{a, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[c*d^2 + a*e^2, 0] && (Po
sQ[a*c] || !IntegerQ[n])
```

Rule 364

```
Int[((c_)*(x_)^(m_))*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)]]/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(d+ex^n)(a+cx^{2n})^2} dx &= \int \left( \frac{e^4}{(cd^2+ae^2)^2(d+ex^n)} - \frac{c(-d+ex^n)}{(cd^2+ae^2)(a+cx^{2n})^2} - \frac{ce^2(-d+ex^n)}{(cd^2+ae^2)^2(a+cx^{2n})} \right) dx \\
&= -\frac{(ce^2) \int \frac{-d+ex^n}{a+cx^{2n}} dx}{(cd^2+ae^2)^2} + \frac{e^4 \int \frac{1}{d+ex^n} dx}{(cd^2+ae^2)^2} - \frac{c \int \frac{-d+ex^n}{(a+cx^{2n})^2} dx}{cd^2+ae^2} \\
&= \frac{cx(d-ex^n)}{2a(cd^2+ae^2)n(a+cx^{2n})} + \frac{e^4 x {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{ex^n}{d}\right)}{d(cd^2+ae^2)^2} + \frac{(cde^2) \int \frac{1}{a+cx^{2n}} dx}{(cd^2+ae^2)^2} - \frac{(ce^3) \int \frac{x^n}{a+cx^{2n}} dx}{(cd^2+ae^2)^2} \\
&= \frac{cx(d-ex^n)}{2a(cd^2+ae^2)n(a+cx^{2n})} + \frac{cde^2 x {}_2F_1\left(1, \frac{1}{2n}; \frac{1}{2}\left(2 + \frac{1}{n}\right); -\frac{cx^{2n}}{a}\right)}{a(cd^2+ae^2)^2} + \frac{e^4 x {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{ex^n}{d}\right)}{d(cd^2+ae^2)^2} \\
&= \frac{cx(d-ex^n)}{2a(cd^2+ae^2)n(a+cx^{2n})} + \frac{cde^2 x {}_2F_1\left(1, \frac{1}{2n}; \frac{1}{2}\left(2 + \frac{1}{n}\right); -\frac{cx^{2n}}{a}\right)}{a(cd^2+ae^2)^2} - \frac{cd(1-2n)x {}_2F_1\left(1, \frac{1}{2n}; \frac{1}{2}\left(2 + \frac{1}{n}\right); -\frac{cx^{2n}}{a}\right)}{2a^2(cd^2+ae^2)^2}
\end{aligned}$$

**Mathematica [A]** time = 0.290809, size = 227, normalized size = 0.68

$$\frac{x \left( a^2 e^4 (n+1) {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{ex^n}{d}\right) + acd^2 e^2 (n+1) {}_2F_1\left(1, \frac{1}{2n}; \frac{1}{2}\left(2 + \frac{1}{n}\right); -\frac{cx^{2n}}{a}\right) + cd \left( (ae^2 + cd^2) \left( d(n+1) {}_2F_1\left(2, \frac{1}{2n}; \frac{1}{2}\left(2 + \frac{1}{n}\right); -\frac{cx^{2n}}{a}\right) \right) \right) \right)}{a^2 d (n+1) (ae^2 + cd^2)^2}$$

Antiderivative was successfully verified.

[In] Integrate[1/((d + e\*x^n)\*(a + c\*x^(2\*n))^2),x]

[Out] (x\*(a\*c\*d^2\*e^2\*(1+n)\*Hypergeometric2F1[1, 1/(2\*n), (2+n^(-1))/2, -((c\*x^(2\*n))/a)] + a^2\*e^4\*(1+n)\*Hypergeometric2F1[1, n^(-1), 1+n^(-1), -((e\*x^n)/d)] + c\*d\*(-(a\*e^3\*x^n\*Hypergeometric2F1[1, (1+n)/(2\*n), (3+n^(-1))/2, -((c\*x^(2\*n))/a)]) + (c\*d^2 + a\*e^2)\*(d\*(1+n)\*Hypergeometric2F1[2, 1/(2\*n), (2+n^(-1))/2, -((c\*x^(2\*n))/a)] - e\*x^n\*Hypergeometric2F1[2, (1+n)/(2\*n), (3+n^(-1))/2, -((c\*x^(2\*n))/a)])))/((a^2\*d\*(c\*d^2 + a\*e^2)^2\*(1+n))

**Maple [F]** time = 0.139, size = 0, normalized size = 0.

$$\int \frac{1}{(d+ex^n)(a+cx^{2n})^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(d+e*x^n)/(a+c*x^(2*n))^2,x)`

[Out] `int(1/(d+e*x^n)/(a+c*x^(2*n))^2,x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$e^4 \int \frac{1}{c^2 d^5 + 2acd^3 e^2 + a^2 d e^4 + (c^2 d^4 e + 2acd^2 e^3 + a^2 e^5) x^n} dx - \frac{cexx^n - cdx}{2(a^2 cd^2 n + a^3 e^2 n + (ac^2 d^2 n + a^2 ce^2 n)x^{2n})} - \int -\frac{1}{2(a^2 cd^2 n + a^3 e^2 n + (ac^2 d^2 n + a^2 ce^2 n)x^{2n})} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(d+e*x^n)/(a+c*x^(2*n))^2,x, algorithm="maxima")`

[Out] `e^4*integrate(1/(c^2*d^5 + 2*a*c*d^3*e^2 + a^2*d*e^4 + (c^2*d^4*e + 2*a*c*d^2*e^3 + a^2*e^5)*x^n), x) - 1/2*(c*e*x*x^n - c*d*x)/(a^2*c*d^2*n + a^3*e^2*n + (a*c^2*d^2*n + a^2*c*e^2*n)*x^(2*n)) - integrate(-1/2*(a*c*d*e^2*(4*n - 1) + c^2*d^3*(2*n - 1) - (a*c*e^3*(3*n - 1) + c^2*d^2*e*(n - 1))*x^n)/(a^2*c^2*d^4*n + 2*a^3*c*d^2*e^2*n + a^4*e^4*n + (a*c^3*d^4*n + 2*a^2*c^2*d^2*e^2*n + a^3*c*e^4*n)*x^(2*n)), x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{a^2 ex^n + a^2 d + (c^2 ex^n + c^2 d)x^{4n} + 2(acex^n + acd)x^{2n}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(d+e*x^n)/(a+c*x^(2*n))^2,x, algorithm="fricas")`

[Out] `integral(1/(a^2*e*x^n + a^2*d + (c^2*e*x^n + c^2*d)*x^(4*n) + 2*(a*c*e*x^n + a*c*d)*x^(2*n)), x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d+e\*x\*\*n)/(a+c\*x\*\*(2\*n))\*\*2,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(cx^{2n} + a)^2 (ex^n + d)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d+e\*x^n)/(a+c\*x^(2\*n))^2,x, algorithm="giac")

[Out] integrate(1/((c\*x^(2\*n) + a)^2\*(e\*x^n + d)), x)

$$3.52 \quad \int \frac{1}{(d+ex^n)^2(a+cx^{2n})^2} dx$$

**Optimal.** Leaf size=410

$$\frac{c^2de(1-n)x^{n+1} {}_2F_1\left(1, \frac{n+1}{2n}; \frac{1}{2}\left(3+\frac{1}{n}\right); -\frac{cx^{2n}}{a}\right)}{a^2n(n+1)(ae^2+cd^2)^2} - \frac{c(1-2n)x(cd^2-ae^2) {}_2F_1\left(1, \frac{1}{2n}; \frac{1}{2}\left(2+\frac{1}{n}\right); -\frac{cx^{2n}}{a}\right)}{2a^2n(ae^2+cd^2)^2} - \frac{4c^2de^3x^{n+1} {}_2F_1\left(1, \frac{1}{2n}; \frac{1}{2}\left(2+\frac{1}{n}\right); -\frac{cx^{2n}}{a}\right)}{a(n+1)}$$

[Out] (c\*x\*(c\*d^2 - a\*e^2 - 2\*c\*d\*e\*x^n))/(2\*a\*(c\*d^2 + a\*e^2)^2\*n\*(a + c\*x^(2\*n))) + (c\*e^2\*(3\*c\*d^2 - a\*e^2)\*x\*Hypergeometric2F1[1, 1/(2\*n), (2 + n^(-1))/2, -((c\*x^(2\*n))/a)])/ (a\*(c\*d^2 + a\*e^2)^3) - (c\*(c\*d^2 - a\*e^2)\*(1 - 2\*n)\*x\*Hypergeometric2F1[1, 1/(2\*n), (2 + n^(-1))/2, -((c\*x^(2\*n))/a)])/ (2\*a^2\*(c\*d^2 + a\*e^2)^2\*n) + (4\*c\*e^4\*x\*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((e\*x^n)/d)])/ (c\*d^2 + a\*e^2)^3 - (4\*c^2\*d\*e^3\*x^(1 + n)\*Hypergeometric2F1[1, (1 + n)/(2\*n), (3 + n^(-1))/2, -((c\*x^(2\*n))/a)])/ (a\*(c\*d^2 + a\*e^2)^3\*(1 + n)) + (c^2\*d\*e\*(1 - n)\*x^(1 + n)\*Hypergeometric2F1[1, (1 + n)/(2\*n), (3 + n^(-1))/2, -((c\*x^(2\*n))/a)])/ (a^2\*(c\*d^2 + a\*e^2)^2\*n\*(1 + n)) + (e^4\*x\*Hypergeometric2F1[2, n^(-1), 1 + n^(-1), -((e\*x^n)/d)])/ (d^2\*(c\*d^2 + a\*e^2)^2)

**Rubi [A]** time = 0.378725, antiderivative size = 410, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 5, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$ , Rules used = {1437, 245, 1431, 1418, 364}

$$\frac{c^2de(1-n)x^{n+1} {}_2F_1\left(1, \frac{n+1}{2n}; \frac{1}{2}\left(3+\frac{1}{n}\right); -\frac{cx^{2n}}{a}\right)}{a^2n(n+1)(ae^2+cd^2)^2} - \frac{c(1-2n)x(cd^2-ae^2) {}_2F_1\left(1, \frac{1}{2n}; \frac{1}{2}\left(2+\frac{1}{n}\right); -\frac{cx^{2n}}{a}\right)}{2a^2n(ae^2+cd^2)^2} - \frac{4c^2de^3x^{n+1} {}_2F_1\left(1, \frac{1}{2n}; \frac{1}{2}\left(2+\frac{1}{n}\right); -\frac{cx^{2n}}{a}\right)}{a(n+1)}$$

Antiderivative was successfully verified.

[In] Int[1/((d + e\*x^n)^2\*(a + c\*x^(2\*n))^2), x]

[Out] (c\*x\*(c\*d^2 - a\*e^2 - 2\*c\*d\*e\*x^n))/(2\*a\*(c\*d^2 + a\*e^2)^2\*n\*(a + c\*x^(2\*n))) + (c\*e^2\*(3\*c\*d^2 - a\*e^2)\*x\*Hypergeometric2F1[1, 1/(2\*n), (2 + n^(-1))/2, -((c\*x^(2\*n))/a)])/ (a\*(c\*d^2 + a\*e^2)^3) - (c\*(c\*d^2 - a\*e^2)\*(1 - 2\*n)\*x\*Hypergeometric2F1[1, 1/(2\*n), (2 + n^(-1))/2, -((c\*x^(2\*n))/a)])/ (2\*a^2\*(c\*d^2 + a\*e^2)^2\*n) + (4\*c\*e^4\*x\*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((e\*x^n)/d)])/ (c\*d^2 + a\*e^2)^3 - (4\*c^2\*d\*e^3\*x^(1 + n)\*Hypergeometric2F1[1, (1 + n)/(2\*n), (3 + n^(-1))/2, -((c\*x^(2\*n))/a)])/ (a\*(c\*d^2 + a\*e^2)^3\*(1 + n)) + (c^2\*d\*e\*(1 - n)\*x^(1 + n)\*Hypergeometric2F1[1, (1 + n)/(2\*n), (3 + n^(-1))/2, -((c\*x^(2\*n))/a)])/ (a^2\*(c\*d^2 + a\*e^2)^2\*n\*(1 + n)) + (e^4\*x\*Hypergeometric2F1[2, n^(-1), 1 + n^(-1), -((e\*x^n)/d)])/ (d^2\*(c\*d^2 + a\*e^2)^2)

+ n<sup>(-1)</sup>)/2, -((c\*x<sup>(2\*n)</sup>)/a)]/(a<sup>2</sup>\*(c\*d<sup>2</sup> + a\*e<sup>2</sup>)<sup>2\*n</sup>\*(1 + n)) + (e<sup>4</sup>\*x\*Hypergeometric2F1[2, n<sup>(-1)</sup>, 1 + n<sup>(-1)</sup>, -((e\*x<sup>n</sup>)/d)]/(d<sup>2</sup>\*(c\*d<sup>2</sup> + a\*e<sup>2</sup>)<sup>2</sup>)

### Rule 1437

Int[((d\_) + (e\_.)\*(x\_)^(n\_))^(q\_)\*((a\_) + (c\_.)\*(x\_)^(n2\_))^(p\_), x\_Symbol] := Int[ExpandIntegrand[(d + e\*x<sup>n</sup>)<sup>q</sup>\*(a + c\*x<sup>(2\*n)</sup>)<sup>p</sup>, x], x] /; FreeQ[{a, c, d, e, n, p, q}, x] && EqQ[n2, 2\*n] && NeQ[c\*d<sup>2</sup> + a\*e<sup>2</sup>, 0] && ((IntegersQ[p, q] && !IntegerQ[n]) || IGtQ[p, 0] || (IGtQ[q, 0] && !IntegerQ[n]))

### Rule 245

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[a<sup>p</sup>\*x\*Hypergeometric2F1[-p, 1/n, 1/n + 1, -((b\*x<sup>n</sup>)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] || GtQ[a, 0])

### Rule 1431

Int[((d\_) + (e\_.)\*(x\_)^(n\_))\*((a\_) + (c\_.)\*(x\_)^(n2\_))^(p\_), x\_Symbol] := -Simp[(x\*(d + e\*x<sup>n</sup>)\*(a + c\*x<sup>(2\*n)</sup>)<sup>(p + 1)</sup>/(2\*a\*n\*(p + 1)), x] + Dist[1/(2\*a\*n\*(p + 1)), Int[(d\*(2\*n\*p + 2\*n + 1) + e\*(2\*n\*p + 3\*n + 1)\*x<sup>n</sup>)\*(a + c\*x<sup>(2\*n)</sup>)<sup>(p + 1)</sup>, x], x] /; FreeQ[{a, c, d, e, n}, x] && EqQ[n2, 2\*n] && ILtQ[p, -1]

### Rule 1418

Int[((d\_) + (e\_.)\*(x\_)^(n\_))/((a\_) + (c\_.)\*(x\_)^(n2\_)), x\_Symbol] := Dist[d, Int[1/(a + c\*x<sup>(2\*n)</sup>), x], x] + Dist[e, Int[x<sup>n</sup>/(a + c\*x<sup>(2\*n)</sup>), x], x] /; FreeQ[{a, c, d, e, n}, x] && EqQ[n2, 2\*n] && NeQ[c\*d<sup>2</sup> + a\*e<sup>2</sup>, 0] && (PosQ[a\*c] || !IntegerQ[n])

### Rule 364

Int[((c\_.)\*(x\_)^(m\_.))\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[(a<sup>p</sup>\*c\*x<sup>(m + 1)</sup>\*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b\*x<sup>n</sup>)/a)])/(c\*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])

### Rubi steps



$$\begin{aligned}
\int \frac{1}{(d+ex^n)^2(a+cx^{2n})^2} dx &= \int \left( \frac{e^4}{(cd^2+ae^2)^2(d+ex^n)^2} + \frac{4cde^4}{(cd^2+ae^2)^3(d+ex^n)} - \frac{c(-cd^2+ae^2+2cdex^n)}{(cd^2+ae^2)^2(a+cx^{2n})^2} - \frac{ce^2(-)}{(cd^2+ae^2)^3} \right) dx \\
&= -\frac{(ce^2) \int \frac{-3cd^2+ae^2+4cdex^n}{a+cx^{2n}} dx}{(cd^2+ae^2)^3} + \frac{(4cde^4) \int \frac{1}{d+ex^n} dx}{(cd^2+ae^2)^3} - \frac{c \int \frac{-cd^2+ae^2+2cdex^n}{(a+cx^{2n})^2} dx}{(cd^2+ae^2)^2} + \frac{e^4 \int \frac{1}{(d+ex^n)^2} dx}{(cd^2+ae^2)^3} \\
&= \frac{cx(cd^2-ae^2-2cdex^n)}{2a(cd^2+ae^2)^2 n(a+cx^{2n})} + \frac{4ce^4 x {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{ex^n}{d}\right)}{(cd^2+ae^2)^3} + \frac{e^4 x {}_2F_1\left(2, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{ex^n}{d}\right)}{d^2(cd^2+ae^2)^2} \\
&= \frac{cx(cd^2-ae^2-2cdex^n)}{2a(cd^2+ae^2)^2 n(a+cx^{2n})} + \frac{ce^2(3cd^2-ae^2)x {}_2F_1\left(1, \frac{1}{2n}; \frac{1}{2}\left(2 + \frac{1}{n}\right); -\frac{cx^{2n}}{a}\right)}{a(cd^2+ae^2)^3} + \frac{4ce^4 x {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{ex^n}{d}\right)}{d^2(cd^2+ae^2)^2} \\
&= \frac{cx(cd^2-ae^2-2cdex^n)}{2a(cd^2+ae^2)^2 n(a+cx^{2n})} + \frac{ce^2(3cd^2-ae^2)x {}_2F_1\left(1, \frac{1}{2n}; \frac{1}{2}\left(2 + \frac{1}{n}\right); -\frac{cx^{2n}}{a}\right)}{a(cd^2+ae^2)^3} - \frac{c(cd^2-ae^2)}{a(cd^2+ae^2)^3}
\end{aligned}$$

**Mathematica [A]** time = 0.502782, size = 298, normalized size = 0.73

$$x \left( -\frac{2c^2 dex^n (ae^2 + cd^2) {}_2F_1\left(2, \frac{n+1}{2n}; \frac{1}{2}\left(3 + \frac{1}{n}\right); -\frac{cx^{2n}}{a}\right)}{a^2(n+1)} + \frac{c(cd^2 - ae^2)(ae^2 + cd^2) {}_2F_1\left(2, \frac{1}{2n}; \frac{1}{2}\left(2 + \frac{1}{n}\right); -\frac{cx^{2n}}{a}\right)}{a^2} - \frac{4c^2 de^3 x^n {}_2F_1\left(1, \frac{n+1}{2n}; \frac{1}{2}\left(3 + \frac{1}{n}\right); -\frac{cx^{2n}}{a}\right)}{a(n+1)} + \frac{ce^2(3cd^2 - ae^2)}{(ae^2 + cd^2)^3} \right)$$

Antiderivative was successfully verified.

[In] Integrate[1/((d + e\*x^n)^2\*(a + c\*x^(2\*n))^2), x]

[Out] (x\*((c\*e^2\*(3\*c\*d^2 - a\*e^2)\*Hypergeometric2F1[1, 1/(2\*n), (2 + n^(-1))/2, -((c\*x^(2\*n))/a)])/a + 4\*c\*e^4\*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((e\*x^n)/d)] - (4\*c^2\*d\*e^3\*x^n\*Hypergeometric2F1[1, (1 + n)/(2\*n), (3 + n^(-1))/2, -((c\*x^(2\*n))/a)])/a\*(1 + n)) + (c\*(c\*d^2 - a\*e^2)\*(c\*d^2 + a\*e^2)\*Hypergeometric2F1[2, 1/(2\*n), (2 + n^(-1))/2, -((c\*x^(2\*n))/a)])/a^2 + (e^4\*(c\*d^2 + a\*e^2)\*Hypergeometric2F1[2, n^(-1), 1 + n^(-1), -((e\*x^n)/d)])/d^2 - (2\*c^2\*d\*e\*(c\*d^2 + a\*e^2)\*x^n\*Hypergeometric2F1[2, (1 + n)/(2\*n), (3 + n^(-1))/2, -((c\*x^(2\*n))/a)])/a^2\*(1 + n)))/(c\*d^2 + a\*e^2)^3

**Maple [F]** time = 0.146, size = 0, normalized size = 0.

$$\int \frac{1}{(d + ex^n)^2 (a + cx^{2n})^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(d+e\*x^n)^2/(a+c\*x^(2\*n))^2,x)

[Out] int(1/(d+e\*x^n)^2/(a+c\*x^(2\*n))^2,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$(cd^2e^4(5n-1) + ae^6(n-1)) \int \frac{1}{c^3d^8n + 3ac^2d^6e^2n + 3a^2cd^4e^4n + a^3d^2e^6n + (c^3d^7en + 3ac^2d^5e^3n + 3a^2cd^3e^5n + a^3de^7n)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d+e\*x^n)^2/(a+c\*x^(2\*n))^2,x, algorithm="maxima")

[Out] (c\*d^2\*e^4\*(5\*n - 1) + a\*e^6\*(n - 1))\*integrate(1/(c^3\*d^8\*n + 3\*a\*c^2\*d^6\*e^2\*n + 3\*a^2\*c\*d^4\*e^4\*n + a^3\*d^2\*e^6\*n + (c^3\*d^7\*e\*n + 3\*a\*c^2\*d^5\*e^3\*n + 3\*a^2\*c\*d^3\*e^5\*n + a^3\*d\*e^7\*n)\*x^n), x) - 1/2\*(2\*(c^2\*d^2\*e^2 - a\*c\*e^4)\*x\*x^(2\*n) + (c^2\*d^3\*e + a\*c\*d\*e^3)\*x\*x^n - (c^2\*d^4 - a\*c\*d^2\*e^2 + 2\*a^2\*e^4)\*x)/(a^2\*c^2\*d^6\*n + 2\*a^3\*c\*d^4\*e^2\*n + a^4\*d^2\*e^4\*n + (a\*c^3\*d^5\*e\*n + 2\*a^2\*c^2\*d^3\*e^3\*n + a^3\*c\*d\*e^5\*n)\*x^(3\*n) + (a\*c^3\*d^6\*n + 2\*a^2\*c^2\*d^4\*e^2\*n + a^3\*c\*d^2\*e^4\*n)\*x^(2\*n) + (a^2\*c^2\*d^5\*e\*n + 2\*a^3\*c\*d^3\*e^3\*n + a^4\*d\*e^5\*n)\*x^n) - integrate(1/2\*(a^2\*c\*e^4\*(4\*n - 1) - c^3\*d^4\*(2\*n - 1) - 6\*a\*c^2\*d^2\*e^2\*n + 2\*(a\*c^2\*d\*e^3\*(5\*n - 1) + c^3\*d^3\*e\*(n - 1))\*x^n)/(a^2\*c^3\*d^6\*n + 3\*a^3\*c^2\*d^4\*e^2\*n + 3\*a^4\*c\*d^2\*e^4\*n + a^5\*e^6\*n + (a\*c^4\*d^6\*n + 3\*a^2\*c^3\*d^4\*e^2\*n + 3\*a^3\*c^2\*d^2\*e^4\*n + a^4\*c\*e^6\*n)\*x^(2\*n)), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{1}{a^2e^2x^{2n} + 2a^2dex^n + a^2d^2 + (c^2e^2x^{2n} + 2c^2dex^n + c^2d^2)x^{4n} + 2(ace^2x^{2n} + 2acdex^n + acd^2)x^{2n}}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x^n)^2/(a+c*x^(2*n))^2,x, algorithm="fricas")
```

```
[Out] integral(1/(a^2*e^2*x^(2*n) + 2*a^2*d*e*x^n + a^2*d^2 + (c^2*e^2*x^(2*n) +
2*c^2*d*e*x^n + c^2*d^2)*x^(4*n) + 2*(a*c*e^2*x^(2*n) + 2*a*c*d*e*x^n + a*c
*d^2)*x^(2*n)), x)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x**n)**2/(a+c*x**(2*n))**2,x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(cx^{2n} + a)^2 (ex^n + d)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x^n)^2/(a+c*x^(2*n))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^(2*n) + a)^2*(e*x^n + d)^2), x)
```

### 3.53

$$\int \frac{(d+ex^n)^3}{(a+cx^{2n})^3} dx$$

**Optimal.** Leaf size=424

$$\frac{e(1-3n)(1-n)x^{n+1} \left(3cd^2 - ae^2\right) {}_2F_1\left(1, \frac{n+1}{2n}; \frac{1}{2}\left(3 + \frac{1}{n}\right); -\frac{cx^{2n}}{a}\right)}{8a^3cn^2(n+1)} + \frac{d(1-4n)(1-2n)x \left(cd^2 - 3ae^2\right) {}_2F_1\left(1, \frac{1}{2n}; \frac{1}{2}\left(2 + \frac{1}{n}\right); -\frac{cx^{2n}}{a}\right)}{8a^3cn^2}$$

[Out] (x\*(d\*(c\*d^2 - 3\*a\*e^2) + e\*(3\*c\*d^2 - a\*e^2)\*x^n))/(4\*a\*c\*n\*(a + c\*x^(2\*n))^2) + (e^2\*x\*(3\*d + e\*x^n))/(2\*a\*c\*n\*(a + c\*x^(2\*n))) - (x\*(d\*(c\*d^2 - 3\*a\*e^2)\*(1 - 4\*n) + e\*(3\*c\*d^2 - a\*e^2)\*(1 - 3\*n)\*x^n))/(8\*a^2\*c\*n^2\*(a + c\*x^(2\*n))) + (d\*(c\*d^2 - 3\*a\*e^2)\*(1 - 4\*n)\*(1 - 2\*n)\*x\*Hypergeometric2F1[1, 1/(2\*n), (2 + n^(-1))/2, -((c\*x^(2\*n))/a)])/(8\*a^3\*c\*n^2) - (3\*d\*e^2\*(1 - 2\*n)\*x\*Hypergeometric2F1[1, 1/(2\*n), (2 + n^(-1))/2, -((c\*x^(2\*n))/a)])/(2\*a^2\*c\*n) + (e\*(3\*c\*d^2 - a\*e^2)\*(1 - 3\*n)\*(1 - n)\*x^(1 + n)\*Hypergeometric2F1[1, (1 + n)/(2\*n), (3 + n^(-1))/2, -((c\*x^(2\*n))/a)])/(8\*a^3\*c\*n^2\*(1 + n)) - (e^3\*(1 - n)\*x^(1 + n)\*Hypergeometric2F1[1, (1 + n)/(2\*n), (3 + n^(-1))/2, -((c\*x^(2\*n))/a)])/(2\*a^2\*c\*n\*(1 + n))

**Rubi [A]** time = 0.382556, antiderivative size = 424, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 5, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$ , Rules used = {1437, 1431, 1418, 245, 364}

$$\frac{e(1-3n)(1-n)x^{n+1} \left(3cd^2 - ae^2\right) {}_2F_1\left(1, \frac{n+1}{2n}; \frac{1}{2}\left(3 + \frac{1}{n}\right); -\frac{cx^{2n}}{a}\right)}{8a^3cn^2(n+1)} + \frac{d(1-4n)(1-2n)x \left(cd^2 - 3ae^2\right) {}_2F_1\left(1, \frac{1}{2n}; \frac{1}{2}\left(2 + \frac{1}{n}\right); -\frac{cx^{2n}}{a}\right)}{8a^3cn^2}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x^n)^3/(a + c\*x^(2\*n))^3,x]

[Out] (x\*(d\*(c\*d^2 - 3\*a\*e^2) + e\*(3\*c\*d^2 - a\*e^2)\*x^n))/(4\*a\*c\*n\*(a + c\*x^(2\*n))^2) + (e^2\*x\*(3\*d + e\*x^n))/(2\*a\*c\*n\*(a + c\*x^(2\*n))) - (x\*(d\*(c\*d^2 - 3\*a\*e^2)\*(1 - 4\*n) + e\*(3\*c\*d^2 - a\*e^2)\*(1 - 3\*n)\*x^n))/(8\*a^2\*c\*n^2\*(a + c\*x^(2\*n))) + (d\*(c\*d^2 - 3\*a\*e^2)\*(1 - 4\*n)\*(1 - 2\*n)\*x\*Hypergeometric2F1[1, 1/(2\*n), (2 + n^(-1))/2, -((c\*x^(2\*n))/a)])/(8\*a^3\*c\*n^2) - (3\*d\*e^2\*(1 - 2\*n)\*x\*Hypergeometric2F1[1, 1/(2\*n), (2 + n^(-1))/2, -((c\*x^(2\*n))/a)])/(2\*a^2\*c\*n) + (e\*(3\*c\*d^2 - a\*e^2)\*(1 - 3\*n)\*(1 - n)\*x^(1 + n)\*Hypergeometric2F1[1, (1 + n)/(2\*n), (3 + n^(-1))/2, -((c\*x^(2\*n))/a)])/(8\*a^3\*c\*n^2\*(1 + n)) - (e^3\*(1 - n)\*x^(1 + n)\*Hypergeometric2F1[1, (1 + n)/(2\*n), (3 + n^(-1))/2, -((c\*x^(2\*n))/a)])/(2\*a^2\*c\*n\*(1 + n))

/2, -((c\*x^(2\*n))/a)]/(2\*a^2\*c\*n\*(1 + n))

### Rule 1437

```
Int[((d_) + (e_)*(x_)^(n_))^(q_)*((a_) + (c_)*(x_)^(n2_))^(p_), x_Symbol]
  := Int[ExpandIntegrand[(d + e*x^n)^q*(a + c*x^(2*n))^p, x], x] /; FreeQ[{a
, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && NeQ[c*d^2 + a*e^2, 0] && ((Integ
ersQ[p, q] && !IntegerQ[n]) || IGtQ[p, 0] || (IGtQ[q, 0] && !IntegerQ[n])
)
```

### Rule 1431

```
Int[((d_) + (e_)*(x_)^(n_))*((a_) + (c_)*(x_)^(n2_))^(p_), x_Symbol] := -
Simp[(x*(d + e*x^n)*(a + c*x^(2*n))^(p + 1))/(2*a*n*(p + 1)), x] + Dist[1/(
2*a*n*(p + 1)), Int[(d*(2*n*p + 2*n + 1) + e*(2*n*p + 3*n + 1)*x^n)*(a + c*
x^(2*n))^(p + 1), x], x] /; FreeQ[{a, c, d, e, n}, x] && EqQ[n2, 2*n] && IL
tQ[p, -1]
```

### Rule 1418

```
Int[((d_) + (e_)*(x_)^(n_))/((a_) + (c_)*(x_)^(n2_)), x_Symbol] := Dist[d
, Int[1/(a + c*x^(2*n)), x], x] + Dist[e, Int[x^n/(a + c*x^(2*n)), x], x] /
; FreeQ[{a, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[c*d^2 + a*e^2, 0] && (Po
sQ[a*c] || !IntegerQ[n])
```

### Rule 245

```
Int[((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

### Rule 364

```
Int[((c_)*(x_)^(m_))*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/((c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

### Rubi steps

$$\begin{aligned}
\int \frac{(d + ex^n)^3}{(a + cx^{2n})^3} dx &= \int \left( \frac{cd^3 - 3ade^2 + (3cd^2e - ae^3)x^n}{c(a + cx^{2n})^3} + \frac{e^2(3d + ex^n)}{c(a + cx^{2n})^2} \right) dx \\
&= \frac{\int \frac{cd^3 - 3ade^2 + (3cd^2e - ae^3)x^n}{(a + cx^{2n})^3} dx}{c} + \frac{e^2 \int \frac{3d + ex^n}{(a + cx^{2n})^2} dx}{c} \\
&= \frac{x(d(cd^2 - 3ae^2) + e(3cd^2 - ae^2)x^n)}{4acn(a + cx^{2n})^2} + \frac{e^2x(3d + ex^n)}{2acn(a + cx^{2n})} - \frac{\int \frac{(cd^3 - 3ade^2)(1-4n) + (3cd^2e - ae^3)(1-3n)x^n}{(a + cx^{2n})^2} dx}{4acn} \\
&= \frac{x(d(cd^2 - 3ae^2) + e(3cd^2 - ae^2)x^n)}{4acn(a + cx^{2n})^2} + \frac{e^2x(3d + ex^n)}{2acn(a + cx^{2n})} - \frac{x(d(cd^2 - 3ae^2)(1 - 4n) + e(3cd^2 - ae^2)x^n)}{8a^2cn^2(a + cx^{2n})} \\
&= \frac{x(d(cd^2 - 3ae^2) + e(3cd^2 - ae^2)x^n)}{4acn(a + cx^{2n})^2} + \frac{e^2x(3d + ex^n)}{2acn(a + cx^{2n})} - \frac{x(d(cd^2 - 3ae^2)(1 - 4n) + e(3cd^2 - ae^2)x^n)}{8a^2cn^2(a + cx^{2n})} \\
&= \frac{x(d(cd^2 - 3ae^2) + e(3cd^2 - ae^2)x^n)}{4acn(a + cx^{2n})^2} + \frac{e^2x(3d + ex^n)}{2acn(a + cx^{2n})} - \frac{x(d(cd^2 - 3ae^2)(1 - 4n) + e(3cd^2 - ae^2)x^n)}{8a^2cn^2(a + cx^{2n})}
\end{aligned}$$

**Mathematica [A]** time = 0.306757, size = 188, normalized size = 0.44

$$\frac{x \left( \frac{ex^n(3cd^2 - ae^2) {}_2F_1\left(3, \frac{n+1}{2n}; \frac{1}{2}\left(3 + \frac{1}{n}\right); -\frac{cx^{2n}}{a}\right)}{n+1} + d(cd^2 - 3ae^2) {}_2F_1\left(3, \frac{1}{2n}; \frac{1}{2}\left(2 + \frac{1}{n}\right); -\frac{cx^{2n}}{a}\right) + 3ade^2 {}_2F_1\left(2, \frac{1}{2n}; \frac{1}{2}\left(2 + \frac{1}{n}\right); -\frac{cx^{2n}}{a}\right) \right)}{a^3c}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e\*x^n)^3/(a + c\*x^(2\*n))^3,x]

[Out] (x\*(3\*a\*d\*e^2\*Hypergeometric2F1[2, 1/(2\*n), (2 + n^(-1))/2, -((c\*x^(2\*n))/a)]) + (a\*e^3\*x^n\*Hypergeometric2F1[2, (1 + n)/(2\*n), (3 + n^(-1))/2, -((c\*x^(2\*n))/a)]))/(1 + n) + d\*(c\*d^2 - 3\*a\*e^2)\*Hypergeometric2F1[3, 1/(2\*n), (2 + n^(-1))/2, -((c\*x^(2\*n))/a)] + (e\*(3\*c\*d^2 - a\*e^2)\*x^n\*Hypergeometric2F1[3, (1 + n)/(2\*n), (3 + n^(-1))/2, -((c\*x^(2\*n))/a)]))/(1 + n))/(a^3\*c)

**Maple [F]** time = 0.088, size = 0, normalized size = 0.

$$\int \frac{(d + ex^n)^3}{(a + cx^{2n})^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d+e\*x^n)^3/(a+c\*x^(2\*n))^3,x)

[Out] int((d+e\*x^n)^3/(a+c\*x^(2\*n))^3,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{(3c^2d^2e(3n-1) + ace^3(n+1))xx^{3n} + (c^2d^3(4n-1) + 3acde^2)xx^{2n} + (3acd^2e(5n-1) - a^2e^3(n-1))xx^n + (acd^3(6n-1))}{8(a^2c^3n^2x^{4n} + 2a^3c^2n^2x^{2n} + a^4cn^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e\*x^n)^3/(a+c\*x^(2\*n))^3,x, algorithm="maxima")

[Out] 1/8\*((3\*c^2\*d^2\*e\*(3\*n - 1) + a\*c\*e^3\*(n + 1))\*x\*x^(3\*n) + (c^2\*d^3\*(4\*n - 1) + 3\*a\*c\*d\*e^2)\*x\*x^(2\*n) + (3\*a\*c\*d^2\*e\*(5\*n - 1) - a^2\*e^3\*(n - 1))\*x\*x^n + (a\*c\*d^3\*(6\*n - 1) - 3\*a^2\*d\*e^2\*(2\*n - 1))\*x)/(a^2\*c^3\*n^2\*x^(4\*n) + 2\*a^3\*c^2\*n^2\*x^(2\*n) + a^4\*c\*n^2) + integrate(1/8\*((8\*n^2 - 6\*n + 1)\*c\*d^3 + 3\*a\*d\*e^2\*(2\*n - 1) + (3\*(3\*n^2 - 4\*n + 1)\*c\*d^2\*e + (n^2 - 1)\*a\*e^3)\*x^n)/(a^2\*c^2\*n^2\*x^(2\*n) + a^3\*c\*n^2), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{e^3x^{3n} + 3de^2x^{2n} + 3d^2ex^n + d^3}{c^3x^{6n} + 3ac^2x^{4n} + 3a^2cx^{2n} + a^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e\*x^n)^3/(a+c\*x^(2\*n))^3,x, algorithm="fricas")

[Out] integral((e^3\*x^(3\*n) + 3\*d\*e^2\*x^(2\*n) + 3\*d^2\*e\*x^n + d^3)/(c^3\*x^(6\*n) + 3\*a\*c^2\*x^(4\*n) + 3\*a^2\*c\*x^(2\*n) + a^3), x)

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e\*x\*\*n)\*\*3/(a+c\*x\*\*(2\*n))\*\*3,x)

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^n + d)^3}{(cx^{2n} + a)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e\*x^n)^3/(a+c\*x^(2\*n))^3,x, algorithm="giac")

[Out] integrate((e\*x^n + d)^3/(c\*x^(2\*n) + a)^3, x)



$$3.54 \quad \int \frac{(d+ex^n)^2}{(a+cx^{2n})^3} dx$$

**Optimal.** Leaf size=272

$$\frac{(1-4n)(1-2n)x(cd^2 - ae^2) {}_2F_1\left(1, \frac{1}{2n}; \frac{1}{2}\left(2 + \frac{1}{n}\right); -\frac{cx^{2n}}{a}\right)}{8a^3cn^2} - \frac{x((1-4n)(cd^2 - ae^2) + 2cde(1-3n)x^n)}{8a^2cn^2(a+cx^{2n})} + \frac{de(1-3n)(1-2n)}{8a^2cn^2}$$

[Out] (x\*(c\*d^2 - a\*e^2 + 2\*c\*d\*e\*x^n))/(4\*a\*c\*n\*(a + c\*x^(2\*n))^2) - (x\*((c\*d^2 - a\*e^2)\*(1 - 4\*n) + 2\*c\*d\*e\*(1 - 3\*n)\*x^n))/(8\*a^2\*c\*n^2\*(a + c\*x^(2\*n))) + ((c\*d^2 - a\*e^2)\*(1 - 4\*n)\*(1 - 2\*n)\*x\*Hypergeometric2F1[1, 1/(2\*n), (2 + n^(-1))/2, -((c\*x^(2\*n))/a)])/(8\*a^3\*c\*n^2) + (d\*e\*(1 - 3\*n)\*(1 - n)\*x^(1 + n)\*Hypergeometric2F1[1, (1 + n)/(2\*n), (3 + n^(-1))/2, -((c\*x^(2\*n))/a)])/(4\*a^3\*n^2\*(1 + n)) + (e^2\*x\*Hypergeometric2F1[2, 1/(2\*n), (2 + n^(-1))/2, -((c\*x^(2\*n))/a)])/(a^2\*c)

**Rubi [A]** time = 0.246966, antiderivative size = 272, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$ , Rules used = {1437, 1431, 1418, 245, 364}

$$\frac{(1-4n)(1-2n)x(cd^2 - ae^2) {}_2F_1\left(1, \frac{1}{2n}; \frac{1}{2}\left(2 + \frac{1}{n}\right); -\frac{cx^{2n}}{a}\right)}{8a^3cn^2} - \frac{x((1-4n)(cd^2 - ae^2) + 2cde(1-3n)x^n)}{8a^2cn^2(a+cx^{2n})} + \frac{de(1-3n)(1-2n)}{8a^2cn^2}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x^n)^2/(a + c\*x^(2\*n))^3, x]

[Out] (x\*(c\*d^2 - a\*e^2 + 2\*c\*d\*e\*x^n))/(4\*a\*c\*n\*(a + c\*x^(2\*n))^2) - (x\*((c\*d^2 - a\*e^2)\*(1 - 4\*n) + 2\*c\*d\*e\*(1 - 3\*n)\*x^n))/(8\*a^2\*c\*n^2\*(a + c\*x^(2\*n))) + ((c\*d^2 - a\*e^2)\*(1 - 4\*n)\*(1 - 2\*n)\*x\*Hypergeometric2F1[1, 1/(2\*n), (2 + n^(-1))/2, -((c\*x^(2\*n))/a)])/(8\*a^3\*c\*n^2) + (d\*e\*(1 - 3\*n)\*(1 - n)\*x^(1 + n)\*Hypergeometric2F1[1, (1 + n)/(2\*n), (3 + n^(-1))/2, -((c\*x^(2\*n))/a)])/(4\*a^3\*n^2\*(1 + n)) + (e^2\*x\*Hypergeometric2F1[2, 1/(2\*n), (2 + n^(-1))/2, -((c\*x^(2\*n))/a)])/(a^2\*c)

**Rule 1437**

Int[((d\_) + (e\_.)\*(x\_)^(n\_))^(q\_)\*((a\_) + (c\_.)\*(x\_)^(n2\_))^(p\_), x\_Symbol]  
 :> Int[ExpandIntegrand[(d + e\*x^n)^q\*(a + c\*x^(2\*n))^p, x], x] /; FreeQ[{a

```
, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && NeQ[c*d^2 + a*e^2, 0] && ((Integ
ersQ[p, q] && !IntegerQ[n]) || IGtQ[p, 0] || (IGtQ[q, 0] && !IntegerQ[n]
))
```

### Rule 1431

```
Int[((d_) + (e_.)*(x_)^(n_))*((a_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol] := -
Simp[(x*(d + e*x^n)*(a + c*x^(2*n))^(p + 1))/(2*a*n*(p + 1)), x] + Dist[1/(
2*a*n*(p + 1)), Int[(d*(2*n*p + 2*n + 1) + e*(2*n*p + 3*n + 1)*x^n)*(a + c*
x^(2*n))^(p + 1), x], x] /; FreeQ[{a, c, d, e, n}, x] && EqQ[n2, 2*n] && ILt
Q[p, -1]
```

### Rule 1418

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (c_.)*(x_)^(n2_)), x_Symbol] := Dist[d
, Int[1/(a + c*x^(2*n)), x], x] + Dist[e, Int[x^n/(a + c*x^(2*n)), x], x] /
; FreeQ[{a, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[c*d^2 + a*e^2, 0] && (Po
sQ[a*c] || !IntegerQ[n])
```

### Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -(b*x^n)/a], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

### Rule 364

```
Int[((c_.)*(x_)^(m_.))*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -(b*x^n)/a
])/ (c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

### Rubi steps

$$\begin{aligned}
\int \frac{(d + ex^n)^2}{(a + cx^{2n})^3} dx &= \int \left( \frac{cd^2 - ae^2 + 2cdex^n}{c(a + cx^{2n})^3} + \frac{e^2}{c(a + cx^{2n})^2} \right) dx \\
&= \frac{\int \frac{cd^2 - ae^2 + 2cdex^n}{(a + cx^{2n})^3} dx}{c} + \frac{e^2 \int \frac{1}{(a + cx^{2n})^2} dx}{c} \\
&= \frac{x(cd^2 - ae^2 + 2cdex^n)}{4acn(a + cx^{2n})^2} + \frac{e^2 x {}_2F_1\left(2, \frac{1}{2n}; \frac{1}{2}\left(2 + \frac{1}{n}\right); -\frac{cx^{2n}}{a}\right)}{a^2c} - \frac{\int \frac{(cd^2 - ae^2)^{(1-4n) + 2cde(1-3n)x^n}}{(a + cx^{2n})^2} dx}{4acn} \\
&= \frac{x(cd^2 - ae^2 + 2cdex^n)}{4acn(a + cx^{2n})^2} - \frac{x\left((cd^2 - ae^2)(1 - 4n) + 2cde(1 - 3n)x^n\right)}{8a^2cn^2(a + cx^{2n})} + \frac{e^2 x {}_2F_1\left(2, \frac{1}{2n}; \frac{1}{2}\left(2 + \frac{1}{n}\right); -\frac{cx^{2n}}{a}\right)}{a^2c} \\
&= \frac{x(cd^2 - ae^2 + 2cdex^n)}{4acn(a + cx^{2n})^2} - \frac{x\left((cd^2 - ae^2)(1 - 4n) + 2cde(1 - 3n)x^n\right)}{8a^2cn^2(a + cx^{2n})} + \frac{e^2 x {}_2F_1\left(2, \frac{1}{2n}; \frac{1}{2}\left(2 + \frac{1}{n}\right); -\frac{cx^{2n}}{a}\right)}{a^2c} \\
&= \frac{x(cd^2 - ae^2 + 2cdex^n)}{4acn(a + cx^{2n})^2} - \frac{x\left((cd^2 - ae^2)(1 - 4n) + 2cde(1 - 3n)x^n\right)}{8a^2cn^2(a + cx^{2n})} + \frac{(cd^2 - ae^2)(1 - 4n)(1 - 2n)x}{8a^3}
\end{aligned}$$

**Mathematica [A]** time = 0.183434, size = 136, normalized size = 0.5

$$\frac{x\left((n+1)(cd^2 - ae^2) {}_2F_1\left(3, \frac{1}{2n}; \frac{1}{2}\left(2 + \frac{1}{n}\right); -\frac{cx^{2n}}{a}\right) + 2cdex^n {}_2F_1\left(3, \frac{n+1}{2n}; \frac{1}{2}\left(3 + \frac{1}{n}\right); -\frac{cx^{2n}}{a}\right) + ae^2(n+1) {}_2F_1\left(2, \frac{1}{2n}; \frac{1}{2}\left(2 + \frac{1}{n}\right); -\frac{cx^{2n}}{a}\right)\right)}{a^3c(n+1)}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e\*x^n)^2/(a + c\*x^(2\*n))^3, x]

[Out] (x\*(a\*e^2\*(1 + n)\*Hypergeometric2F1[2, 1/(2\*n), (2 + n^(-1))/2, -((c\*x^(2\*n))/a)] + (c\*d^2 - a\*e^2)\*(1 + n)\*Hypergeometric2F1[3, 1/(2\*n), (2 + n^(-1))/2, -((c\*x^(2\*n))/a)] + 2\*c\*d\*e\*x^n\*Hypergeometric2F1[3, (1 + n)/(2\*n), (3 + n^(-1))/2, -((c\*x^(2\*n))/a)]))/(a^3\*c\*(1 + n))

**Maple [F]** time = 0.081, size = 0, normalized size = 0.

$$\int \frac{(d + ex^n)^2}{(a + cx^{2n})^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d+e*x^n)^2/(a+c*x^(2*n))^3,x)`

[Out] `int((d+e*x^n)^2/(a+c*x^(2*n))^3,x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{2c^2de(3n-1)xx^{3n} + 2acde(5n-1)xx^n + (c^2d^2(4n-1) + ace^2)xx^{2n} + (acd^2(6n-1) - a^2e^2(2n-1))x}{8(a^2c^3n^2x^{4n} + 2a^3c^2n^2x^{2n} + a^4cn^2)} + \int \frac{2(3n^2 - 4n + 1)c^2d^2e^2x^{2n}}{8(a^2c^3n^2x^{4n} + 2a^3c^2n^2x^{2n} + a^4cn^2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d+e*x^n)^2/(a+c*x^(2*n))^3,x, algorithm="maxima")`

[Out] `1/8*(2*c^2*d*e*(3*n - 1)*x*x^(3*n) + 2*a*c*d*e*(5*n - 1)*x*x^n + (c^2*d^2*(4*n - 1) + a*c*e^2)*x*x^(2*n) + (a*c*d^2*(6*n - 1) - a^2*e^2*(2*n - 1))*x)/ (a^2*c^3*n^2*x^(4*n) + 2*a^3*c^2*n^2*x^(2*n) + a^4*c*n^2) + integrate(1/8*(2*(3*n^2 - 4*n + 1)*c*d*e*x^n + (8*n^2 - 6*n + 1)*c*d^2 + a*e^2*(2*n - 1))/ (a^2*c^2*n^2*x^(2*n) + a^3*c*n^2), x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{e^2x^{2n} + 2dex^n + d^2}{c^3x^{6n} + 3ac^2x^{4n} + 3a^2cx^{2n} + a^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d+e*x^n)^2/(a+c*x^(2*n))^3,x, algorithm="fricas")`

[Out] `integral((e^2*x^(2*n) + 2*d*e*x^n + d^2)/(c^3*x^(6*n) + 3*a*c^2*x^(4*n) + 3*a^2*c*x^(2*n) + a^3), x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e\*x\*\*n)\*\*2/(a+c\*x\*\*(2\*n))\*\*3,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^n + d)^2}{(cx^{2n} + a)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e\*x^n)^2/(a+c\*x^(2\*n))^3,x, algorithm="giac")

[Out] integrate((e\*x^n + d)^2/(c\*x^(2\*n) + a)^3, x)

$$3.55 \quad \int \frac{d+ex^n}{(a+cx^{2n})^3} dx$$

**Optimal.** Leaf size=184

$$-\frac{x(d(1-4n)+e(1-3n)x^n)}{8a^2n^2(a+cx^{2n})} + \frac{d(1-4n)(1-2n)x {}_2F_1\left(1, \frac{1}{2n}; \frac{1}{2}\left(2+\frac{1}{n}\right); -\frac{cx^{2n}}{a}\right)}{8a^3n^2} + \frac{e(1-3n)(1-n)x^{n+1} {}_2F_1\left(1, \frac{n+1}{2n}; \frac{1}{2}\left(3+\frac{1}{n}\right); -\frac{cx^{2n}}{a}\right)}{8a^3n^2(n+1)}$$

[Out] (x\*(d + e\*x^n))/(4\*a\*n\*(a + c\*x^(2\*n))^2) - (x\*(d\*(1 - 4\*n) + e\*(1 - 3\*n)\*x^n))/(8\*a^2\*n^2\*(a + c\*x^(2\*n))) + (d\*(1 - 4\*n)\*(1 - 2\*n)\*x\*Hypergeometric2F1[1, 1/(2\*n), (2 + n^(-1))/2, -((c\*x^(2\*n))/a)])/(8\*a^3\*n^2) + (e\*(1 - 3\*n)\*(1 - n)\*x^(1 + n)\*Hypergeometric2F1[1, (1 + n)/(2\*n), (3 + n^(-1))/2, -((c\*x^(2\*n))/a)])/(8\*a^3\*n^2\*(1 + n))

**Rubi [A]** time = 0.10074, antiderivative size = 184, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.21$ , Rules used = {1431, 1418, 245, 364}

$$-\frac{x(d(1-4n)+e(1-3n)x^n)}{8a^2n^2(a+cx^{2n})} + \frac{d(1-4n)(1-2n)x {}_2F_1\left(1, \frac{1}{2n}; \frac{1}{2}\left(2+\frac{1}{n}\right); -\frac{cx^{2n}}{a}\right)}{8a^3n^2} + \frac{e(1-3n)(1-n)x^{n+1} {}_2F_1\left(1, \frac{n+1}{2n}; \frac{1}{2}\left(3+\frac{1}{n}\right); -\frac{cx^{2n}}{a}\right)}{8a^3n^2(n+1)}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x^n)/(a + c\*x^(2\*n))^3, x]

[Out] (x\*(d + e\*x^n))/(4\*a\*n\*(a + c\*x^(2\*n))^2) - (x\*(d\*(1 - 4\*n) + e\*(1 - 3\*n)\*x^n))/(8\*a^2\*n^2\*(a + c\*x^(2\*n))) + (d\*(1 - 4\*n)\*(1 - 2\*n)\*x\*Hypergeometric2F1[1, 1/(2\*n), (2 + n^(-1))/2, -((c\*x^(2\*n))/a)])/(8\*a^3\*n^2) + (e\*(1 - 3\*n)\*(1 - n)\*x^(1 + n)\*Hypergeometric2F1[1, (1 + n)/(2\*n), (3 + n^(-1))/2, -((c\*x^(2\*n))/a)])/(8\*a^3\*n^2\*(1 + n))

**Rule 1431**

Int[((d\_) + (e\_.)\*(x\_)^(n\_))\*((a\_) + (c\_.)\*(x\_)^(n2\_))^(p\_), x\_Symbol] :> -Simp[(x\*(d + e\*x^n)\*(a + c\*x^(2\*n))^(p + 1))/(2\*a\*n\*(p + 1)), x] + Dist[1/(2\*a\*n\*(p + 1)), Int[(d\*(2\*n\*p + 2\*n + 1) + e\*(2\*n\*p + 3\*n + 1)\*x^n)\*(a + c\*x^(2\*n))^(p + 1), x], x] /; FreeQ[{a, c, d, e, n}, x] && EqQ[n2, 2\*n] && ILtQ[p, -1]

Rule 1418

```
Int[((d_) + (e_)*(x_)^(n_))/((a_) + (c_)*(x_)^(n2_)), x_Symbol] := Dist[d
, Int[1/(a + c*x^(2*n)), x], x] + Dist[e, Int[x^n/(a + c*x^(2*n)), x], x] /
; FreeQ[{a, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[c*d^2 + a*e^2, 0] && (Po
sQ[a*c] || !IntegerQ[n])
```

Rule 245

```
Int[((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 364

```
Int[((c_)*(x_)^(m_))*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/ (c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{d + ex^n}{(a + cx^{2n})^3} dx &= \frac{x(d + ex^n)}{4an(a + cx^{2n})^2} - \frac{\int \frac{d(1-4n) + e(1-3n)x^n}{(a + cx^{2n})^2} dx}{4an} \\ &= \frac{x(d + ex^n)}{4an(a + cx^{2n})^2} - \frac{x(d(1-4n) + e(1-3n)x^n)}{8a^2n^2(a + cx^{2n})} + \frac{\int \frac{d(1-4n)(1-2n) + e(1-3n)(1-n)x^n}{a + cx^{2n}} dx}{8a^2n^2} \\ &= \frac{x(d + ex^n)}{4an(a + cx^{2n})^2} - \frac{x(d(1-4n) + e(1-3n)x^n)}{8a^2n^2(a + cx^{2n})} + \frac{(d(1-4n)(1-2n)) \int \frac{1}{a + cx^{2n}} dx}{8a^2n^2} + \frac{(e(1-3n)(1-n))}{8a^2n^2} \\ &= \frac{x(d + ex^n)}{4an(a + cx^{2n})^2} - \frac{x(d(1-4n) + e(1-3n)x^n)}{8a^2n^2(a + cx^{2n})} + \frac{d(1-4n)(1-2n)x {}_2F_1\left(1, \frac{1}{2n}; \frac{1}{2}\left(2 + \frac{1}{n}\right); -\frac{cx^{2n}}{a}\right)}{8a^3n^2} + \end{aligned}$$

**Mathematica [A]** time = 0.047879, size = 83, normalized size = 0.45

$$\frac{dx {}_2F_1\left(3, \frac{1}{2n}; \frac{1}{2}\left(2 + \frac{1}{n}\right); -\frac{cx^{2n}}{a}\right)}{a^3} + \frac{ex^{n+1} {}_2F_1\left(3, \frac{n+1}{2n}; \frac{1}{2}\left(3 + \frac{1}{n}\right); -\frac{cx^{2n}}{a}\right)}{a^3(n+1)}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e\*x^n)/(a + c\*x^(2\*n))^3,x]

[Out] (d\*x\*Hypergeometric2F1[3, 1/(2\*n), (2 + n^(-1))/2, -((c\*x^(2\*n))/a)]/a^3 + (e\*x^(1 + n)\*Hypergeometric2F1[3, (1 + n)/(2\*n), (3 + n^(-1))/2, -((c\*x^(2\*n))/a)])/(a^3\*(1 + n))

**Maple [F]** time = 0.079, size = 0, normalized size = 0.

$$\int \frac{d + ex^n}{(a + cx^{2n})^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d+e\*x^n)/(a+c\*x^(2\*n))^3,x)

[Out] int((d+e\*x^n)/(a+c\*x^(2\*n))^3,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{ce(3n-1)xx^{3n} + cd(4n-1)xx^{2n} + ae(5n-1)xx^n + ad(6n-1)x}{8(a^2c^2n^2x^{4n} + 2a^3cn^2x^{2n} + a^4n^2)} + \int \frac{(3n^2 - 4n + 1)ex^n + (8n^2 - 6n + 1)d}{8(a^2cn^2x^{2n} + a^3n^2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e\*x^n)/(a+c\*x^(2\*n))^3,x, algorithm="maxima")

[Out] 1/8\*(c\*e\*(3\*n - 1)\*x\*x^(3\*n) + c\*d\*(4\*n - 1)\*x\*x^(2\*n) + a\*e\*(5\*n - 1)\*x\*x^n + a\*d\*(6\*n - 1)\*x)/(a^2\*c^2\*n^2\*x^(4\*n) + 2\*a^3\*c\*n^2\*x^(2\*n) + a^4\*n^2) + integrate(1/8\*((3\*n^2 - 4\*n + 1)\*e\*x^n + (8\*n^2 - 6\*n + 1)\*d)/(a^2\*c\*n^2\*x^(2\*n) + a^3\*n^2), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{ex^n + d}{c^3x^{6n} + 3ac^2x^{4n} + 3a^2cx^{2n} + a^3}, x\right)$$



Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d+e*x^n)/(a+c*x^(2*n))^3,x, algorithm="fricas")`

[Out] `integral((e*x^n + d)/(c^3*x^(6*n) + 3*a*c^2*x^(4*n) + 3*a^2*c*x^(2*n) + a^3), x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d+e*x**n)/(a+c*x**(2*n))**3,x)`

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{ex^n + d}{(cx^{2n} + a)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d+e*x^n)/(a+c*x^(2*n))^3,x, algorithm="giac")`

[Out] `integrate((e*x^n + d)/(c*x^(2*n) + a)^3, x)`

$$3.56 \quad \int \frac{1}{(d+ex^n)(a+cx^{2n})^3} dx$$

**Optimal.** Leaf size=582

$$-\frac{ce(1-3n)(1-n)x^{n+1} {}_2F_1\left(1, \frac{n+1}{2n}; \frac{1}{2}\left(3+\frac{1}{n}\right); -\frac{cx^{2n}}{a}\right)}{8a^3n^2(n+1)(ae^2+cd^2)} + \frac{cd(1-4n)(1-2n)x {}_2F_1\left(1, \frac{1}{2n}; \frac{1}{2}\left(2+\frac{1}{n}\right); -\frac{cx^{2n}}{a}\right)}{8a^3n^2(ae^2+cd^2)} + \frac{ce^3(1-n)x^{n+1}}{2a^2n}$$

[Out] (c\*x\*(d - e\*x^n))/(4\*a\*(c\*d^2 + a\*e^2)\*n\*(a + c\*x^(2\*n))^2) + (c\*e^2\*x\*(d - e\*x^n))/(2\*a\*(c\*d^2 + a\*e^2)^2\*n\*(a + c\*x^(2\*n))) - (c\*x\*(d\*(1 - 4\*n) - e\*(1 - 3\*n)\*x^n))/(8\*a^2\*(c\*d^2 + a\*e^2)\*n^2\*(a + c\*x^(2\*n))) + (c\*d\*e^4\*x\*Hypergeometric2F1[1, 1/(2\*n), (2 + n^(-1))/2, -((c\*x^(2\*n))/a)])/(a\*(c\*d^2 + a\*e^2)^3) + (c\*d\*(1 - 4\*n)\*(1 - 2\*n)\*x\*Hypergeometric2F1[1, 1/(2\*n), (2 + n^(-1))/2, -((c\*x^(2\*n))/a)])/(8\*a^3\*(c\*d^2 + a\*e^2)\*n^2) - (c\*d\*e^2\*(1 - 2\*n)\*x\*Hypergeometric2F1[1, 1/(2\*n), (2 + n^(-1))/2, -((c\*x^(2\*n))/a)])/(2\*a^2\*(c\*d^2 + a\*e^2)^2\*n) + (e^6\*x\*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -(e\*x^n)/d])/(d\*(c\*d^2 + a\*e^2)^3) - (c\*e^5\*x^(1 + n)\*Hypergeometric2F1[1, (1 + n)/(2\*n), (3 + n^(-1))/2, -((c\*x^(2\*n))/a)])/(a\*(c\*d^2 + a\*e^2)^3\*(1 + n)) - (c\*e\*(1 - 3\*n)\*(1 - n)\*x^(1 + n)\*Hypergeometric2F1[1, (1 + n)/(2\*n), (3 + n^(-1))/2, -((c\*x^(2\*n))/a)])/(8\*a^3\*(c\*d^2 + a\*e^2)\*n^2\*(1 + n)) + (c\*e^3\*(1 - n)\*x^(1 + n)\*Hypergeometric2F1[1, (1 + n)/(2\*n), (3 + n^(-1))/2, -((c\*x^(2\*n))/a)])/(2\*a^2\*(c\*d^2 + a\*e^2)^2\*n\*(1 + n))

**Rubi [A]** time = 0.416065, antiderivative size = 582, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 5, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$ , Rules used = {1437, 245, 1431, 1418, 364}

$$-\frac{ce(1-3n)(1-n)x^{n+1} {}_2F_1\left(1, \frac{n+1}{2n}; \frac{1}{2}\left(3+\frac{1}{n}\right); -\frac{cx^{2n}}{a}\right)}{8a^3n^2(n+1)(ae^2+cd^2)} + \frac{cd(1-4n)(1-2n)x {}_2F_1\left(1, \frac{1}{2n}; \frac{1}{2}\left(2+\frac{1}{n}\right); -\frac{cx^{2n}}{a}\right)}{8a^3n^2(ae^2+cd^2)} + \frac{ce^3(1-n)x^{n+1}}{2a^2n}$$

Antiderivative was successfully verified.

[In] Int[1/((d + e\*x^n)\*(a + c\*x^(2\*n))^3), x]

[Out] (c\*x\*(d - e\*x^n))/(4\*a\*(c\*d^2 + a\*e^2)\*n\*(a + c\*x^(2\*n))^2) + (c\*e^2\*x\*(d - e\*x^n))/(2\*a\*(c\*d^2 + a\*e^2)^2\*n\*(a + c\*x^(2\*n))) - (c\*x\*(d\*(1 - 4\*n) - e\*(1 - 3\*n)\*x^n))/(8\*a^2\*(c\*d^2 + a\*e^2)\*n^2\*(a + c\*x^(2\*n))) + (c\*d\*e^4\*x\*Hypergeometric2F1[1, 1/(2\*n), (2 + n^(-1))/2, -((c\*x^(2\*n))/a)])/(a\*(c\*d^2 + a\*e^2)^3) + (c\*d\*(1 - 4\*n)\*(1 - 2\*n)\*x\*Hypergeometric2F1[1, 1/(2\*n), (2 + n

$$\begin{aligned} & \text{Int}[\dots] / (8a^3(c^2d + ae^2)n^2) - (cde^2(1 - 2n) \times \text{Hypergeometric2F1}[1, 1/(2n), (2 + n^{-1})/2, -(cx^{2n})/a]) / (2a^2(c^2d + ae^2)^{2n} + e^6 \times \text{Hypergeometric2F1}[1, n^{-1}, 1 + n^{-1}, -(ex^n)/d]) / (d(c^2d + ae^2)^3) - (c^5e^5x^{1+n} \times \text{Hypergeometric2F1}[1, (1+n)/(2n), (3 + n^{-1})/2, -(cx^{2n})/a]) / (a(c^2d + ae^2)^3(1+n)) - (ce(1 - 3n)(1 - n)x^{1+n} \times \text{Hypergeometric2F1}[1, (1+n)/(2n), (3 + n^{-1})/2, -(cx^{2n})/a]) / (8a^3(c^2d + ae^2)n^2(1+n)) + (c^3e^3(1 - n)x^{1+n} \times \text{Hypergeometric2F1}[1, (1+n)/(2n), (3 + n^{-1})/2, -(cx^{2n})/a]) / (2a^2(c^2d + ae^2)^2n(1+n)) \end{aligned}$$

### Rule 1437

```
Int[((d_) + (e_)*(x_)^(n_))^(q_)*((a_) + (c_)*(x_)^(n2_))^(p_), x_Symbol]
  := Int[ExpandIntegrand[(d + e*x^n)^q*(a + c*x^(2*n))^p, x], x] /; FreeQ[{a, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && NeQ[c*d^2 + a*e^2, 0] && ((IntegersQ[p, q] && !IntegerQ[n]) || IGtQ[p, 0] || (IGtQ[q, 0] && !IntegerQ[n]))
```

### Rule 245

```
Int[((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Simp[a^p*x*Hypergeometric2F1[-p, 1/n, 1/n + 1, -(b*x^n)/a], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] || GtQ[a, 0])
```

### Rule 1431

```
Int[((d_) + (e_)*(x_)^(n_))*((a_) + (c_)*(x_)^(n2_))^(p_), x_Symbol] := -Simp[(x*(d + e*x^n)*(a + c*x^(2*n))^(p + 1))/(2*a*n*(p + 1)), x] + Dist[1/(2*a*n*(p + 1)), Int[(d*(2*n*p + 2*n + 1) + e*(2*n*p + 3*n + 1)*x^n)*(a + c*x^(2*n))^(p + 1), x], x] /; FreeQ[{a, c, d, e, n}, x] && EqQ[n2, 2*n] && ILtQ[p, -1]
```

### Rule 1418

```
Int[((d_) + (e_)*(x_)^(n_))/((a_) + (c_)*(x_)^(n2_)), x_Symbol] := Dist[d, Int[1/(a + c*x^(2*n)), x], x] + Dist[e, Int[x^n/(a + c*x^(2*n)), x], x] /; FreeQ[{a, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[c*d^2 + a*e^2, 0] && (PosQ[a*c] || !IntegerQ[n])
```

### Rule 364

```
Int[((c_)*(x_)^(m_))*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Simp[(a^p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -(b*x^n)/a]) / (c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
```

Q[p, 0] || GtQ[a, 0])

### Rubi steps

$$\begin{aligned}
 \int \frac{1}{(d + ex^n)(a + cx^{2n})^3} dx &= \int \left( \frac{e^6}{(cd^2 + ae^2)^3 (d + ex^n)} - \frac{c(-d + ex^n)}{(cd^2 + ae^2)(a + cx^{2n})^3} - \frac{ce^2(-d + ex^n)}{(cd^2 + ae^2)^2 (a + cx^{2n})^2} - \frac{ce^4}{(cd^2 + ae^2)^3} \right) dx \\
 &= -\frac{(ce^4) \int \frac{-d+ex^n}{a+cx^{2n}} dx}{(cd^2 + ae^2)^3} + \frac{e^6 \int \frac{1}{d+ex^n} dx}{(cd^2 + ae^2)^3} - \frac{(ce^2) \int \frac{-d+ex^n}{(a+cx^{2n})^2} dx}{(cd^2 + ae^2)^2} - \frac{c \int \frac{-d+ex^n}{(a+cx^{2n})^3} dx}{cd^2 + ae^2} \\
 &= \frac{cx(d - ex^n)}{4a(cd^2 + ae^2)n(a + cx^{2n})^2} + \frac{ce^2x(d - ex^n)}{2a(cd^2 + ae^2)^2n(a + cx^{2n})} + \frac{e^6x {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{ex^n}{d}\right)}{d(cd^2 + ae^2)^3} + \dots \\
 &= \frac{cx(d - ex^n)}{4a(cd^2 + ae^2)n(a + cx^{2n})^2} + \frac{ce^2x(d - ex^n)}{2a(cd^2 + ae^2)^2n(a + cx^{2n})} - \frac{cx(d(1 - 4n) - e(1 - 3n)x^n)}{8a^2(cd^2 + ae^2)n^2(a + cx^{2n})} \\
 &= \frac{cx(d - ex^n)}{4a(cd^2 + ae^2)n(a + cx^{2n})^2} + \frac{ce^2x(d - ex^n)}{2a(cd^2 + ae^2)^2n(a + cx^{2n})} - \frac{cx(d(1 - 4n) - e(1 - 3n)x^n)}{8a^2(cd^2 + ae^2)n^2(a + cx^{2n})} \\
 &= \frac{cx(d - ex^n)}{4a(cd^2 + ae^2)n(a + cx^{2n})^2} + \frac{ce^2x(d - ex^n)}{2a(cd^2 + ae^2)^2n(a + cx^{2n})} - \frac{cx(d(1 - 4n) - e(1 - 3n)x^n)}{8a^2(cd^2 + ae^2)n^2(a + cx^{2n})}
 \end{aligned}$$

**Mathematica [A]** time = 0.464, size = 346, normalized size = 0.59

$$x \left( -\frac{ce^3x^n(ae^2+cd^2) {}_2F_1\left(2, \frac{n+1}{2n}; \frac{1}{2}\left(3+\frac{1}{n}\right); -\frac{cx^{2n}}{a}\right)}{a^2(n+1)} - \frac{cex^n(ae^2+cd^2)^2 {}_2F_1\left(3, \frac{n+1}{2n}; \frac{1}{2}\left(3+\frac{1}{n}\right); -\frac{cx^{2n}}{a}\right)}{a^3(n+1)} + \frac{cd^2(ae^2+cd^2) {}_2F_1\left(2, \frac{1}{2n}; \frac{1}{2}\left(2+\frac{1}{n}\right); -\frac{cx^{2n}}{a}\right)}{a^2} + \frac{cd(ae^2+cd^2)^2}{a^2} \right) \frac{1}{(ae^2 + cd^2)^3}$$

Antiderivative was successfully verified.

[In] Integrate[1/((d + e\*x^n)\*(a + c\*x^(2\*n)))^3, x]

[Out] (x\*((c\*d\*e^4\*Hypergeometric2F1[1, 1/(2\*n), (2 + n^(-1))/2, -((c\*x^(2\*n))/a)])/a + (e^6\*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((e\*x^n)/d)])/d - (c\*e^5\*x^n\*Hypergeometric2F1[1, (1 + n)/(2\*n), (3 + n^(-1))/2, -((c\*x^(2\*n))/a)]))/(a\*(1 + n)) + (c\*d\*e^2\*(c\*d^2 + a\*e^2)\*Hypergeometric2F1[2, 1/(2\*n), (2

+ n<sup>(-1)</sup>)/2, -((c\*x<sup>(2\*n)</sup>)/a)]/a<sup>2</sup> - (c\*e<sup>3</sup>(c\*d<sup>2</sup> + a\*e<sup>2</sup>)\*x<sup>n</sup>\*Hypergeometric2F1[2, (1 + n)/(2\*n), (3 + n<sup>(-1)</sup>)/2, -((c\*x<sup>(2\*n)</sup>)/a)]/(a<sup>2</sup>\*(1 + n)) + (c\*d\*(c\*d<sup>2</sup> + a\*e<sup>2</sup>)<sup>2</sup>\*Hypergeometric2F1[3, 1/(2\*n), (2 + n<sup>(-1)</sup>)/2, -((c\*x<sup>(2\*n)</sup>)/a)]/a<sup>3</sup> - (c\*e\*(c\*d<sup>2</sup> + a\*e<sup>2</sup>)<sup>2</sup>\*x<sup>n</sup>\*Hypergeometric2F1[3, (1 + n)/(2\*n), (3 + n<sup>(-1)</sup>)/2, -((c\*x<sup>(2\*n)</sup>)/a)]/(a<sup>3</sup>\*(1 + n)))/(c\*d<sup>2</sup> + a\*e<sup>2</sup>)<sup>3</sup>

**Maple [F]** time = 0.17, size = 0, normalized size = 0.

$$\int \frac{1}{(d + ex^n)(a + cx^{2n})^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(d+e\*x^n)/(a+c\*x^(2\*n))^3,x)

[Out] int(1/(d+e\*x^n)/(a+c\*x^(2\*n))^3,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$e^6 \int \frac{1}{c^3 d^7 + 3 a c^2 d^5 e^2 + 3 a^2 c d^3 e^4 + a^3 d e^6 + (c^3 d^6 e + 3 a c^2 d^4 e^3 + 3 a^2 c d^2 e^5 + a^3 e^7) x^n} dx - \frac{(a c^2 e^3 (7 n - 1) + c^3 d^2 e (3 n - 1)) x^{3 n} - (a c^2 d e^2 (8 n - 1) + c^3 d^3 (4 n - 1)) x^{2 n} + (a^2 c e^3 (9 n - 1) + a c^2 d^2 e (5 n - 1)) x^n - (a^2 c d e^2 (10 n - 1) + a c^2 d^3 (6 n - 1)) x}{8 (a^4 c^2 d^4 n^2 + \dots)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d+e\*x^n)/(a+c\*x^(2\*n))^3,x, algorithm="maxima")

[Out] e<sup>6</sup>\*integrate(1/(c<sup>3</sup>\*d<sup>7</sup> + 3\*a\*c<sup>2</sup>\*d<sup>5</sup>\*e<sup>2</sup> + 3\*a<sup>2</sup>\*c\*d<sup>3</sup>\*e<sup>4</sup> + a<sup>3</sup>\*d\*e<sup>6</sup> + (c<sup>3</sup>\*d<sup>6</sup>\*e + 3\*a\*c<sup>2</sup>\*d<sup>4</sup>\*e<sup>3</sup> + 3\*a<sup>2</sup>\*c\*d<sup>2</sup>\*e<sup>5</sup> + a<sup>3</sup>\*e<sup>7</sup>)\*x<sup>n</sup>), x) - 1/8\*((a\*c<sup>2</sup>\*e<sup>3</sup>\*(7\*n - 1) + c<sup>3</sup>\*d<sup>2</sup>\*e\*(3\*n - 1))\*x\*x<sup>(3\*n)</sup> - (a\*c<sup>2</sup>\*d\*e<sup>2</sup>\*(8\*n - 1) + c<sup>3</sup>\*d<sup>3</sup>\*(4\*n - 1))\*x\*x<sup>(2\*n)</sup> + (a<sup>2</sup>\*c\*e<sup>3</sup>\*(9\*n - 1) + a\*c<sup>2</sup>\*d<sup>2</sup>\*e\*(5\*n - 1))\*x\*x<sup>n</sup> - (a<sup>2</sup>\*c\*d\*e<sup>2</sup>\*(10\*n - 1) + a\*c<sup>2</sup>\*d<sup>3</sup>\*(6\*n - 1))\*x)/(a<sup>4</sup>\*c<sup>2</sup>\*d<sup>4</sup>\*n<sup>2</sup> + 2\*a<sup>5</sup>\*c\*d<sup>2</sup>\*e<sup>2</sup>\*n<sup>2</sup> + a<sup>6</sup>\*e<sup>4</sup>\*n<sup>2</sup> + (a<sup>2</sup>\*c<sup>4</sup>\*d<sup>4</sup>\*n<sup>2</sup> + 2\*a<sup>3</sup>\*c<sup>3</sup>\*d<sup>2</sup>\*e<sup>2</sup>\*n<sup>2</sup> + a<sup>4</sup>\*c<sup>2</sup>\*e<sup>4</sup>\*n<sup>2</sup>)\*x<sup>(4\*n)</sup> + 2\*(a<sup>3</sup>\*c<sup>3</sup>\*d<sup>4</sup>\*n<sup>2</sup> + 2\*a<sup>4</sup>\*c<sup>2</sup>\*d<sup>2</sup>\*e<sup>2</sup>\*n<sup>2</sup> + a<sup>5</sup>\*c\*e<sup>4</sup>\*n<sup>2</sup>)\*x<sup>(2\*n)</sup>) - integrate(-1/8\*((8\*n<sup>2</sup> - 6\*n + 1)\*c<sup>3</sup>\*d<sup>5</sup> + 2\*(12\*n<sup>2</sup> - 8\*n + 1)\*a\*c<sup>2</sup>\*d<sup>3</sup>\*e<sup>2</sup> + (24\*n<sup>2</sup> - 10\*n + 1)\*a<sup>2</sup>\*c\*d\*e<sup>4</sup> - ((3\*n<sup>2</sup> - 4\*n + 1)\*c<sup>3</sup>\*d<sup>4</sup>\*e + 2\*(5\*n<sup>2</sup> - 6\*n + 1)\*a\*c<sup>2</sup>\*d<sup>2</sup>\*e<sup>3</sup> + (15\*n<sup>2</sup> - 8\*n + 1)\*a<sup>2</sup>\*c\*e<sup>5</sup>)\*x<sup>n</sup>)/(a<sup>3</sup>\*c<sup>3</sup>\*d<sup>6</sup>\*n<sup>2</sup> + 3\*a<sup>4</sup>\*c<sup>2</sup>\*d<sup>4</sup>\*e<sup>2</sup>\*n<sup>2</sup> + 3\*a<sup>5</sup>\*c\*d<sup>2</sup>\*e<sup>4</sup>\*n<sup>2</sup> + a<sup>6</sup>\*e<sup>6</sup>\*n<sup>2</sup> + (a<sup>2</sup>\*c<sup>4</sup>\*d<sup>6</sup>\*n<sup>2</sup> + 3\*a<sup>3</sup>\*c<sup>3</sup>\*d<sup>4</sup>\*e<sup>2</sup>\*n<sup>2</sup> +

$3*a^4*c^2*d^2*e^4*n^2 + a^5*c*e^6*n^2)*x^(2*n)), x)$

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{a^3ex^n + a^3d + (c^3ex^n + c^3d)x^{6n} + 3(ac^2ex^n + ac^2d)x^{4n} + 3(a^2cex^n + a^2cd)x^{2n}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d+e\*x^n)/(a+c\*x^(2\*n))^3,x, algorithm="fricas")

[Out] integral(1/(a^3\*e\*x^n + a^3\*d + (c^3\*e\*x^n + c^3\*d)\*x^(6\*n) + 3\*(a\*c^2\*e\*x^n + a\*c^2\*d)\*x^(4\*n) + 3\*(a^2\*c\*e\*x^n + a^2\*c\*d)\*x^(2\*n)), x)

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d+e\*x\*\*n)/(a+c\*x\*\*(2\*n))\*\*3,x)

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(cx^{2n} + a)^3 (ex^n + d)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d+e\*x^n)/(a+c\*x^(2\*n))^3,x, algorithm="giac")

[Out] integrate(1/((c\*x^(2\*n) + a)^3\*(e\*x^n + d)), x)

$$3.57 \quad \int \frac{1}{(d+ex^n)^2(a+cx^{2n})^3} dx$$

**Optimal.** Leaf size=701

$$\frac{c^2de(1-3n)(1-n)x^{n+1} {}_2F_1\left(1, \frac{n+1}{2n}; \frac{1}{2}\left(3+\frac{1}{n}\right); -\frac{cx^{2n}}{a}\right)}{4a^3n^2(n+1)(ae^2+cd^2)^2} + \frac{2c^2de^3(1-n)x^{n+1} {}_2F_1\left(1, \frac{n+1}{2n}; \frac{1}{2}\left(3+\frac{1}{n}\right); -\frac{cx^{2n}}{a}\right)}{a^2n(n+1)(ae^2+cd^2)^3} + \frac{c(1-4n)(1-n)x^{n+1} {}_2F_1\left(1, \frac{n+1}{2n}; \frac{1}{2}\left(3+\frac{1}{n}\right); -\frac{cx^{2n}}{a}\right)}{a^2n(n+1)(ae^2+cd^2)^3}$$

```
[Out] (c*x*(c*d^2 - a*e^2 - 2*c*d*e*x^n))/(4*a*(c*d^2 + a*e^2)^2*n*(a + c*x^(2*n))^2) + (c*e^2*x*(3*c*d^2 - a*e^2 - 4*c*d*e*x^n))/(2*a*(c*d^2 + a*e^2)^3*n*(a + c*x^(2*n))) - (c*x*((c*d^2 - a*e^2)*(1 - 4*n) - 2*c*d*e*(1 - 3*n)*x^n))/(8*a^2*(c*d^2 + a*e^2)^2*n^2*(a + c*x^(2*n))) + (c*e^4*(5*c*d^2 - a*e^2)*x*Hypergeometric2F1[1, 1/(2*n), (2 + n^(-1))/2, -((c*x^(2*n))/a)])/(a*(c*d^2 + a*e^2)^4) + (c*(c*d^2 - a*e^2)*(1 - 4*n)*(1 - 2*n)*x*Hypergeometric2F1[1, 1/(2*n), (2 + n^(-1))/2, -((c*x^(2*n))/a)])/(8*a^3*(c*d^2 + a*e^2)^2*n^2) - (c*e^2*(3*c*d^2 - a*e^2)*(1 - 2*n)*x*Hypergeometric2F1[1, 1/(2*n), (2 + n^(-1))/2, -((c*x^(2*n))/a)])/(2*a^2*(c*d^2 + a*e^2)^3*n) + (6*c*e^6*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((e*x^n)/d)])/(c*d^2 + a*e^2)^4 - (6*c^2*d*e^5*x^(1 + n)*Hypergeometric2F1[1, (1 + n)/(2*n), (3 + n^(-1))/2, -((c*x^(2*n))/a)])/(a*(c*d^2 + a*e^2)^4*(1 + n)) - (c^2*d*e*(1 - 3*n)*(1 - n)*x^(1 + n)*Hypergeometric2F1[1, (1 + n)/(2*n), (3 + n^(-1))/2, -((c*x^(2*n))/a)])/(4*a^3*(c*d^2 + a*e^2)^2*n^2*(1 + n)) + (2*c^2*d*e^3*(1 - n)*x^(1 + n)*Hypergeometric2F1[1, (1 + n)/(2*n), (3 + n^(-1))/2, -((c*x^(2*n))/a)])/(a^2*(c*d^2 + a*e^2)^3*n*(1 + n)) + (e^6*x*Hypergeometric2F1[2, n^(-1), 1 + n^(-1), -((e*x^n)/d)])/(d^2*(c*d^2 + a*e^2)^3)
```

**Rubi [A]** time = 0.692386, antiderivative size = 701, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 5, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$ , Rules used = {1437, 245, 1431, 1418, 364}

$$\frac{c^2de(1-3n)(1-n)x^{n+1} {}_2F_1\left(1, \frac{n+1}{2n}; \frac{1}{2}\left(3+\frac{1}{n}\right); -\frac{cx^{2n}}{a}\right)}{4a^3n^2(n+1)(ae^2+cd^2)^2} + \frac{2c^2de^3(1-n)x^{n+1} {}_2F_1\left(1, \frac{n+1}{2n}; \frac{1}{2}\left(3+\frac{1}{n}\right); -\frac{cx^{2n}}{a}\right)}{a^2n(n+1)(ae^2+cd^2)^3} + \frac{c(1-4n)(1-n)x^{n+1} {}_2F_1\left(1, \frac{n+1}{2n}; \frac{1}{2}\left(3+\frac{1}{n}\right); -\frac{cx^{2n}}{a}\right)}{a^2n(n+1)(ae^2+cd^2)^3}$$

Antiderivative was successfully verified.

[In] Int[1/((d + e\*x^n)^2\*(a + c\*x^(2\*n))^3), x]

```
[Out] (c*x*(c*d^2 - a*e^2 - 2*c*d*e*x^n))/(4*a*(c*d^2 + a*e^2)^2*n*(a + c*x^(2*n))^2) + (c*e^2*x*(3*c*d^2 - a*e^2 - 4*c*d*e*x^n))/(2*a*(c*d^2 + a*e^2)^3*n*(a + c*x^(2*n))) - (c*x*((c*d^2 - a*e^2)*(1 - 4*n) - 2*c*d*e*(1 - 3*n)*x^n))/(8*a^2*(c*d^2 + a*e^2)^2*n^2*(a + c*x^(2*n))) + (c*e^4*(5*c*d^2 - a*e^2)*x*Hypergeometric2F1[1, 1/(2*n), (2 + n^(-1))/2, -((c*x^(2*n))/a)])/(a*(c*d^2 + a*e^2)^4) + (c*(c*d^2 - a*e^2)*(1 - 4*n)*(1 - 2*n)*x*Hypergeometric2F1[1, 1/(2*n), (2 + n^(-1))/2, -((c*x^(2*n))/a)])/(8*a^3*(c*d^2 + a*e^2)^2*n^2) - (c*e^2*(3*c*d^2 - a*e^2)*(1 - 2*n)*x*Hypergeometric2F1[1, 1/(2*n), (2 + n^(-1))/2, -((c*x^(2*n))/a)])/(2*a^2*(c*d^2 + a*e^2)^3*n) + (6*c*e^6*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((e*x^n)/d)])/(c*d^2 + a*e^2)^4 - (6*c^2*d*e^5*x^(1 + n)*Hypergeometric2F1[1, (1 + n)/(2*n), (3 + n^(-1))/2, -((c*x^(2*n))/a)])/(a*(c*d^2 + a*e^2)^4*(1 + n)) - (c^2*d*e*(1 - 3*n)*(1 - n)*x^(1 + n)*Hypergeometric2F1[1, (1 + n)/(2*n), (3 + n^(-1))/2, -((c*x^(2*n))/a)])/(4*a^3*(c*d^2 + a*e^2)^2*n^2*(1 + n)) + (2*c^2*d*e^3*(1 - n)*x^(1 + n)*Hypergeometric2F1[1, (1 + n)/(2*n), (3 + n^(-1))/2, -((c*x^(2*n))/a)])/(a^2*(c*d^2 + a*e^2)^3*n*(1 + n)) + (e^6*x*Hypergeometric2F1[2, n^(-1), 1 + n^(-1), -((e*x^n)/d)])/(d^2*(c*d^2 + a*e^2)^3)
```

$$\begin{aligned}
& a + c*x^{(2*n)}) - (c*x*((c*d^2 - a*e^2)*(1 - 4*n) - 2*c*d*e*(1 - 3*n)*x^n) \\
& / (8*a^2*(c*d^2 + a*e^2)^2*n^2*(a + c*x^{(2*n)})) + (c*e^4*(5*c*d^2 - a*e^2)*x \\
& *Hypergeometric2F1[1, 1/(2*n), (2 + n^{(-1)})/2, -((c*x^{(2*n)})/a)] / (a*(c*d^2 \\
& + a*e^2)^4) + (c*(c*d^2 - a*e^2)*(1 - 4*n)*(1 - 2*n)*x*Hypergeometric2F1[1 \\
& , 1/(2*n), (2 + n^{(-1)})/2, -((c*x^{(2*n)})/a)] / (8*a^3*(c*d^2 + a*e^2)^2*n^2) \\
& - (c*e^2*(3*c*d^2 - a*e^2)*(1 - 2*n)*x*Hypergeometric2F1[1, 1/(2*n), (2 + \\
& n^{(-1)})/2, -((c*x^{(2*n)})/a)] / (2*a^2*(c*d^2 + a*e^2)^3*n) + (6*c*e^6*x*Hype \\
& rgeometric2F1[1, n^{(-1)}, 1 + n^{(-1)}, -((e*x^n)/d)] / (c*d^2 + a*e^2)^4 - (6* \\
& c^2*d*e^5*x^{(1 + n)}*Hypergeometric2F1[1, (1 + n)/(2*n), (3 + n^{(-1)})/2, -(( \\
& c*x^{(2*n)})/a)] / (a*(c*d^2 + a*e^2)^4*(1 + n)) - (c^2*d*e*(1 - 3*n)*(1 - n)* \\
& x^{(1 + n)}*Hypergeometric2F1[1, (1 + n)/(2*n), (3 + n^{(-1)})/2, -((c*x^{(2*n)}) \\
& /a)] / (4*a^3*(c*d^2 + a*e^2)^2*n^2*(1 + n)) + (2*c^2*d*e^3*(1 - n)*x^{(1 + n)} \\
& )*Hypergeometric2F1[1, (1 + n)/(2*n), (3 + n^{(-1)})/2, -((c*x^{(2*n)})/a)] / (a \\
& ^2*(c*d^2 + a*e^2)^3*n*(1 + n)) + (e^6*x*Hypergeometric2F1[2, n^{(-1)}, 1 + n \\
& ^{(-1)}, -((e*x^n)/d)] / (d^2*(c*d^2 + a*e^2)^3)
\end{aligned}$$

### Rule 1437

```

Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x^n)^q*(a + c*x^(2*n))^p, x], x] /; FreeQ[{a
, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && NeQ[c*d^2 + a*e^2, 0] && ((Integ
ersQ[p, q] && !IntegerQ[n]) || IGtQ[p, 0] || (IGtQ[q, 0] && !IntegerQ[n])
)

```

### Rule 245

```

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])

```

### Rule 1431

```

Int[((d_) + (e_.)*(x_)^(n_))*((a_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol] := -
Simp[(x*(d + e*x^n)*(a + c*x^(2*n))^(p + 1))/(2*a*n*(p + 1)), x] + Dist[1/(
2*a*n*(p + 1)), Int[(d*(2*n*p + 2*n + 1) + e*(2*n*p + 3*n + 1)*x^n)*(a + c*
x^(2*n))^(p + 1), x], x] /; FreeQ[{a, c, d, e, n}, x] && EqQ[n2, 2*n] && IL
tQ[p, -1]

```

### Rule 1418

```

Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (c_.)*(x_)^(n2_)), x_Symbol] := Dist[d
, Int[1/(a + c*x^(2*n)), x], x] + Dist[e, Int[x^n/(a + c*x^(2*n)), x], x] /
; FreeQ[{a, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[c*d^2 + a*e^2, 0] && (Po

```



sQ[a\*c] || !IntegerQ[n])

### Rule 364

Int[((c\_.)\*(x\_))^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[(a^p\*(c\*x)^(m + 1)\*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -(b\*x^n)/a])/((c\*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])

### Rubi steps

$$\begin{aligned}
 \int \frac{1}{(d + ex^n)^2 (a + cx^{2n})^3} dx &= \int \left( \frac{e^6}{(cd^2 + ae^2)^3 (d + ex^n)^2} + \frac{6cde^6}{(cd^2 + ae^2)^4 (d + ex^n)} - \frac{c(-cd^2 + ae^2 + 2cdex^n)}{(cd^2 + ae^2)^2 (a + cx^{2n})^3} - \frac{ce^2(-)}{(cd^2 + ae^2)^2} \right) dx \\
 &= -\frac{(ce^4) \int \frac{-5cd^2 + ae^2 + 6cdex^n}{a + cx^{2n}} dx}{(cd^2 + ae^2)^4} + \frac{(6cde^6) \int \frac{1}{d + ex^n} dx}{(cd^2 + ae^2)^4} - \frac{(ce^2) \int \frac{-3cd^2 + ae^2 + 4cdex^n}{(a + cx^{2n})^2} dx}{(cd^2 + ae^2)^3} + \frac{e^6 \int \frac{1}{d + ex^n} dx}{(cd^2 + ae^2)^2} \\
 &= \frac{cx(cd^2 - ae^2 - 2cdex^n)}{4a(cd^2 + ae^2)^2 n(a + cx^{2n})^2} + \frac{ce^2x(3cd^2 - ae^2 - 4cdex^n)}{2a(cd^2 + ae^2)^3 n(a + cx^{2n})} + \frac{6ce^6x {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{cx^n}{a}\right)}{(cd^2 + ae^2)^4} \\
 &= \frac{cx(cd^2 - ae^2 - 2cdex^n)}{4a(cd^2 + ae^2)^2 n(a + cx^{2n})^2} + \frac{ce^2x(3cd^2 - ae^2 - 4cdex^n)}{2a(cd^2 + ae^2)^3 n(a + cx^{2n})} - \frac{cx((cd^2 - ae^2)(1 - 4n) - ce^2x)}{8a^2(cd^2 + ae^2)^2 n^2} \\
 &= \frac{cx(cd^2 - ae^2 - 2cdex^n)}{4a(cd^2 + ae^2)^2 n(a + cx^{2n})^2} + \frac{ce^2x(3cd^2 - ae^2 - 4cdex^n)}{2a(cd^2 + ae^2)^3 n(a + cx^{2n})} - \frac{cx((cd^2 - ae^2)(1 - 4n) - ce^2x)}{8a^2(cd^2 + ae^2)^2 n^2} \\
 &= \frac{cx(cd^2 - ae^2 - 2cdex^n)}{4a(cd^2 + ae^2)^2 n(a + cx^{2n})^2} + \frac{ce^2x(3cd^2 - ae^2 - 4cdex^n)}{2a(cd^2 + ae^2)^3 n(a + cx^{2n})} - \frac{cx((cd^2 - ae^2)(1 - 4n) - ce^2x)}{8a^2(cd^2 + ae^2)^2 n^2}
 \end{aligned}$$

**Mathematica [A]** time = 0.742218, size = 426, normalized size = 0.61

$$x \left( -\frac{4c^2de^3x^n(ae^2 + cd^2) {}_2F_1\left(2, \frac{n+1}{2n}; \frac{1}{2}\left(3 + \frac{1}{n}\right); -\frac{cx^{2n}}{a}\right)}{a^{2(n+1)}} - \frac{2c^2dex^n(ae^2 + cd^2)^2 {}_2F_1\left(3, \frac{n+1}{2n}; \frac{1}{2}\left(3 + \frac{1}{n}\right); -\frac{cx^{2n}}{a}\right)}{a^{3(n+1)}} + \frac{ce^2(3cd^2 - ae^2)(ae^2 + cd^2) {}_2F_1\left(2, \frac{1}{2n}; \frac{1}{2}\left(2 + \frac{1}{n}\right); -\frac{cx^{2n}}{a}\right)}{a^2} \right)$$

Antiderivative was successfully verified.

[In] Integrate[1/((d + e\*x^n)^2\*(a + c\*x^(2\*n))^3),x]

[Out] (x\*((c\*e^4\*(5\*c\*d^2 - a\*e^2)\*Hypergeometric2F1[1, 1/(2\*n), (2 + n^(-1))/2, -((c\*x^(2\*n))/a)])/a + 6\*c\*e^6\*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((e\*x^n)/d)] - (6\*c^2\*d\*e^5\*x^n\*Hypergeometric2F1[1, (1 + n)/(2\*n), (3 + n^(-1))/2, -((c\*x^(2\*n))/a)])/(a\*(1 + n)) + (c\*e^2\*(3\*c\*d^2 - a\*e^2)\*(c\*d^2 + a\*e^2)\*Hypergeometric2F1[2, 1/(2\*n), (2 + n^(-1))/2, -((c\*x^(2\*n))/a)])/a^2 + (e^6\*(c\*d^2 + a\*e^2)\*Hypergeometric2F1[2, n^(-1), 1 + n^(-1), -((e\*x^n)/d)])/d^2 - (4\*c^2\*d\*e^3\*(c\*d^2 + a\*e^2)\*x^n\*Hypergeometric2F1[2, (1 + n)/(2\*n), (3 + n^(-1))/2, -((c\*x^(2\*n))/a)])/(a^2\*(1 + n)) + (c\*(c\*d^2 - a\*e^2)\*(c\*d^2 + a\*e^2)^2\*Hypergeometric2F1[3, 1/(2\*n), (2 + n^(-1))/2, -((c\*x^(2\*n))/a)])/a^3 - (2\*c^2\*d\*e\*(c\*d^2 + a\*e^2)^2\*x^n\*Hypergeometric2F1[3, (1 + n)/(2\*n), (3 + n^(-1))/2, -((c\*x^(2\*n))/a)])/(a^3\*(1 + n)))/(c\*d^2 + a\*e^2)^4

**Maple [F]** time = 0.197, size = 0, normalized size = 0.

$$\int \frac{1}{(d + ex^n)^2 (a + cx^{2n})^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(d+e\*x^n)^2/(a+c\*x^(2\*n))^3,x)

[Out] int(1/(d+e\*x^n)^2/(a+c\*x^(2\*n))^3,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d+e\*x^n)^2/(a+c\*x^(2\*n))^3,x, algorithm="maxima")

[Out] (c\*d^2\*e^6\*(7\*n - 1) + a\*e^8\*(n - 1))\*integrate(1/(c^4\*d^10\*n + 4\*a\*c^3\*d^8\*e^2\*n + 6\*a^2\*c^2\*d^6\*e^4\*n + 4\*a^3\*c\*d^4\*e^6\*n + a^4\*d^2\*e^8\*n + (c^4\*d^9\*e\*n + 4\*a\*c^3\*d^7\*e^3\*n + 6\*a^2\*c^2\*d^5\*e^5\*n + 4\*a^3\*c\*d^3\*e^7\*n + a^4\*d\*e^9\*n)\*x^n), x) - 1/8\*(2\*(a\*c^3\*d^2\*e^4\*(11\*n - 1) + c^4\*d^4\*e^2\*(3\*n - 1) - 4\*a^2\*c^2\*e^6\*n)\*x\*x^(4\*n) + (a^2\*c^2\*d\*e^5\*(8\*n - 1) + 2\*a\*c^3\*d^3\*e^3\*(5\*n - 1) + c^4\*d^5\*e\*(2\*n - 1))\*x\*x^(3\*n) + (a^2\*c^2\*d^2\*e^4\*(34\*n - 3) - c

$$\begin{aligned} &^4d^6(4n - 1) - 2ac^3d^4e^2(n + 1) - 16a^3c^3e^6n) * x^{(2n)} + (a \\ &^3c^3d^5e^5(10n - 1) + 2a^2c^2d^3e^3(7n - 1) + ac^3d^5e^5(4n - 1) \\ &) * x^n + (a^3c^3d^2e^4(10n - 1) - ac^3d^6(6n - 1) - 12a^2c^2d^4e^2n \\ &- 8a^4e^6n) * x) / (a^4c^3d^8n^2 + 3a^5c^2d^6e^2n^2 + 3a^6c^3d^4e^4n^2 \\ &+ a^7d^2e^6n^2 + (a^2c^5d^7e^5n^2 + 3a^3c^4d^5e^3n^2 \\ &+ 3a^4c^3d^3e^5n^2 + a^5c^2d^2e^7n^2) * x^{(5n)} + (a^2c^5d^8n^2 + 3 \\ &a^3c^4d^6e^2n^2 + 3a^4c^3d^4e^4n^2 + a^5c^2d^2e^6n^2) * x^{(4n)} \\ &+ 2(a^3c^4d^7e^5n^2 + 3a^4c^3d^5e^3n^2 + 3a^5c^2d^3e^5n^2 + a \\ &^6c^3d^4e^7n^2) * x^{(3n)} + 2(a^3c^4d^8n^2 + 3a^4c^3d^6e^2n^2 + 3a^5 \\ &c^2d^4e^4n^2 + a^6c^3d^2e^6n^2) * x^{(2n)} + (a^4c^3d^7e^5n^2 + 3a^5 \\ &c^2d^5e^3n^2 + 3a^6c^3d^3e^5n^2 + a^7d^2e^7n^2) * x^n) - \text{integrate}(-1 \\ &/ 8 * ((8n^2 - 6n + 1) * c^4d^6 + (32n^2 - 18n + 1) * ac^3d^4e^2 + (48n^2 \\ &- 2n - 1) * a^2c^2d^2e^4 - (24n^2 - 10n + 1) * a^3c^3e^6 - 2 * ((3n^2 - 4 \\ &n + 1) * c^4d^5e + 2 * (7n^2 - 8n + 1) * ac^3d^3e^3 + (35n^2 - 12n + 1) \\ &* a^2c^2d^2e^5) * x^n) / (a^3c^4d^8n^2 + 4a^4c^3d^6e^2n^2 + 6a^5c^2d^4e^4n^2 \\ &+ 4a^6c^3d^2e^6n^2 + a^7e^8n^2 + (a^2c^5d^8n^2 + 4a^3c^4d^6e^2n^2 \\ &+ 6a^4c^3d^4e^4n^2 + 4a^5c^2d^2e^6n^2 + a^6c^3e^8n^2) * x^{(2n)}), x) \end{aligned}$$

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{1}{a^3e^2x^{2n} + 2a^3dex^n + a^3d^2 + (c^3e^2x^{2n} + 2c^3dex^n + c^3d^2)x^{6n} + 3(ac^2e^2x^{2n} + 2ac^2dex^n + ac^2d^2)x^{4n} + 3(a^2ce^2x^{2n} + 2a^2dex^n + a^2d^2)x^{2n}}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d+e\*x^n)^2/(a+c\*x^(2\*n))^3,x, algorithm="fricas")

[Out] integral(1/(a^3e^2\*x^(2\*n) + 2\*a^3\*d\*e\*x^n + a^3\*d^2 + (c^3e^2\*x^(2\*n) + 2\*c^3\*d\*e\*x^n + c^3\*d^2)\*x^(6\*n) + 3\*(a\*c^2\*e^2\*x^(2\*n) + 2\*a\*c^2\*d\*e\*x^n + a\*c^2\*d^2)\*x^(4\*n) + 3\*(a^2\*c\*e^2\*x^(2\*n) + 2\*a^2\*c\*d\*e\*x^n + a^2\*c\*d^2)\*x^(2\*n)), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d+e\*x\*\*n)\*\*2/(a+c\*x\*\*(2\*n))\*\*3,x)

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(cx^{2n} + a)^3 (ex^n + d)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d+e\*x^n)^2/(a+c\*x^(2\*n))^3,x, algorithm="giac")

[Out] integrate(1/((c\*x^(2\*n) + a)^3\*(e\*x^n + d)^2), x)

$$3.58 \quad \int \frac{1}{(d+ex^n)\sqrt{a+cx^{2n}}} dx$$

**Optimal.** Leaf size=171

$$\frac{x\sqrt{\frac{cx^{2n}}{a}} + {}_1F_1\left(\frac{1}{2n}; \frac{1}{2}, 1; \frac{1}{2}\left(2 + \frac{1}{n}\right); -\frac{cx^{2n}}{a}, \frac{e^2x^{2n}}{d^2}\right)}{d\sqrt{a+cx^{2n}}} - \frac{ex^{n+1}\sqrt{\frac{cx^{2n}}{a}} + {}_1F_1\left(\frac{n+1}{2n}; \frac{1}{2}, 1; \frac{1}{2}\left(3 + \frac{1}{n}\right); -\frac{cx^{2n}}{a}, \frac{e^2x^{2n}}{d^2}\right)}{d^2(n+1)\sqrt{a+cx^{2n}}}$$

[Out] (x\*Sqrt[1 + (c\*x^(2\*n))/a]\*AppellF1[1/(2\*n), 1/2, 1, (2 + n^(-1))/2, -((c\*x^(2\*n))/a), (e^2\*x^(2\*n))/d^2])/(d\*Sqrt[a + c\*x^(2\*n)]) - (e\*x^(1 + n)\*Sqrt[1 + (c\*x^(2\*n))/a]\*AppellF1[(1 + n)/(2\*n), 1/2, 1, (3 + n^(-1))/2, -((c\*x^(2\*n))/a), (e^2\*x^(2\*n))/d^2])/(d^2\*(1 + n)\*Sqrt[a + c\*x^(2\*n)])

**Rubi [A]** time = 0.166221, antiderivative size = 171, normalized size of antiderivative = 1, number of steps used = 6, number of rules used = 5, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$ , Rules used = {1438, 430, 429, 511, 510}

$$\frac{x\sqrt{\frac{cx^{2n}}{a}} + {}_1F_1\left(\frac{1}{2n}; \frac{1}{2}, 1; \frac{1}{2}\left(2 + \frac{1}{n}\right); -\frac{cx^{2n}}{a}, \frac{e^2x^{2n}}{d^2}\right)}{d\sqrt{a+cx^{2n}}} - \frac{ex^{n+1}\sqrt{\frac{cx^{2n}}{a}} + {}_1F_1\left(\frac{n+1}{2n}; \frac{1}{2}, 1; \frac{1}{2}\left(3 + \frac{1}{n}\right); -\frac{cx^{2n}}{a}, \frac{e^2x^{2n}}{d^2}\right)}{d^2(n+1)\sqrt{a+cx^{2n}}}$$

Antiderivative was successfully verified.

[In] Int[1/((d + e\*x^n)\*Sqrt[a + c\*x^(2\*n)]), x]

[Out] (x\*Sqrt[1 + (c\*x^(2\*n))/a]\*AppellF1[1/(2\*n), 1/2, 1, (2 + n^(-1))/2, -((c\*x^(2\*n))/a), (e^2\*x^(2\*n))/d^2])/(d\*Sqrt[a + c\*x^(2\*n)]) - (e\*x^(1 + n)\*Sqrt[1 + (c\*x^(2\*n))/a]\*AppellF1[(1 + n)/(2\*n), 1/2, 1, (3 + n^(-1))/2, -((c\*x^(2\*n))/a), (e^2\*x^(2\*n))/d^2])/(d^2\*(1 + n)\*Sqrt[a + c\*x^(2\*n)])

### Rule 1438

Int[((d\_) + (e\_.)\*(x\_)^(n\_))^(q\_)\*((a\_) + (c\_.)\*(x\_)^(n2\_))^(p\_), x\_Symbol]
 :> Int[ExpandIntegrand[(a + c\*x^(2\*n))^p, (d/(d^2 - e^2\*x^(2\*n)) - (e\*x^n)/(d^2 - e^2\*x^(2\*n)))^(-q), x], x] /; FreeQ[{a, c, d, e, n, p}, x] && EqQ[n 2, 2\*n] && NeQ[c\*d^2 + a\*e^2, 0] && !IntegerQ[p] && ILtQ[q, 0]

### Rule 430

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol]
 :> Dist[(a^IntPart[p]\*(a + b\*x^n)^FracPart[p])/(1 + (b\*x^n)/a)^FracPart[p],

Int[(1 + (b\*x^n)/a)^p\*(c + d\*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b\*c - a\*d, 0] && NeQ[n, -1] && !(IntegerQ[p] || GtQ[a, 0])

### Rule 429

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol] := Simp[a^p\*c^q\*x\*AppellF1[1/n, -p, -q, 1 + 1/n, -((b\*x^n)/a), -((d\*x^n)/c)], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b\*c - a\*d, 0] && NeQ[n, -1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])

### Rule 511

Int[((e\_.)\*(x\_))^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol] := Dist[(a^IntPart[p]\*(a + b\*x^n)^FracPart[p])/(1 + (b\*x^n)/a)^FracPart[p], Int[(e\*x)^m\*(1 + (b\*x^n)/a)^p\*(c + d\*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b\*c - a\*d, 0] && NeQ[m, -1] && NeQ[m, n - 1] && !(IntegerQ[p] || GtQ[a, 0])

### Rule 510

Int[((e\_.)\*(x\_))^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol] := Simp[(a^p\*c^q\*(e\*x)^(m + 1)\*AppellF1[(m + 1)/n, -p, -q, 1 + (m + 1)/n, -((b\*x^n)/a), -((d\*x^n)/c)]/(e\*(m + 1)), x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b\*c - a\*d, 0] && NeQ[m, -1] && NeQ[m, n - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])

### Rubi steps

$$\begin{aligned}
 \int \frac{1}{(d + ex^n)\sqrt{a + cx^{2n}}} dx &= \int \left( \frac{d}{\sqrt{a + cx^{2n}}(d^2 - e^2x^{2n})} + \frac{ex^n}{\sqrt{a + cx^{2n}}(-d^2 + e^2x^{2n})} \right) dx \\
 &= d \int \frac{1}{\sqrt{a + cx^{2n}}(d^2 - e^2x^{2n})} dx + e \int \frac{x^n}{\sqrt{a + cx^{2n}}(-d^2 + e^2x^{2n})} dx \\
 &= \frac{\left( d\sqrt{1 + \frac{cx^{2n}}{a}} \right) \int \frac{1}{\sqrt{1 + \frac{cx^{2n}}{a}}(d^2 - e^2x^{2n})} dx}{\sqrt{a + cx^{2n}}} + \frac{\left( e\sqrt{1 + \frac{cx^{2n}}{a}} \right) \int \frac{x^n}{\sqrt{1 + \frac{cx^{2n}}{a}}(-d^2 + e^2x^{2n})} dx}{\sqrt{a + cx^{2n}}} \\
 &= \frac{x\sqrt{1 + \frac{cx^{2n}}{a}} F_1\left(\frac{1}{2n}; \frac{1}{2}, 1; \frac{1}{2}\left(2 + \frac{1}{n}\right); -\frac{cx^{2n}}{a}, \frac{e^2x^{2n}}{d^2}\right)}{d\sqrt{a + cx^{2n}}} - \frac{ex^{1+n}\sqrt{1 + \frac{cx^{2n}}{a}} F_1\left(\frac{1+n}{2n}; \frac{1}{2}, 1; \frac{1}{2}\left(3 + \frac{1}{n}\right); -\frac{cx^{2n}}{a}, \frac{e^2x^{2n}}{d^2}\right)}{d^2(1+n)\sqrt{a + cx^{2n}}}
 \end{aligned}$$

**Mathematica [F]** time = 0.122433, size = 0, normalized size = 0.

$$\int \frac{1}{(d + ex^n) \sqrt{a + cx^{2n}}} dx$$

Verification is Not applicable to the result.

[In] Integrate[1/((d + e\*x^n)\*Sqrt[a + c\*x^(2\*n)]), x]

[Out] Integrate[1/((d + e\*x^n)\*Sqrt[a + c\*x^(2\*n)]), x]

**Maple [F]** time = 0.062, size = 0, normalized size = 0.

$$\int \frac{1}{d + ex^n} \frac{1}{\sqrt{a + cx^{2n}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(d+e\*x^n)/(a+c\*x^(2\*n))^(1/2), x)

[Out] int(1/(d+e\*x^n)/(a+c\*x^(2\*n))^(1/2), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^{2n} + a}(ex^n + d)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d+e\*x^n)/(a+c\*x^(2\*n))^(1/2), x, algorithm="maxima")

[Out] integrate(1/(sqrt(c\*x^(2\*n) + a)\*(e\*x^n + d)), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^{2n} + a}}{aex^n + ad + (cex^n + cd)x^{2n}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x^n)/(a+c*x^(2*n))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^(2*n) + a)/(a*e*x^n + a*d + (c*e*x^n + c*d)*x^(2*n)), x)
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a + cx^{2n}} (d + ex^n)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x**n)/(a+c*x**(2*n))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a + c*x**(2*n))*(d + e*x**n)), x)
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^{2n} + a}(ex^n + d)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x^n)/(a+c*x^(2*n))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c*x^(2*n) + a)*(e*x^n + d)), x)
```



$$3.59 \quad \int (d + ex^n)^q (a + cx^{2n})^p dx$$

Optimal. Leaf size=23

$$\text{Unintegrable}\left(\left(a + cx^{2n}\right)^p (d + ex^n)^q, x\right)$$

[Out] Defer[Int] [(d + e\*x^n)^q\*(a + c\*x^(2\*n))^p, x]

**Rubi [A]** time = 0.0080332, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int (d + ex^n)^q (a + cx^{2n})^p dx$$

Verification is Not applicable to the result.

[In] Int[(d + e\*x^n)^q\*(a + c\*x^(2\*n))^p, x]

[Out] Defer[Int] [(d + e\*x^n)^q\*(a + c\*x^(2\*n))^p, x]

Rubi steps

$$\int (d + ex^n)^q (a + cx^{2n})^p dx = \int (d + ex^n)^q (a + cx^{2n})^p dx$$

**Mathematica [A]** time = 0.161738, size = 0, normalized size = 0.

$$\int (d + ex^n)^q (a + cx^{2n})^p dx$$

Verification is Not applicable to the result.

[In] Integrate[(d + e\*x^n)^q\*(a + c\*x^(2\*n))^p, x]

[Out] Integrate[(d + e\*x^n)^q\*(a + c\*x^(2\*n))^p, x]

**Maple [A]** time = 0.148, size = 0, normalized size = 0.

$$\int (d + ex^n)^q (a + cx^{2n})^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d+e*x^n)^q*(a+c*x^(2*n))^p,x)`

[Out] `int((d+e*x^n)^q*(a+c*x^(2*n))^p,x)`

---

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int (cx^{2n} + a)^p (ex^n + d)^q dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d+e*x^n)^q*(a+c*x^(2*n))^p,x, algorithm="maxima")`

[Out] `integrate((c*x^(2*n) + a)^p*(e*x^n + d)^q, x)`

---

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(cx^{2n} + a\right)^p \left(ex^n + d\right)^q, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d+e*x^n)^q*(a+c*x^(2*n))^p,x, algorithm="fricas")`

[Out] `integral((c*x^(2*n) + a)^p*(e*x^n + d)^q, x)`

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e\*x\*\*n)\*\*q\*(a+c\*x\*\*(2\*n))\*\*p,x)

[Out] Timed out

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int (cx^{2n} + a)^p (ex^n + d)^q dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e\*x^n)^q\*(a+c\*x^(2\*n))^p,x, algorithm="giac")

[Out] integrate((c\*x^(2\*n) + a)^p\*(e\*x^n + d)^q, x)

### 3.60 $\int (d + ex^n)^3 (a + cx^{2n})^p dx$

**Optimal.** Leaf size=299

$$\frac{3d^2ex^{n+1}(a+cx^{2n})^p\left(\frac{cx^{2n}}{a}+1\right)^{-p}{}_2F_1\left(\frac{n+1}{2n},-p;\frac{1}{2}\left(3+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{n+1}+d^3x(a+cx^{2n})^p\left(\frac{cx^{2n}}{a}+1\right)^{-p}{}_2F_1\left(\frac{1}{2n},-p;\frac{1}{2}\left(2+\frac{1}{n}\right)\right)$$

```
[Out] (3*d*e^2*x^(1+2*n)*(a+c*x^(2*n))^p*Hypergeometric2F1[(2+n^(-1))/2,-p,
(4+n^(-1))/2,-((c*x^(2*n))/a)]/((1+2*n)*(1+(c*x^(2*n))/a)^p)+(e^3*x^(1+3*n)*
(a+c*x^(2*n))^p*Hypergeometric2F1[(3+n^(-1))/2,-p,(5+n^(-1))/2,-((c*x^(2*n))/a)]/
((1+3*n)*(1+(c*x^(2*n))/a)^p)+(d^3*x*(a+c*x^(2*n))^p*Hypergeometric2F1[1/(2*n),-p,
(2+n^(-1))/2,-((c*x^(2*n))/a)]/((1+(c*x^(2*n))/a)^p)+(3*d^2*e*x^(1+n)*(a+c*x^(2*n))^p*Hyperg
eometric2F1[(1+n)/(2*n),-p,(3+n^(-1))/2,-((c*x^(2*n))/a)]/((1+n)*(1+(c*x^(2*n))/a)^p)
```

**Rubi [A]** time = 0.160287, antiderivative size = 299, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 5, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$ , Rules used = {1437, 246, 245, 365, 364}

$$\frac{3d^2ex^{n+1}(a+cx^{2n})^p\left(\frac{cx^{2n}}{a}+1\right)^{-p}{}_2F_1\left(\frac{n+1}{2n},-p;\frac{1}{2}\left(3+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{n+1}+d^3x(a+cx^{2n})^p\left(\frac{cx^{2n}}{a}+1\right)^{-p}{}_2F_1\left(\frac{1}{2n},-p;\frac{1}{2}\left(2+\frac{1}{n}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^n)^3*(a + c*x^(2*n))^p,x]
```

```
[Out] (3*d*e^2*x^(1+2*n)*(a+c*x^(2*n))^p*Hypergeometric2F1[(2+n^(-1))/2,-p,
(4+n^(-1))/2,-((c*x^(2*n))/a)]/((1+2*n)*(1+(c*x^(2*n))/a)^p)+(e^3*x^(1+3*n)*
(a+c*x^(2*n))^p*Hypergeometric2F1[(3+n^(-1))/2,-p,(5+n^(-1))/2,-((c*x^(2*n))/a)]/
((1+3*n)*(1+(c*x^(2*n))/a)^p)+(d^3*x*(a+c*x^(2*n))^p*Hypergeometric2F1[1/(2*n),-p,
(2+n^(-1))/2,-((c*x^(2*n))/a)]/((1+(c*x^(2*n))/a)^p)+(3*d^2*e*x^(1+n)*(a+c*x^(2*n))^p*Hyperg
eometric2F1[(1+n)/(2*n),-p,(3+n^(-1))/2,-((c*x^(2*n))/a)]/((1+n)*(1+(c*x^(2*n))/a)^p)
```

#### Rule 1437

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x^n)^q*(a + c*x^(2*n))^p, x], x] /; FreeQ[{a
```

```
, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && NeQ[c*d^2 + a*e^2, 0] && ((Integ
ersQ[p, q] && !IntegerQ[n]) || IGtQ[p, 0] || (IGtQ[q, 0] && !IntegerQ[n])
)
```

### Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
/; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Simp
lify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

### Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

### Rule 365

```
Int[((c_.)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)
^m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
&& !(ILtQ[p, 0] || GtQ[a, 0])
```

### Rule 364

```
Int[((c_.)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)]]/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

### Rubi steps

$$\begin{aligned}
\int (d + ex^n)^3 (a + cx^{2n})^p dx &= \int \left( d^3 (a + cx^{2n})^p + 3d^2 ex^n (a + cx^{2n})^p + 3de^2 x^{2n} (a + cx^{2n})^p + e^3 x^{3n} (a + cx^{2n})^p \right) dx \\
&= d^3 \int (a + cx^{2n})^p dx + (3d^2 e) \int x^n (a + cx^{2n})^p dx + (3de^2) \int x^{2n} (a + cx^{2n})^p dx + e^3 \int x^{3n} (a + cx^{2n})^p dx \\
&= \left( d^3 (a + cx^{2n})^p \left( 1 + \frac{cx^{2n}}{a} \right)^{-p} \right) \int \left( 1 + \frac{cx^{2n}}{a} \right)^p dx + \left( 3d^2 e (a + cx^{2n})^p \left( 1 + \frac{cx^{2n}}{a} \right)^{-p} \right) \int x^n \left( 1 + \frac{cx^{2n}}{a} \right)^p dx \\
&= \frac{3de^2 x^{1+2n} (a + cx^{2n})^p \left( 1 + \frac{cx^{2n}}{a} \right)^{-p} {}_2F_1\left(\frac{1}{2}\left(2 + \frac{1}{n}\right), -p; \frac{1}{2}\left(4 + \frac{1}{n}\right); -\frac{cx^{2n}}{a}\right)}{1 + 2n} + \frac{e^3 x^{1+3n} (a + cx^{2n})^p \left( 1 + \frac{cx^{2n}}{a} \right)^{-p} {}_2F_1\left(\frac{1}{2}\left(3 + \frac{1}{n}\right), -p; \frac{1}{2}\left(5 + \frac{1}{n}\right); -\frac{cx^{2n}}{a}\right)}{1 + 3n}
\end{aligned}$$

**Mathematica [A]** time = 0.200395, size = 213, normalized size = 0.71

$$x(a + cx^{2n})^p \left( \frac{cx^{2n}}{a} + 1 \right)^{-p} \left( d^2 \left( d {}_2F_1\left(\frac{1}{2n}, -p; \frac{1}{2}\left(2 + \frac{1}{n}\right); -\frac{cx^{2n}}{a}\right) + \frac{3ex^n {}_2F_1\left(\frac{n+1}{2n}, -p; \frac{1}{2}\left(3 + \frac{1}{n}\right); -\frac{cx^{2n}}{a}\right)}{n+1} \right) + \frac{3de^2 x^{2n} {}_2F_1\left(\frac{1}{2}\left(2 + \frac{1}{n}\right), -p; \frac{1}{2}\left(4 + \frac{1}{n}\right); -\frac{cx^{2n}}{a}\right)}{1+2n} + \frac{e^3 x^{3n} {}_2F_1\left(\frac{1}{2}\left(3 + \frac{1}{n}\right), -p; \frac{1}{2}\left(5 + \frac{1}{n}\right); -\frac{cx^{2n}}{a}\right)}{1+3n} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(d + e\*x^n)^3\*(a + c\*x^(2\*n))^p,x]

[Out] (x\*(a + c\*x^(2\*n))^p\*((3\*d\*e^2\*x^(2\*n)\*Hypergeometric2F1[(2 + n^(-1))/2, -p, (4 + n^(-1))/2, -((c\*x^(2\*n))/a)])/(1 + 2\*n) + (e^3\*x^(3\*n)\*Hypergeometric2F1[(3 + n^(-1))/2, -p, (5 + n^(-1))/2, -((c\*x^(2\*n))/a)])/(1 + 3\*n) + d^2\*(d\*Hypergeometric2F1[1/(2\*n), -p, (2 + n^(-1))/2, -((c\*x^(2\*n))/a)] + (3\*e\*x^n\*Hypergeometric2F1[(1 + n)/(2\*n), -p, (3 + n^(-1))/2, -((c\*x^(2\*n))/a)]))/(1 + n)))/(1 + (c\*x^(2\*n))/a)^p

**Maple [F]** time = 0.112, size = 0, normalized size = 0.

$$\int (d + ex^n)^3 (a + cx^{2n})^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d+e\*x^n)^3\*(a+c\*x^(2\*n))^p,x)

[Out] int((d+e\*x^n)^3\*(a+c\*x^(2\*n))^p,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (ex^n + d)^3 (cx^{2n} + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e\*x^n)^3\*(a+c\*x^(2\*n))^p,x, algorithm="maxima")

[Out] integrate((e\*x^n + d)^3\*(c\*x^(2\*n) + a)^p, x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(e^3x^{3n} + 3de^2x^{2n} + 3d^2ex^n + d^3\right)(cx^{2n} + a)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e\*x^n)^3\*(a+c\*x^(2\*n))^p,x, algorithm="fricas")

[Out] integral((e^3\*x^(3\*n) + 3\*d\*e^2\*x^(2\*n) + 3\*d^2\*e\*x^n + d^3)\*(c\*x^(2\*n) + a)^p, x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e\*x\*\*n)\*\*3\*(a+c\*x\*\*(2\*n))\*\*p,x)

[Out] Timed out

**Giac [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x^n)^3*(a+c*x^(2*n))^p,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```



### 3.61 $\int (d + ex^n)^2 (a + cx^{2n})^p dx$

**Optimal.** Leaf size=217

$$d^2x (a + cx^{2n})^p \left(\frac{cx^{2n}}{a} + 1\right)^{-p} {}_2F_1\left(\frac{1}{2n}, -p; \frac{1}{2}\left(2 + \frac{1}{n}\right); -\frac{cx^{2n}}{a}\right) + \frac{2dex^{n+1} (a + cx^{2n})^p \left(\frac{cx^{2n}}{a} + 1\right)^{-p} {}_2F_1\left(\frac{n+1}{2n}, -p; \frac{1}{2}\left(3 + \frac{1}{n}\right)\right)}{n+1}$$

[Out] (e^2\*x^(1 + 2\*n)\*(a + c\*x^(2\*n))^p\*Hypergeometric2F1[(2 + n^(-1))/2, -p, (4 + n^(-1))/2, -((c\*x^(2\*n))/a)])/((1 + 2\*n)\*(1 + (c\*x^(2\*n))/a)^p) + (d^2\*x\*(a + c\*x^(2\*n))^p\*Hypergeometric2F1[1/(2\*n), -p, (2 + n^(-1))/2, -((c\*x^(2\*n))/a)])/(1 + (c\*x^(2\*n))/a)^p + (2\*d\*e\*x^(1 + n)\*(a + c\*x^(2\*n))^p\*Hypergeometric2F1[(1 + n)/(2\*n), -p, (3 + n^(-1))/2, -((c\*x^(2\*n))/a)])/((1 + n)\*(1 + (c\*x^(2\*n))/a)^p)

**Rubi [A]** time = 0.100881, antiderivative size = 217, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$ , Rules used = {1437, 246, 245, 365, 364}

$$d^2x (a + cx^{2n})^p \left(\frac{cx^{2n}}{a} + 1\right)^{-p} {}_2F_1\left(\frac{1}{2n}, -p; \frac{1}{2}\left(2 + \frac{1}{n}\right); -\frac{cx^{2n}}{a}\right) + \frac{2dex^{n+1} (a + cx^{2n})^p \left(\frac{cx^{2n}}{a} + 1\right)^{-p} {}_2F_1\left(\frac{n+1}{2n}, -p; \frac{1}{2}\left(3 + \frac{1}{n}\right)\right)}{n+1}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x^n)^2\*(a + c\*x^(2\*n))^p,x]

[Out] (e^2\*x^(1 + 2\*n)\*(a + c\*x^(2\*n))^p\*Hypergeometric2F1[(2 + n^(-1))/2, -p, (4 + n^(-1))/2, -((c\*x^(2\*n))/a)])/((1 + 2\*n)\*(1 + (c\*x^(2\*n))/a)^p) + (d^2\*x\*(a + c\*x^(2\*n))^p\*Hypergeometric2F1[1/(2\*n), -p, (2 + n^(-1))/2, -((c\*x^(2\*n))/a)])/(1 + (c\*x^(2\*n))/a)^p + (2\*d\*e\*x^(1 + n)\*(a + c\*x^(2\*n))^p\*Hypergeometric2F1[(1 + n)/(2\*n), -p, (3 + n^(-1))/2, -((c\*x^(2\*n))/a)])/((1 + n)\*(1 + (c\*x^(2\*n))/a)^p)

#### Rule 1437

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol]
  :> Int[ExpandIntegrand[(d + e*x^n)^q*(a + c*x^(2*n))^p, x], x] /; FreeQ[{a
, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && NeQ[c*d^2 + a*e^2, 0] && ((Integ
ersQ[p, q] && !IntegerQ[n]) || IGtQ[p, 0] || (IGtQ[q, 0] && !IntegerQ[n])
)
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x]
/; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Simp
lify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)
^m*(1 + (b*x^n)/a)^p, x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
&& !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/((c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}
\int (d + ex^n)^2 (a + cx^{2n})^p dx &= \int \left( d^2 (a + cx^{2n})^p + 2dex^n (a + cx^{2n})^p + e^2 x^{2n} (a + cx^{2n})^p \right) dx \\
&= d^2 \int (a + cx^{2n})^p dx + (2de) \int x^n (a + cx^{2n})^p dx + e^2 \int x^{2n} (a + cx^{2n})^p dx \\
&= \left( d^2 (a + cx^{2n})^p \left( 1 + \frac{cx^{2n}}{a} \right)^{-p} \right) \int \left( 1 + \frac{cx^{2n}}{a} \right)^p dx + \left( 2de (a + cx^{2n})^p \left( 1 + \frac{cx^{2n}}{a} \right)^{-p} \right) \int x^n \left( 1 + \frac{cx^{2n}}{a} \right)^p dx \\
&= \frac{e^2 x^{1+2n} (a + cx^{2n})^p \left( 1 + \frac{cx^{2n}}{a} \right)^{-p} {}_2F_1 \left( \frac{1}{2} \left( 2 + \frac{1}{n} \right), -p; \frac{1}{2} \left( 4 + \frac{1}{n} \right); -\frac{cx^{2n}}{a} \right)}{1 + 2n} + d^2 x (a + cx^{2n})^p \left( 1 + \frac{cx^{2n}}{a} \right)^{-p}
\end{aligned}$$

**Mathematica [A]** time = 0.108526, size = 171, normalized size = 0.79

$$\frac{x(a + cx^{2n})^p \left(\frac{cx^{2n}}{a} + 1\right)^{-p} \left(d(2n + 1) \left(d(n + 1) {}_2F_1\left(\frac{1}{2n}, -p; \frac{1}{2}\left(2 + \frac{1}{n}\right); -\frac{cx^{2n}}{a}\right) + 2ex^n {}_2F_1\left(\frac{n+1}{2n}, -p; \frac{1}{2}\left(3 + \frac{1}{n}\right); -\frac{cx^{2n}}{a}\right)\right) + e}{(n + 1)(2n + 1)}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e\*x^n)^2\*(a + c\*x^(2\*n))^p,x]

[Out] (x\*(a + c\*x^(2\*n))^p\*(e^2\*(1 + n)\*x^(2\*n)\*Hypergeometric2F1[(2 + n^(-1))/2, -p, (4 + n^(-1))/2, -((c\*x^(2\*n))/a)] + d\*(1 + 2\*n)\*(d\*(1 + n)\*Hypergeometric2F1[1/(2\*n), -p, (2 + n^(-1))/2, -((c\*x^(2\*n))/a)] + 2\*e\*x^n\*Hypergeometric2F1[(1 + n)/(2\*n), -p, (3 + n^(-1))/2, -((c\*x^(2\*n))/a)])))/((1 + n)\*(1 + 2\*n)\*(1 + (c\*x^(2\*n))/a))^p

**Maple [F]** time = 0.094, size = 0, normalized size = 0.

$$\int (d + ex^n)^2 (a + cx^{2n})^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d+e\*x^n)^2\*(a+c\*x^(2\*n))^p,x)

[Out] int((d+e\*x^n)^2\*(a+c\*x^(2\*n))^p,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (ex^n + d)^2 (cx^{2n} + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e\*x^n)^2\*(a+c\*x^(2\*n))^p,x, algorithm="maxima")

[Out] integrate((e\*x^n + d)^2\*(c\*x^(2\*n) + a)^p, x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(e^2x^{2n} + 2dex^n + d^2\right)(cx^{2n} + a)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e\*x^n)^2\*(a+c\*x^(2\*n))^p,x, algorithm="fricas")

[Out] integral((e^2\*x^(2\*n) + 2\*d\*e\*x^n + d^2)\*(c\*x^(2\*n) + a)^p, x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e\*x\*\*n)\*\*2\*(a+c\*x\*\*(2\*n))\*\*p,x)

[Out] Timed out

**Giac [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e\*x^n)^2\*(a+c\*x^(2\*n))^p,x, algorithm="giac")

[Out] Exception raised: TypeError

### 3.62 $\int (d + ex^n) (a + cx^{2n})^p dx$

**Optimal.** Leaf size=135

$$dx (a + cx^{2n})^p \left(\frac{cx^{2n}}{a} + 1\right)^{-p} {}_2F_1\left(\frac{1}{2n}, -p; \frac{1}{2}\left(2 + \frac{1}{n}\right); -\frac{cx^{2n}}{a}\right) + \frac{ex^{n+1} (a + cx^{2n})^p \left(\frac{cx^{2n}}{a} + 1\right)^{-p} {}_2F_1\left(\frac{n+1}{2n}, -p; \frac{1}{2}\left(3 + \frac{1}{n}\right); -\frac{cx^{2n}}{a}\right)}{n+1}$$

[Out] (d\*x\*(a + c\*x^(2\*n))^p\*Hypergeometric2F1[1/(2\*n), -p, (2 + n^(-1))/2, -((c\*x^(2\*n))/a)]/(1 + (c\*x^(2\*n))/a)^p + (e\*x^(1 + n)\*(a + c\*x^(2\*n))^p\*Hypergeometric2F1[(1 + n)/(2\*n), -p, (3 + n^(-1))/2, -((c\*x^(2\*n))/a)])/((1 + n)\*(1 + (c\*x^(2\*n))/a)^p)

**Rubi [A]** time = 0.0604615, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$ , Rules used = {1433, 246, 245, 365, 364}

$$dx (a + cx^{2n})^p \left(\frac{cx^{2n}}{a} + 1\right)^{-p} {}_2F_1\left(\frac{1}{2n}, -p; \frac{1}{2}\left(2 + \frac{1}{n}\right); -\frac{cx^{2n}}{a}\right) + \frac{ex^{n+1} (a + cx^{2n})^p \left(\frac{cx^{2n}}{a} + 1\right)^{-p} {}_2F_1\left(\frac{n+1}{2n}, -p; \frac{1}{2}\left(3 + \frac{1}{n}\right); -\frac{cx^{2n}}{a}\right)}{n+1}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x^n)\*(a + c\*x^(2\*n))^p,x]

[Out] (d\*x\*(a + c\*x^(2\*n))^p\*Hypergeometric2F1[1/(2\*n), -p, (2 + n^(-1))/2, -((c\*x^(2\*n))/a)]/(1 + (c\*x^(2\*n))/a)^p + (e\*x^(1 + n)\*(a + c\*x^(2\*n))^p\*Hypergeometric2F1[(1 + n)/(2\*n), -p, (3 + n^(-1))/2, -((c\*x^(2\*n))/a)])/((1 + n)\*(1 + (c\*x^(2\*n))/a)^p)

#### Rule 1433

Int[((d\_) + (e\_.)\*(x\_)^(n\_))\*((a\_) + (c\_.)\*(x\_)^(n2\_))^(p\_), x\_Symbol] := Int[ExpandIntegrand[(d + e\*x^n)\*(a + c\*x^(2\*n))^p, x], x] /; FreeQ[{a, c, d, e, n}, x] && EqQ[n2, 2\*n]

#### Rule 246

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Dist[(a^IntPart[p]\*(a + b\*x^n)^FracPart[p])/(1 + (b\*x^n)/a)^FracPart[p], Int[(1 + (b\*x^n)/a)^p, x], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim

plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])

### Rule 245

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[a^p\*x\*Hypergeometric2F1[-p, 1/n, 1/n + 1, -((b\*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] || GtQ[a, 0])

### Rule 365

Int[((c\_.)\*(x\_))^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Dist[(a^IntPart[p]\*(a + b\*x^n)^FracPart[p])/(1 + (b\*x^n)/a)^FracPart[p], Int[(c\*x)^m\*(1 + (b\*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && !(ILtQ[p, 0] || GtQ[a, 0])

### Rule 364

Int[((c\_.)\*(x\_))^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[(a^p\*(c\*x)^(m + 1)\*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b\*x^n)/a)])/((c\*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])

### Rubi steps

$$\begin{aligned} \int (d + ex^n)(a + cx^{2n})^p dx &= \int \left( d(a + cx^{2n})^p + ex^n(a + cx^{2n})^p \right) dx \\ &= d \int (a + cx^{2n})^p dx + e \int x^n (a + cx^{2n})^p dx \\ &= \left( d(a + cx^{2n})^p \left( 1 + \frac{cx^{2n}}{a} \right)^{-p} \right) \int \left( 1 + \frac{cx^{2n}}{a} \right)^p dx + \left( e(a + cx^{2n})^p \left( 1 + \frac{cx^{2n}}{a} \right)^{-p} \right) \int x^n \left( 1 + \frac{cx^{2n}}{a} \right)^p dx \\ &= dx (a + cx^{2n})^p \left( 1 + \frac{cx^{2n}}{a} \right)^{-p} {}_2F_1 \left( \frac{1}{2n}, -p; \frac{1}{2} \left( 2 + \frac{1}{n} \right); -\frac{cx^{2n}}{a} \right) + \frac{ex^{1+n} (a + cx^{2n})^p \left( 1 + \frac{cx^{2n}}{a} \right)^p}{n + 1} \end{aligned}$$

**Mathematica [A]** time = 0.0497929, size = 110, normalized size = 0.81

$$\frac{x (a + cx^{2n})^p \left( \frac{cx^{2n}}{a} + 1 \right)^{-p} \left( d(n + 1) {}_2F_1 \left( \frac{1}{2n}, -p; \frac{1}{2} \left( 2 + \frac{1}{n} \right); -\frac{cx^{2n}}{a} \right) + ex^n {}_2F_1 \left( \frac{n+1}{2n}, -p; \frac{1}{2} \left( 3 + \frac{1}{n} \right); -\frac{cx^{2n}}{a} \right) \right)}{n + 1}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e\*x^n)\*(a + c\*x^(2\*n))^p,x]

[Out]  $(x*(a + c*x^{(2*n)})^p*(d*(1 + n)*\text{Hypergeometric2F1}[1/(2*n), -p, (2 + n^{(-1)})/2, -((c*x^{(2*n)})/a)] + e*x^n*\text{Hypergeometric2F1}[(1 + n)/(2*n), -p, (3 + n^{(-1)})/2, -((c*x^{(2*n)})/a)]))/((1 + n)*(1 + (c*x^{(2*n)})/a)^p)$

**Maple [F]** time = 0.093, size = 0, normalized size = 0.

$$\int (d + ex^n)(a + cx^{2n})^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d+e\*x^n)\*(a+c\*x^(2\*n))^p,x)

[Out] int((d+e\*x^n)\*(a+c\*x^(2\*n))^p,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (ex^n + d)(cx^{2n} + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e\*x^n)\*(a+c\*x^(2\*n))^p,x, algorithm="maxima")

[Out] integrate((e\*x^n + d)\*(c\*x^(2\*n) + a)^p, x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((ex^n + d)(cx^{2n} + a)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e\*x^n)\*(a+c\*x^(2\*n))^p,x, algorithm="fricas")

[Out] integral((e\*x^n + d)\*(c\*x^(2\*n) + a)^p, x)

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e\*x\*\*n)\*(a+c\*x\*\*(2\*n))\*\*p,x)

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (ex^n + d)(cx^{2n} + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e\*x^n)\*(a+c\*x^(2\*n))^p,x, algorithm="giac")

[Out] integrate((e\*x^n + d)\*(c\*x^(2\*n) + a)^p, x)



$$3.63 \quad \int \frac{(a+cx^{2n})^p}{d+ex^n} dx$$

**Optimal.** Leaf size=167

$$\frac{x(a+cx^{2n})^p \left(\frac{cx^{2n}}{a} + 1\right)^{-p} F_1\left(\frac{1}{2n}; -p, 1; \frac{1}{2}\left(2 + \frac{1}{n}\right); -\frac{cx^{2n}}{a}, \frac{e^2 x^{2n}}{d^2}\right)}{d} - \frac{ex^{n+1}(a+cx^{2n})^p \left(\frac{cx^{2n}}{a} + 1\right)^{-p} F_1\left(\frac{n+1}{2n}; -p, 1; \frac{1}{2}\left(3 + \frac{1}{n}\right)\right)}{d^2(n+1)}$$

[Out] (x\*(a + c\*x^(2\*n))^p\*AppellF1[1/(2\*n), -p, 1, (2 + n^(-1))/2, -((c\*x^(2\*n))/a), (e^2\*x^(2\*n))/d^2])/(d\*(1 + (c\*x^(2\*n))/a)^p) - (e\*x^(1 + n)\*(a + c\*x^(2\*n))^p\*AppellF1[(1 + n)/(2\*n), -p, 1, (3 + n^(-1))/2, -((c\*x^(2\*n))/a), (e^2\*x^(2\*n))/d^2])/(d^2\*(1 + n)\*(1 + (c\*x^(2\*n))/a)^p)

**Rubi [A]** time = 0.141518, antiderivative size = 167, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$ , Rules used = {1438, 430, 429, 511, 510}

$$\frac{x(a+cx^{2n})^p \left(\frac{cx^{2n}}{a} + 1\right)^{-p} F_1\left(\frac{1}{2n}; -p, 1; \frac{1}{2}\left(2 + \frac{1}{n}\right); -\frac{cx^{2n}}{a}, \frac{e^2 x^{2n}}{d^2}\right)}{d} - \frac{ex^{n+1}(a+cx^{2n})^p \left(\frac{cx^{2n}}{a} + 1\right)^{-p} F_1\left(\frac{n+1}{2n}; -p, 1; \frac{1}{2}\left(3 + \frac{1}{n}\right)\right)}{d^2(n+1)}$$

Antiderivative was successfully verified.

[In] Int[(a + c\*x^(2\*n))^p/(d + e\*x^n), x]

[Out] (x\*(a + c\*x^(2\*n))^p\*AppellF1[1/(2\*n), -p, 1, (2 + n^(-1))/2, -((c\*x^(2\*n))/a), (e^2\*x^(2\*n))/d^2])/(d\*(1 + (c\*x^(2\*n))/a)^p) - (e\*x^(1 + n)\*(a + c\*x^(2\*n))^p\*AppellF1[(1 + n)/(2\*n), -p, 1, (3 + n^(-1))/2, -((c\*x^(2\*n))/a), (e^2\*x^(2\*n))/d^2])/(d^2\*(1 + n)\*(1 + (c\*x^(2\*n))/a)^p)

### Rule 1438

Int[((d\_) + (e\_.)\*(x\_)^(n\_))^(q\_)\*((a\_) + (c\_.)\*(x\_)^(n2\_))^(p\_), x\_Symbol]  
 :> Int[ExpandIntegrand[(a + c\*x^(2\*n))^p, (d/(d^2 - e^2\*x^(2\*n)) - (e\*x^n)/(d^2 - e^2\*x^(2\*n)))^(-q), x], x] /; FreeQ[{a, c, d, e, n, p}, x] && EqQ[n, 2, 2\*n] && NeQ[c\*d^2 + a\*e^2, 0] && !IntegerQ[p] && ILtQ[q, 0]

### Rule 430

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol]  
 :> Dist[(a^IntPart[p]\*(a + b\*x^n)^FracPart[p])/(1 + (b\*x^n)/a)^FracPart[p],

Int[(1 + (b\*x^n)/a)^p\*(c + d\*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b\*c - a\*d, 0] && NeQ[n, -1] && !(IntegerQ[p] || GtQ[a, 0])

### Rule 429

Int[((a\_) + (b\_)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_)\*(x\_)^(n\_))^(q\_), x\_Symbol] :> Simp[a^p\*c^q\*x\*AppellF1[1/n, -p, -q, 1 + 1/n, -((b\*x^n)/a), -((d\*x^n)/c)], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b\*c - a\*d, 0] && NeQ[n, -1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])

### Rule 511

Int[((e\_)\*(x\_))^(m\_)\*((a\_) + (b\_)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_)\*(x\_)^(n\_))^(q\_), x\_Symbol] :> Dist[(a^IntPart[p]\*(a + b\*x^n)^FracPart[p])/(1 + (b\*x^n)/a)^FracPart[p], Int[(e\*x)^m\*(1 + (b\*x^n)/a)^p\*(c + d\*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b\*c - a\*d, 0] && NeQ[m, -1] && NeQ[m, n - 1] && !(IntegerQ[p] || GtQ[a, 0])

### Rule 510

Int[((e\_)\*(x\_))^(m\_)\*((a\_) + (b\_)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_)\*(x\_)^(n\_))^(q\_), x\_Symbol] :> Simp[(a^p\*c^q\*(e\*x)^(m + 1)\*AppellF1[(m + 1)/n, -p, -q, 1 + (m + 1)/n, -((b\*x^n)/a), -((d\*x^n)/c)]/(e\*(m + 1)), x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b\*c - a\*d, 0] && NeQ[m, -1] && NeQ[m, n - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])

### Rubi steps

$$\begin{aligned}
 \int \frac{(a + cx^{2n})^p}{d + ex^n} dx &= \int \left( \frac{d(a + cx^{2n})^p}{d^2 - e^2x^{2n}} + \frac{ex^n(a + cx^{2n})^p}{-d^2 + e^2x^{2n}} \right) dx \\
 &= d \int \frac{(a + cx^{2n})^p}{d^2 - e^2x^{2n}} dx + e \int \frac{x^n(a + cx^{2n})^p}{-d^2 + e^2x^{2n}} dx \\
 &= \left( d(a + cx^{2n})^p \left( 1 + \frac{cx^{2n}}{a} \right)^{-p} \right) \int \frac{\left( 1 + \frac{cx^{2n}}{a} \right)^p}{d^2 - e^2x^{2n}} dx + \left( e(a + cx^{2n})^p \left( 1 + \frac{cx^{2n}}{a} \right)^{-p} \right) \int \frac{x^n \left( 1 + \frac{cx^{2n}}{a} \right)^p}{-d^2 + e^2x^{2n}} dx \\
 &= \frac{x(a + cx^{2n})^p \left( 1 + \frac{cx^{2n}}{a} \right)^{-p} F_1\left(\frac{1}{2n}; -p, 1; \frac{1}{2} \left( 2 + \frac{1}{n} \right); -\frac{cx^{2n}}{a}, \frac{e^2x^{2n}}{d^2}\right)}{d} - \frac{ex^{1+n}(a + cx^{2n})^p \left( 1 + \frac{cx^{2n}}{a} \right)^{-p} F_1\left(\frac{1}{2n}; -p, 1; \frac{1}{2} \left( 2 + \frac{1}{n} \right); -\frac{cx^{2n}}{a}, \frac{e^2x^{2n}}{d^2}\right)}{d^2(1 + \frac{cx^{2n}}{a})}
 \end{aligned}$$

**Mathematica [F]** time = 0.080604, size = 0, normalized size = 0.

$$\int \frac{(a + cx^{2n})^p}{d + ex^n} dx$$

Verification is Not applicable to the result.

[In] Integrate[(a + c\*x^(2\*n))^p/(d + e\*x^n), x]

[Out] Integrate[(a + c\*x^(2\*n))^p/(d + e\*x^n), x]

**Maple [F]** time = 0.092, size = 0, normalized size = 0.

$$\int \frac{(a + cx^{2n})^p}{d + ex^n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+c\*x^(2\*n))^p/(d+e\*x^n), x)

[Out] int((a+c\*x^(2\*n))^p/(d+e\*x^n), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx^{2n} + a)^p}{ex^n + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+c\*x^(2\*n))^p/(d+e\*x^n), x, algorithm="maxima")

[Out] integrate((c\*x^(2\*n) + a)^p/(e\*x^n + d), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(cx^{2n} + a)^p}{ex^n + d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+c\*x^(2\*n))^p/(d+e\*x^n),x, algorithm="fricas")

[Out] integral((c\*x^(2\*n) + a)^p/(e\*x^n + d), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+c\*x\*\*(2\*n))\*\*p/(d+e\*x\*\*n),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx^{2n} + a)^p}{ex^n + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+c\*x^(2\*n))^p/(d+e\*x^n),x, algorithm="giac")

[Out] integrate((c\*x^(2\*n) + a)^p/(e\*x^n + d), x)

$$3.64 \quad \int \frac{(a+cx^{2n})^p}{(d+ex^n)^2} dx$$

**Optimal.** Leaf size=261

$$\frac{2ex^{n+1}(a+cx^{2n})^p \left(\frac{cx^{2n}}{a} + 1\right)^{-p} F_1\left(\frac{n+1}{2n}; -p, 2; \frac{1}{2}\left(3 + \frac{1}{n}\right); -\frac{cx^{2n}}{a}, \frac{e^2x^{2n}}{d^2}\right)}{d^3(n+1)} + \frac{e^2x^{2n+1}(a+cx^{2n})^p \left(\frac{cx^{2n}}{a} + 1\right)^{-p} F_1\left(\frac{1}{2}\left(2 + \frac{1}{n}\right)\right)}{d^4(2n+1)}$$

[Out]  $(e^{2x^{1+2n}}(a+c x^{2n}))^p \text{AppellF1}\left[\frac{2+n(-1)}{2}, -p, 2, \frac{4+n(-1)}{2}, -\left(\frac{c x^{2n}}{a}\right), \frac{e^{2x^{2n}}}{d^2}\right] / (d^4(1+2n)(1+(c x^{2n})/a)^p) + (x(a+c x^{2n}))^p \text{AppellF1}\left[\frac{1}{2n}, -p, 2, \frac{2+n(-1)}{2}, -\left(\frac{c x^{2n}}{a}\right), \frac{e^{2x^{2n}}}{d^2}\right] / (d^2(1+(c x^{2n})/a)^p) - (2e x^{1+n}(a+c x^{2n}))^p \text{AppellF1}\left[\frac{1+n}{2n}, -p, 2, \frac{3+n(-1)}{2}, -\left(\frac{c x^{2n}}{a}\right), \frac{e^{2x^{2n}}}{d^2}\right] / (d^3(1+n)(1+(c x^{2n})/a)^p)$

**Rubi [A]** time = 0.235468, antiderivative size = 261, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$ , Rules used = {1438, 430, 429, 511, 510}

$$\frac{2ex^{n+1}(a+cx^{2n})^p \left(\frac{cx^{2n}}{a} + 1\right)^{-p} F_1\left(\frac{n+1}{2n}; -p, 2; \frac{1}{2}\left(3 + \frac{1}{n}\right); -\frac{cx^{2n}}{a}, \frac{e^2x^{2n}}{d^2}\right)}{d^3(n+1)} + \frac{e^2x^{2n+1}(a+cx^{2n})^p \left(\frac{cx^{2n}}{a} + 1\right)^{-p} F_1\left(\frac{1}{2}\left(2 + \frac{1}{n}\right)\right)}{d^4(2n+1)}$$

Antiderivative was successfully verified.

[In] Int[(a + c\*x^(2\*n))^p/(d + e\*x^n)^2,x]

[Out]  $(e^{2x^{1+2n}}(a+c x^{2n}))^p \text{AppellF1}\left[\frac{2+n(-1)}{2}, -p, 2, \frac{4+n(-1)}{2}, -\left(\frac{c x^{2n}}{a}\right), \frac{e^{2x^{2n}}}{d^2}\right] / (d^4(1+2n)(1+(c x^{2n})/a)^p) + (x(a+c x^{2n}))^p \text{AppellF1}\left[\frac{1}{2n}, -p, 2, \frac{2+n(-1)}{2}, -\left(\frac{c x^{2n}}{a}\right), \frac{e^{2x^{2n}}}{d^2}\right] / (d^2(1+(c x^{2n})/a)^p) - (2e x^{1+n}(a+c x^{2n}))^p \text{AppellF1}\left[\frac{1+n}{2n}, -p, 2, \frac{3+n(-1)}{2}, -\left(\frac{c x^{2n}}{a}\right), \frac{e^{2x^{2n}}}{d^2}\right] / (d^3(1+n)(1+(c x^{2n})/a)^p)$

### Rule 1438

Int[((d\_) + (e\_.)\*(x\_)^(n\_))^(q\_)\*((a\_) + (c\_.)\*(x\_)^(n2\_))^(p\_), x\_Symbol] :> Int[ExpandIntegrand[(a + c\*x^(2\*n))^p, (d/(d^2 - e^2\*x^(2\*n)) - (e\*x^n)/(d^2 - e^2\*x^(2\*n)))^(-q), x], x] /; FreeQ[{a, c, d, e, n, p}, x] && EqQ[n, 2, 2\*n] && NeQ[c\*d^2 + a\*e^2, 0] && !IntegerQ[p] && ILtQ[q, 0]

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
  Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q},
  x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
&& (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
NeQ[m, n - 1] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
  b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
  - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(a + cx^{2n})^p}{(d + ex^n)^2} dx &= \int \left( \frac{d^2 (a + cx^{2n})^p}{(d^2 - e^2 x^{2n})^2} - \frac{2dex^n (a + cx^{2n})^p}{(-d^2 + e^2 x^{2n})^2} + \frac{e^2 x^{2n} (a + cx^{2n})^p}{(-d^2 + e^2 x^{2n})^2} \right) dx \\
&= d^2 \int \frac{(a + cx^{2n})^p}{(d^2 - e^2 x^{2n})^2} dx - (2de) \int \frac{x^n (a + cx^{2n})^p}{(-d^2 + e^2 x^{2n})^2} dx + e^2 \int \frac{x^{2n} (a + cx^{2n})^p}{(-d^2 + e^2 x^{2n})^2} dx \\
&= \left( d^2 (a + cx^{2n})^p \left( 1 + \frac{cx^{2n}}{a} \right)^{-p} \right) \int \frac{\left( 1 + \frac{cx^{2n}}{a} \right)^p}{(d^2 - e^2 x^{2n})^2} dx - \left( 2de (a + cx^{2n})^p \left( 1 + \frac{cx^{2n}}{a} \right)^{-p} \right) \int \frac{x^n \left( 1 + \frac{cx^{2n}}{a} \right)^p}{(-d^2 + e^2 x^{2n})^2} dx \\
&= \frac{e^2 x^{1+2n} (a + cx^{2n})^p \left( 1 + \frac{cx^{2n}}{a} \right)^{-p} F_1 \left( \frac{1}{2} \left( 2 + \frac{1}{n} \right); -p, 2; \frac{1}{2} \left( 4 + \frac{1}{n} \right); -\frac{cx^{2n}}{a}, \frac{e^2 x^{2n}}{d^2} \right)}{d^4 (1 + 2n)} + \frac{x (a + cx^{2n})^p \left( 1 + \frac{cx^{2n}}{a} \right)^{-p}}{d^4 (1 + 2n)}
\end{aligned}$$

**Mathematica [F]** time = 0.102836, size = 0, normalized size = 0.

$$\int \frac{(a + cx^{2n})^p}{(d + ex^n)^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[(a + c\*x^(2\*n))^p/(d + e\*x^n)^2,x]

[Out] Integrate[(a + c\*x^(2\*n))^p/(d + e\*x^n)^2, x]

**Maple [F]** time = 0.076, size = 0, normalized size = 0.

$$\int \frac{(a + cx^{2n})^p}{(d + ex^n)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+c\*x^(2\*n))^p/(d+e\*x^n)^2,x)

[Out] int((a+c\*x^(2\*n))^p/(d+e\*x^n)^2,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx^{2n} + a)^p}{(ex^n + d)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+c\*x^(2\*n))^p/(d+e\*x^n)^2,x, algorithm="maxima")

[Out] integrate((c\*x^(2\*n) + a)^p/(e\*x^n + d)^2, x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(cx^{2n} + a)^p}{e^2x^{2n} + 2dex^n + d^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+c\*x^(2\*n))^p/(d+e\*x^n)^2,x, algorithm="fricas")

[Out] integral((c\*x^(2\*n) + a)^p/(e^2\*x^(2\*n) + 2\*d\*e\*x^n + d^2), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+c\*x\*\*(2\*n))\*\*p/(d+e\*x\*\*n)\*\*2,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx^{2n} + a)^p}{(ex^n + d)^2} dx$$



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c*x^(2*n))^p/(d+e*x^n)^2,x, algorithm="giac")
```

```
[Out] integrate((c*x^(2*n) + a)^p/(e*x^n + d)^2, x)
```

$$3.65 \quad \int \frac{(a+cx^{2n})^p}{(d+ex^n)^3} dx$$

**Optimal.** Leaf size=357

$$\frac{3ex^{n+1}(a+cx^{2n})^p\left(\frac{cx^{2n}}{a}+1\right)^{-p}F_1\left(\frac{n+1}{2n};-p,3;\frac{1}{2}\left(3+\frac{1}{n}\right);-\frac{cx^{2n}}{a},\frac{e^2x^{2n}}{d^2}\right)}{d^4(n+1)} + \frac{3e^2x^{2n+1}(a+cx^{2n})^p\left(\frac{cx^{2n}}{a}+1\right)^{-p}F_1\left(\frac{1}{2}\left(2+\frac{1}{n}\right)\right)}{d^5(2n+1)}$$

[Out] (3\*e^2\*x^(1+2\*n)\*(a+c\*x^(2\*n))^p\*AppellF1[(2+n^(-1))/2,-p,3,(4+n^(-1))/2,-((c\*x^(2\*n))/a),(e^2\*x^(2\*n))/d^2])/(d^5\*(1+2\*n)\*(1+(c\*x^(2\*n))/a)^p) - (e^3\*x^(1+3\*n)\*(a+c\*x^(2\*n))^p\*AppellF1[(3+n^(-1))/2,-p,3,(5+n^(-1))/2,-((c\*x^(2\*n))/a),(e^2\*x^(2\*n))/d^2])/(d^6\*(1+3\*n)\*(1+(c\*x^(2\*n))/a)^p) + (x\*(a+c\*x^(2\*n))^p\*AppellF1[1/(2\*n),-p,3,(2+n^(-1))/2,-((c\*x^(2\*n))/a),(e^2\*x^(2\*n))/d^2])/(d^3\*(1+(c\*x^(2\*n))/a)^p) - (3\*e\*x^(1+n)\*(a+c\*x^(2\*n))^p\*AppellF1[(1+n)/(2\*n),-p,3,(3+n^(-1))/2,-((c\*x^(2\*n))/a),(e^2\*x^(2\*n))/d^2])/(d^4\*(1+n)\*(1+(c\*x^(2\*n))/a)^p)

**Rubi [A]** time = 0.338036, antiderivative size = 357, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 5, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$ , Rules used = {1438, 430, 429, 511, 510}

$$\frac{3ex^{n+1}(a+cx^{2n})^p\left(\frac{cx^{2n}}{a}+1\right)^{-p}F_1\left(\frac{n+1}{2n};-p,3;\frac{1}{2}\left(3+\frac{1}{n}\right);-\frac{cx^{2n}}{a},\frac{e^2x^{2n}}{d^2}\right)}{d^4(n+1)} + \frac{3e^2x^{2n+1}(a+cx^{2n})^p\left(\frac{cx^{2n}}{a}+1\right)^{-p}F_1\left(\frac{1}{2}\left(2+\frac{1}{n}\right)\right)}{d^5(2n+1)}$$

Antiderivative was successfully verified.

[In] Int[(a + c\*x^(2\*n))^p/(d + e\*x^n)^3,x]

[Out] (3\*e^2\*x^(1+2\*n)\*(a+c\*x^(2\*n))^p\*AppellF1[(2+n^(-1))/2,-p,3,(4+n^(-1))/2,-((c\*x^(2\*n))/a),(e^2\*x^(2\*n))/d^2])/(d^5\*(1+2\*n)\*(1+(c\*x^(2\*n))/a)^p) - (e^3\*x^(1+3\*n)\*(a+c\*x^(2\*n))^p\*AppellF1[(3+n^(-1))/2,-p,3,(5+n^(-1))/2,-((c\*x^(2\*n))/a),(e^2\*x^(2\*n))/d^2])/(d^6\*(1+3\*n)\*(1+(c\*x^(2\*n))/a)^p) + (x\*(a+c\*x^(2\*n))^p\*AppellF1[1/(2\*n),-p,3,(2+n^(-1))/2,-((c\*x^(2\*n))/a),(e^2\*x^(2\*n))/d^2])/(d^3\*(1+(c\*x^(2\*n))/a)^p) - (3\*e\*x^(1+n)\*(a+c\*x^(2\*n))^p\*AppellF1[(1+n)/(2\*n),-p,3,(3+n^(-1))/2,-((c\*x^(2\*n))/a),(e^2\*x^(2\*n))/d^2])/(d^4\*(1+n)\*(1+(c\*x^(2\*n))/a)^p)

**Rule 1438**

```
Int[((d_) + (e_)*(x_)^(n_))^(q_)*((a_) + (c_)*(x_)^(n2_))^(p_), x_Symbol]
:> Int[ExpandIntegrand[(a + c*x^(2*n))^p, (d/(d^2 - e^2*x^(2*n)) - (e*x^n)/(d^2 - e^2*x^(2*n)))^(-q), x], x] /; FreeQ[{a, c, d, e, n, p}, x] && EqQ[n 2, 2*n] && NeQ[c*d^2 + a*e^2, 0] && !IntegerQ[p] && ILtQ[q, 0]
```

### Rule 430

```
Int[((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q},
x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] && !(IntegerQ[p] || GtQ[a, 0])
```

### Rule 429

```
Int[((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
&& (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

### Rule 511

```
Int[((e_)*(x_)^(m_))*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
NeQ[m, n - 1] && !(IntegerQ[p] || GtQ[a, 0])
```

### Rule 510

```
Int[((e_)*(x_)^(m_))*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n - 1]
&& (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

### Rubi steps

$$\begin{aligned}
\int \frac{(a + cx^{2n})^p}{(d + ex^n)^3} dx &= \int \left( \frac{d^3 (a + cx^{2n})^p}{(d^2 - e^2 x^{2n})^3} + \frac{3d^2 ex^n (a + cx^{2n})^p}{(-d^2 + e^2 x^{2n})^3} - \frac{3de^2 x^{2n} (a + cx^{2n})^p}{(-d^2 + e^2 x^{2n})^3} + \frac{e^3 x^{3n} (a + cx^{2n})^p}{(-d^2 + e^2 x^{2n})^3} \right) dx \\
&= d^3 \int \frac{(a + cx^{2n})^p}{(d^2 - e^2 x^{2n})^3} dx + (3d^2 e) \int \frac{x^n (a + cx^{2n})^p}{(-d^2 + e^2 x^{2n})^3} dx - (3de^2) \int \frac{x^{2n} (a + cx^{2n})^p}{(-d^2 + e^2 x^{2n})^3} dx + e^3 \int \frac{x^{3n} (a + cx^{2n})^p}{(-d^2 + e^2 x^{2n})^3} dx \\
&= \left( d^3 (a + cx^{2n})^p \left( 1 + \frac{cx^{2n}}{a} \right)^{-p} \right) \int \frac{\left( 1 + \frac{cx^{2n}}{a} \right)^p}{(d^2 - e^2 x^{2n})^3} dx + \left( 3d^2 e (a + cx^{2n})^p \left( 1 + \frac{cx^{2n}}{a} \right)^{-p} \right) \int \frac{x^n \left( 1 + \frac{cx^{2n}}{a} \right)^p}{(-d^2 + e^2 x^{2n})^3} dx \\
&= \frac{3e^2 x^{1+2n} (a + cx^{2n})^p \left( 1 + \frac{cx^{2n}}{a} \right)^{-p} F_1 \left( \frac{1}{2} \left( 2 + \frac{1}{n} \right); -p, 3; \frac{1}{2} \left( 4 + \frac{1}{n} \right); -\frac{cx^{2n}}{a}, \frac{e^2 x^{2n}}{d^2} \right)}{d^5 (1 + 2n)} - \frac{e^3 x^{1+3n} (a + cx^{2n})^p}{d^5 (1 + 2n)}
\end{aligned}$$

**Mathematica [F]** time = 0.294008, size = 0, normalized size = 0.

$$\int \frac{(a + cx^{2n})^p}{(d + ex^n)^3} dx$$

Verification is Not applicable to the result.

[In] Integrate[(a + c\*x^(2\*n))^p/(d + e\*x^n)^3,x]

[Out] Integrate[(a + c\*x^(2\*n))^p/(d + e\*x^n)^3, x]

**Maple [F]** time = 0.086, size = 0, normalized size = 0.

$$\int \frac{(a + cx^{2n})^p}{(d + ex^n)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+c\*x^(2\*n))^p/(d+e\*x^n)^3,x)

[Out] int((a+c\*x^(2\*n))^p/(d+e\*x^n)^3,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx^{2n} + a)^p}{(ex^n + d)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+c\*x^(2\*n))^p/(d+e\*x^n)^3,x, algorithm="maxima")

[Out] integrate((c\*x^(2\*n) + a)^p/(e\*x^n + d)^3, x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(cx^{2n} + a)^p}{e^3x^{3n} + 3de^2x^{2n} + 3d^2ex^n + d^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+c\*x^(2\*n))^p/(d+e\*x^n)^3,x, algorithm="fricas")

[Out] integral((c\*x^(2\*n) + a)^p/(e^3\*x^(3\*n) + 3\*d\*e^2\*x^(2\*n) + 3\*d^2\*e\*x^n + d^3), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+c\*x\*\*(2\*n))\*\*p/(d+e\*x\*\*n)\*\*3,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx^{2n} + a)^p}{(ex^n + d)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c*x^(2*n))^p/(d+e*x^n)^3,x, algorithm="giac")
```

```
[Out] integrate((c*x^(2*n) + a)^p/(e*x^n + d)^3, x)
```

### 3.66 $\int (d + ex^n) (a + bx^n + cx^{2n}) dx$

**Optimal.** Leaf size=62

$$\frac{x^{n+1}(ae + bd)}{n + 1} + adx + \frac{x^{2n+1}(be + cd)}{2n + 1} + \frac{cex^{3n+1}}{3n + 1}$$

[Out] a\*d\*x + ((b\*d + a\*e)\*x^(1 + n))/(1 + n) + ((c\*d + b\*e)\*x^(1 + 2\*n))/(1 + 2\*n) + (c\*e\*x^(1 + 3\*n))/(1 + 3\*n)

**Rubi [A]** time = 0.038973, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$ , Rules used = {1407}

$$\frac{x^{n+1}(ae + bd)}{n + 1} + adx + \frac{x^{2n+1}(be + cd)}{2n + 1} + \frac{cex^{3n+1}}{3n + 1}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x^n)\*(a + b\*x^n + c\*x^(2\*n)), x]

[Out] a\*d\*x + ((b\*d + a\*e)\*x^(1 + n))/(1 + n) + ((c\*d + b\*e)\*x^(1 + 2\*n))/(1 + 2\*n) + (c\*e\*x^(1 + 3\*n))/(1 + 3\*n)

#### Rule 1407

Int[((d\_) + (e\_)\*(x\_)^(n\_))^(q\_)\*((a\_) + (b\_)\*(x\_)^(n\_) + (c\_)\*(x\_)^(n2\_)), x\_Symbol] :> Int[ExpandIntegrand[(d + e\*x^n)^q\*(a + b\*x^n + c\*x^(2\*n)), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2\*n] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && IGtQ[q, 0]

#### Rubi steps

$$\begin{aligned} \int (d + ex^n) (a + bx^n + cx^{2n}) dx &= \int (ad + (bd + ae)x^n + (cd + be)x^{2n} + cex^{3n}) dx \\ &= adx + \frac{(bd + ae)x^{1+n}}{1 + n} + \frac{(cd + be)x^{1+2n}}{1 + 2n} + \frac{cex^{1+3n}}{1 + 3n} \end{aligned}$$

**Mathematica [A]** time = 0.147129, size = 57, normalized size = 0.92

$$x \left( \frac{x^n(ae + bd)}{n + 1} + ad + \frac{x^{2n}(be + cd)}{2n + 1} + \frac{cex^{3n}}{3n + 1} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(d + e\*x^n)\*(a + b\*x^n + c\*x^(2\*n)),x]

[Out]  $x*(a*d + ((b*d + a*e)*x^n)/(1 + n) + ((c*d + b*e)*x^(2*n))/(1 + 2*n) + (c*e*x^(3*n))/(1 + 3*n))$

**Maple [A]** time = 0.01, size = 66, normalized size = 1.1

$$adx + \frac{(ae + bd)xe^{n \ln(x)}}{1 + n} + \frac{(be + cd)x(e^{n \ln(x)})^2}{1 + 2n} + \frac{cex(e^{n \ln(x)})^3}{1 + 3n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d+e\*x^n)\*(a+b\*x^n+c\*x^(2\*n)),x)

[Out]  $a*d*x+(a*e+b*d)/(1+n)*x*\exp(n*\ln(x))+(b*e+c*d)/(1+2*n)*x*\exp(n*\ln(x))^2+c*e/(1+3*n)*x*\exp(n*\ln(x))^3$

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e\*x^n)\*(a+b\*x^n+c\*x^(2\*n)),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [B]** time = 1.43046, size = 315, normalized size = 5.08

$$\frac{(2cen^2 + 3cen + ce)xx^{3n} + (3(cd + be)n^2 + cd + be + 4(cd + be)n)xx^{2n} + (6(bd + ae)n^2 + bd + ae + 5(bd + ae)n)xx^n}{6n^3 + 11n^2 + 6n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e\*x^n)\*(a+b\*x^n+c\*x^(2\*n)),x, algorithm="fricas")



[Out]  $((2*c*e*n^2 + 3*c*e*n + c*e)*x*x^(3*n) + (3*(c*d + b*e)*n^2 + c*d + b*e + 4*(c*d + b*e)*n)*x*x^(2*n) + (6*(b*d + a*e)*n^2 + b*d + a*e + 5*(b*d + a*e)*n)*x*x^n + (6*a*d*n^3 + 11*a*d*n^2 + 6*a*d*n + a*d)*x)/(6*n^3 + 11*n^2 + 6*n + 1)$

**Sympy [A]** time = 1.40157, size = 656, normalized size = 10.58

$$\left\{ \begin{array}{l} \text{adx} + \text{ae} \log(x) + \text{bd} \log(x) - \frac{\text{be}}{x} - \frac{\text{cd}}{x} - \frac{\text{ce}}{2x^2} \\ \text{adx} + 2\text{ae}\sqrt{x} + 2\text{bd}\sqrt{x} + \text{be} \log(x) + \text{cd} \log(x) - \frac{2\text{ce}}{\sqrt{x}} \\ \text{adx} + \frac{3\text{aex}^{\frac{2}{3}}}{6\text{adn}^{\frac{2}{3}}x} + \frac{3\text{bdx}^{\frac{2}{3}}}{2} + 3\text{be}\sqrt[3]{x} + 3\text{cd}\sqrt[3]{x} + \text{ce} \log(x) \end{array} \right.$$

$$\frac{\text{adx}}{6n^3+11n^2+6n+1} + \frac{11\text{adn}^2x}{6n^3+11n^2+6n+1} + \frac{6\text{adnx}}{6n^3+11n^2+6n+1} + \frac{\text{adx}}{6n^3+11n^2+6n+1} + \frac{6\text{aen}^2xx^n}{6n^3+11n^2+6n+1} + \frac{5\text{aenxx}^n}{6n^3+11n^2+6n+1} + \frac{\text{aexx}^n}{6n^3+11n^2+6n+1} + \frac{6\text{ce}}{6n^3+11n^2+6n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e\*x\*\*n)\*(a+b\*x\*\*n+c\*x\*\*(2\*n)), x)

[Out] Piecewise((a\*d\*x + a\*e\*log(x) + b\*d\*log(x) - b\*e/x - c\*d/x - c\*e/(2\*x\*\*2), Eq(n, -1)), (a\*d\*x + 2\*a\*e\*sqrt(x) + 2\*b\*d\*sqrt(x) + b\*e\*log(x) + c\*d\*log(x) - 2\*c\*e/sqrt(x), Eq(n, -1/2)), (a\*d\*x + 3\*a\*e\*x\*\*(2/3)/2 + 3\*b\*d\*x\*\*(2/3)/2 + 3\*b\*e\*x\*\*(1/3) + 3\*c\*d\*x\*\*(1/3) + c\*e\*log(x), Eq(n, -1/3)), (6\*a\*d\*n\*\*3\*x/(6\*n\*\*3 + 11\*n\*\*2 + 6\*n + 1) + 11\*a\*d\*n\*\*2\*x/(6\*n\*\*3 + 11\*n\*\*2 + 6\*n + 1) + 6\*a\*d\*n\*x/(6\*n\*\*3 + 11\*n\*\*2 + 6\*n + 1) + a\*d\*x/(6\*n\*\*3 + 11\*n\*\*2 + 6\*n + 1) + 6\*a\*e\*n\*\*2\*x\*x\*\*n/(6\*n\*\*3 + 11\*n\*\*2 + 6\*n + 1) + 5\*a\*e\*n\*x\*x\*\*n/(6\*n\*\*3 + 11\*n\*\*2 + 6\*n + 1) + a\*e\*x\*x\*\*n/(6\*n\*\*3 + 11\*n\*\*2 + 6\*n + 1) + 6\*b\*d\*n\*\*2\*x\*x\*\*n/(6\*n\*\*3 + 11\*n\*\*2 + 6\*n + 1) + 5\*b\*d\*n\*x\*x\*\*n/(6\*n\*\*3 + 11\*n\*\*2 + 6\*n + 1) + b\*d\*x\*x\*\*n/(6\*n\*\*3 + 11\*n\*\*2 + 6\*n + 1) + 3\*b\*e\*n\*\*2\*x\*x\*\*(2\*n)/(6\*n\*\*3 + 11\*n\*\*2 + 6\*n + 1) + 4\*b\*e\*n\*x\*x\*\*(2\*n)/(6\*n\*\*3 + 11\*n\*\*2 + 6\*n + 1) + b\*e\*x\*x\*\*(2\*n)/(6\*n\*\*3 + 11\*n\*\*2 + 6\*n + 1) + 3\*c\*d\*n\*\*2\*x\*x\*\*(2\*n)/(6\*n\*\*3 + 11\*n\*\*2 + 6\*n + 1) + 4\*c\*d\*n\*x\*x\*\*(2\*n)/(6\*n\*\*3 + 11\*n\*\*2 + 6\*n + 1) + c\*d\*x\*x\*\*(2\*n)/(6\*n\*\*3 + 11\*n\*\*2 + 6\*n + 1) + 2\*c\*e\*n\*\*2\*x\*x\*\*(3\*n)/(6\*n\*\*3 + 11\*n\*\*2 + 6\*n + 1) + 3\*c\*e\*n\*x\*x\*\*(3\*n)/(6\*n\*\*3 + 11\*n\*\*2 + 6\*n + 1) + c\*e\*x\*x\*\*(3\*n)/(6\*n\*\*3 + 11\*n\*\*2 + 6\*n + 1), True))

**Giac [B]** time = 1.13671, size = 279, normalized size = 4.5

$$\frac{6\text{adn}^3x + 3\text{cdn}^2xx^{2n} + 6\text{bdn}^2xx^n + 2\text{cn}^2xx^{3n}e + 3\text{bn}^2xx^{2n}e + 6\text{an}^2xx^n e + 11\text{adn}^2x + 4\text{cdnxx}^{2n} + 5\text{bdnxx}^n + 3\text{cnxx}^n}{6n^3 + 11n^2 + 6n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x^n)*(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
```

```
[Out] (6*a*d*n^3*x + 3*c*d*n^2*x*x^(2*n) + 6*b*d*n^2*x*x^n + 2*c*n^2*x*x^(3*n)*e  
+ 3*b*n^2*x*x^(2*n)*e + 6*a*n^2*x*x^n*e + 11*a*d*n^2*x + 4*c*d*n*x*x^(2*n)  
+ 5*b*d*n*x*x^n + 3*c*n*x*x^(3*n)*e + 4*b*n*x*x^(2*n)*e + 5*a*n*x*x^n*e + 6  
*a*d*n*x + c*d*x*x^(2*n) + b*d*x*x^n + c*x*x^(3*n)*e + b*x*x^(2*n)*e + a*x*  
x^n*e + a*d*x)/(6*n^3 + 11*n^2 + 6*n + 1)
```

### 3.67 $\int (d + ex^n) (a + bx^n + cx^{2n})^2 dx$

**Optimal.** Leaf size=132

$$a^2 dx + \frac{x^{2n+1} (2abe + 2acd + b^2 d)}{2n+1} + \frac{x^{3n+1} (2ace + b^2 e + 2bcd)}{3n+1} + \frac{ax^{n+1} (ae + 2bd)}{n+1} + \frac{cx^{4n+1} (2be + cd)}{4n+1} + \frac{c^2 ex^{5n+1}}{5n+1}$$

[Out]  $a^2 d x + (a(2 b d + a e) x^{1+n}) / (1+n) + ((b^2 d + 2 a c d + 2 a b e) x^{1+2 n}) / (1+2 n) + ((2 b c d + b^2 e + 2 a c e) x^{1+3 n}) / (1+3 n) + (c(c d + 2 b e) x^{1+4 n}) / (1+4 n) + (c^2 e x^{1+5 n}) / (1+5 n)$

**Rubi [A]** time = 0.101675, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$ , Rules used = {1432}

$$a^2 dx + \frac{x^{2n+1} (2abe + 2acd + b^2 d)}{2n+1} + \frac{x^{3n+1} (2ace + b^2 e + 2bcd)}{3n+1} + \frac{ax^{n+1} (ae + 2bd)}{n+1} + \frac{cx^{4n+1} (2be + cd)}{4n+1} + \frac{c^2 ex^{5n+1}}{5n+1}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x^n)\*(a + b\*x^n + c\*x^(2\*n))^2,x]

[Out]  $a^2 d x + (a(2 b d + a e) x^{1+n}) / (1+n) + ((b^2 d + 2 a c d + 2 a b e) x^{1+2 n}) / (1+2 n) + ((2 b c d + b^2 e + 2 a c e) x^{1+3 n}) / (1+3 n) + (c(c d + 2 b e) x^{1+4 n}) / (1+4 n) + (c^2 e x^{1+5 n}) / (1+5 n)$

#### Rule 1432

Int[((d\_) + (e\_)\*(x\_)^(n\_))\*((a\_) + (b\_)\*(x\_)^(n\_) + (c\_)\*(x\_)^(2n\_))^(p\_), x\_Symbol] :> Int[ExpandIntegrand[(d + e\*x^n)\*(a + b\*x^n + c\*x^(2\*n))^p, x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2\*n] && NeQ[b^2 - 4\*a\*c, 0]

#### Rubi steps

$$\int (d + ex^n)(a + bx^n + cx^{2n})^2 dx = \int (a^2d + a(2bd + ae)x^n + (b^2d + 2acd + 2abe)x^{2n} + (2bcd + b^2e + 2ace)x^{3n} + c(cd + 2be + cd)x^{4n} + c^2ex^{5n}) dx$$

$$= a^2dx + \frac{a(2bd + ae)x^{1+n}}{1+n} + \frac{(b^2d + 2acd + 2abe)x^{1+2n}}{1+2n} + \frac{(2bcd + b^2e + 2ace)x^{1+3n}}{1+3n} + \frac{c(cd + 2be + cd)x^{1+4n}}{1+4n} + \frac{c^2ex^{5n}}{5n+1}$$

**Mathematica [A]** time = 0.254792, size = 123, normalized size = 0.93

$$x \left( a^2d + \frac{x^{2n}(2abe + 2acd + b^2d)}{2n+1} + \frac{x^{3n}(2ace + b^2e + 2bcd)}{3n+1} + \frac{ax^n(ae + 2bd)}{n+1} + \frac{cx^{4n}(2be + cd)}{4n+1} + \frac{c^2ex^{5n}}{5n+1} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(d + e\*x^n)\*(a + b\*x^n + c\*x^(2\*n))^2,x]

[Out] x\*(a^2\*d + (a\*(2\*b\*d + a\*e)\*x^n)/(1 + n) + ((b^2\*d + 2\*a\*c\*d + 2\*a\*b\*e)\*x^(2\*n))/(1 + 2\*n) + ((2\*b\*c\*d + b^2\*e + 2\*a\*c\*e)\*x^(3\*n))/(1 + 3\*n) + (c\*(c\*d + 2\*b\*e)\*x^(4\*n))/(1 + 4\*n) + (c^2\*e\*x^(5\*n))/(1 + 5\*n))

**Maple [A]** time = 0.013, size = 138, normalized size = 1.1

$$a^2dx + \frac{(2ace + b^2e + 2bcd)x(e^{n\ln(x)})^3}{1+3n} + \frac{(2abe + 2acd + b^2d)x(e^{n\ln(x)})^2}{1+2n} + \frac{a(ae + 2bd)xe^{n\ln(x)}}{1+n} + \frac{c(2be + cd)x(e^{n\ln(x)})}{1+4n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d+e\*x^n)\*(a+b\*x^n+c\*x^(2\*n))^2,x)

[Out] a^2\*d\*x+(2\*a\*c\*e+b^2\*e+2\*b\*c\*d)/(1+3\*n)\*x\*exp(n\*ln(x))^3+(2\*a\*b\*e+2\*a\*c\*d+b^2\*d)/(1+2\*n)\*x\*exp(n\*ln(x))^2+a\*(a\*e+2\*b\*d)/(1+n)\*x\*exp(n\*ln(x))+c\*(2\*b\*e+c\*d)/(1+4\*n)\*x\*exp(n\*ln(x))^4+c^2\*e/(1+5\*n)\*x\*exp(n\*ln(x))^5

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x^n)*(a+b*x^n+c*x^(2*n))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

**Fricas [B]** time = 1.38937, size = 1161, normalized size = 8.8

$$\frac{(24c^2en^4 + 50c^2en^3 + 35c^2en^2 + 10c^2en + c^2e)xx^{5n} + (30(c^2d + 2bce)n^4 + 61(c^2d + 2bce)n^3 + c^2d + 2bce + 41(c^2d + 2bce)n^2 + 11(c^2d + 2bce)n + b^2 + 2a*c)e)n^4 + 78(2*b*c*d + (b^2 + 2*a*c)*e)n^3 + 2*b*c*d + 49(2*b*c*d + (b^2 + 2*a*c)*e)n^2 + (b^2 + 2*a*c)*e + 12(2*b*c*d + (b^2 + 2*a*c)*e)*n)*x^{3n} + (60(2*a*b*e + (b^2 + 2*a*c)*d)n^4 + 107(2*a*b*e + (b^2 + 2*a*c)*d)n^3 + 2*a*b*e + 59(2*a*b*e + (b^2 + 2*a*c)*d)n^2 + (b^2 + 2*a*c)*d + 13(2*a*b*e + (b^2 + 2*a*c)*d)*n)*x^{2n} + (120(2*a*b*d + a^2*e)n^4 + 154(2*a*b*d + a^2*e)n^3 + 2*a*b*d + a^2*e + 71(2*a*b*d + a^2*e)n^2 + 14(2*a*b*d + a^2*e)*n)*x^n + (120*a^2*d*n^5 + 274*a^2*d*n^4 + 225*a^2*d*n^3 + 85*a^2*d*n^2 + 15*a^2*d*n + a^2*d)*x)/(120*n^5 + 274*n^4 + 225*n^3 + 85*n^2 + 15*n + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x^n)*(a+b*x^n+c*x^(2*n))^2,x, algorithm="fricas")
```

```
[Out] ((24*c^2*e*n^4 + 50*c^2*e*n^3 + 35*c^2*e*n^2 + 10*c^2*e*n + c^2*e)*x*x^(5*n)
) + (30*(c^2*d + 2*b*c*e)*n^4 + 61*(c^2*d + 2*b*c*e)*n^3 + c^2*d + 2*b*c*e
+ 41*(c^2*d + 2*b*c*e)*n^2 + 11*(c^2*d + 2*b*c*e)*n)*x*x^(4*n) + (40*(2*b*c
*d + (b^2 + 2*a*c)*e)*n^4 + 78*(2*b*c*d + (b^2 + 2*a*c)*e)*n^3 + 2*b*c*d +
49*(2*b*c*d + (b^2 + 2*a*c)*e)*n^2 + (b^2 + 2*a*c)*e + 12*(2*b*c*d + (b^2 +
2*a*c)*e)*n)*x*x^(3*n) + (60*(2*a*b*e + (b^2 + 2*a*c)*d)*n^4 + 107*(2*a*b*
e + (b^2 + 2*a*c)*d)*n^3 + 2*a*b*e + 59*(2*a*b*e + (b^2 + 2*a*c)*d)*n^2 + (
b^2 + 2*a*c)*d + 13*(2*a*b*e + (b^2 + 2*a*c)*d)*n)*x*x^(2*n) + (120*(2*a*b*
d + a^2*e)*n^4 + 154*(2*a*b*d + a^2*e)*n^3 + 2*a*b*d + a^2*e + 71*(2*a*b*d
+ a^2*e)*n^2 + 14*(2*a*b*d + a^2*e)*n)*x*x^n + (120*a^2*d*n^5 + 274*a^2*d*n
^4 + 225*a^2*d*n^3 + 85*a^2*d*n^2 + 15*a^2*d*n + a^2*d)*x)/(120*n^5 + 274*n
^4 + 225*n^3 + 85*n^2 + 15*n + 1)
```

**Sympy [A]** time = 13.2015, size = 3128, normalized size = 23.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x**n)*(a+b*x**n+c*x**(2*n))**2,x)
```

```
[Out] Piecewise((a**2*d*x + a**2*e*log(x) + 2*a*b*d*log(x) - 2*a*b*e/x - 2*a*c*d/
x - a*c*e/x**2 - b**2*d/x - b**2*e/(2*x**2) - b*c*d/x**2 - 2*b*c*e/(3*x**3)
- c**2*d/(3*x**3) - c**2*e/(4*x**4), Eq(n, -1)), (a**2*d*x + 2*a**2*e*sqrt
(x) + 4*a*b*d*sqrt(x) + 2*a*b*e*log(x) + 2*a*c*d*log(x) - 4*a*c*e/sqrt(x) +
b**2*d*log(x) - 2*b**2*e/sqrt(x) - 4*b*c*d/sqrt(x) - 2*b*c*e/x - c**2*d/x
```

$- 2c^{**2}e/(3x^{**}(3/2)), Eq(n, -1/2)), (a^{**2}d*x + 3a^{**2}e*x^{**}(2/3)/2 + 3a*b*d*x^{**}(2/3) + 6a*b*e*x^{**}(1/3) + 6a*c*d*x^{**}(1/3) + 2a*c*e*log(x) + 3b^{**2}d*x^{**}(1/3) + b^{**2}e*log(x) + 2b*c*d*log(x) - 6b*c*e/x^{**}(1/3) - 3c^{**2}d/x^{**}(1/3) - 3c^{**2}e/(2x^{**}(2/3)), Eq(n, -1/3)), (a^{**2}d*x + 4a^{**2}e*x^{**}(3/4)/3 + 8a*b*d*x^{**}(3/4)/3 + 4a*b*e*sqrt(x) + 4a*c*d*sqrt(x) + 8a*c*e*x^{**}(1/4) + 2b^{**2}d*sqrt(x) + 4b^{**2}e*x^{**}(1/4) + 8b*c*d*x^{**}(1/4) + 2b*c*e*log(x) + c^{**2}d*log(x) - 4c^{**2}e/x^{**}(1/4), Eq(n, -1/4)), (a^{**2}d*x + 5a^{**2}e*x^{**}(4/5)/4 + 5a*b*d*x^{**}(4/5)/2 + 10a*b*e*x^{**}(3/5)/3 + 10a*c*d*x^{**}(3/5)/3 + 5a*c*e*x^{**}(2/5) + 5b^{**2}d*x^{**}(3/5)/3 + 5b^{**2}e*x^{**}(2/5)/2 + 5b*c*d*x^{**}(2/5) + 10b*c*e*x^{**}(1/5) + 5c^{**2}d*x^{**}(1/5) + c^{**2}e*log(x), Eq(n, -1/5)), (120a^{**2}d*n^{**5}x/(120n^{**5} + 274n^{**4} + 225n^{**3} + 85n^{**2} + 15n + 1) + 274a^{**2}d*n^{**4}x/(120n^{**5} + 274n^{**4} + 225n^{**3} + 85n^{**2} + 15n + 1) + 225a^{**2}d*n^{**3}x/(120n^{**5} + 274n^{**4} + 225n^{**3} + 85n^{**2} + 15n + 1) + 85a^{**2}d*n^{**2}x/(120n^{**5} + 274n^{**4} + 225n^{**3} + 85n^{**2} + 15n + 1) + 15a^{**2}d*n*x/(120n^{**5} + 274n^{**4} + 225n^{**3} + 85n^{**2} + 15n + 1) + a^{**2}d*x/(120n^{**5} + 274n^{**4} + 225n^{**3} + 85n^{**2} + 15n + 1) + 120a^{**2}e*n^{**4}x*x^{**n}/(120n^{**5} + 274n^{**4} + 225n^{**3} + 85n^{**2} + 15n + 1) + 154a^{**2}e*n^{**3}x*x^{**n}/(120n^{**5} + 274n^{**4} + 225n^{**3} + 85n^{**2} + 15n + 1) + 71a^{**2}e*n^{**2}x*x^{**n}/(120n^{**5} + 274n^{**4} + 225n^{**3} + 85n^{**2} + 15n + 1) + 14a^{**2}e*n*x*x^{**n}/(120n^{**5} + 274n^{**4} + 225n^{**3} + 85n^{**2} + 15n + 1) + a^{**2}e*x*x^{**n}/(120n^{**5} + 274n^{**4} + 225n^{**3} + 85n^{**2} + 15n + 1) + 240a*b*d*n^{**4}x*x^{**n}/(120n^{**5} + 274n^{**4} + 225n^{**3} + 85n^{**2} + 15n + 1) + 308a*b*d*n^{**3}x*x^{**n}/(120n^{**5} + 274n^{**4} + 225n^{**3} + 85n^{**2} + 15n + 1) + 142a*b*d*n^{**2}x*x^{**n}/(120n^{**5} + 274n^{**4} + 225n^{**3} + 85n^{**2} + 15n + 1) + 28a*b*d*n*x*x^{**n}/(120n^{**5} + 274n^{**4} + 225n^{**3} + 85n^{**2} + 15n + 1) + 2a*b*d*x*x^{**n}/(120n^{**5} + 274n^{**4} + 225n^{**3} + 85n^{**2} + 15n + 1) + 120a*b*e*n^{**4}x*x^{**}(2*n)/(120n^{**5} + 274n^{**4} + 225n^{**3} + 85n^{**2} + 15n + 1) + 214a*b*e*n^{**3}x*x^{**}(2*n)/(120n^{**5} + 274n^{**4} + 225n^{**3} + 85n^{**2} + 15n + 1) + 118a*b*e*n^{**2}x*x^{**}(2*n)/(120n^{**5} + 274n^{**4} + 225n^{**3} + 85n^{**2} + 15n + 1) + 26a*b*e*n*x*x^{**}(2*n)/(120n^{**5} + 274n^{**4} + 225n^{**3} + 85n^{**2} + 15n + 1) + 2a*b*e*x*x^{**}(2*n)/(120n^{**5} + 274n^{**4} + 225n^{**3} + 85n^{**2} + 15n + 1) + 120a*c*d*n^{**4}x*x^{**}(2*n)/(120n^{**5} + 274n^{**4} + 225n^{**3} + 85n^{**2} + 15n + 1) + 214a*c*d*n^{**3}x*x^{**}(2*n)/(120n^{**5} + 274n^{**4} + 225n^{**3} + 85n^{**2} + 15n + 1) + 118a*c*d*n^{**2}x*x^{**}(2*n)/(120n^{**5} + 274n^{**4} + 225n^{**3} + 85n^{**2} + 15n + 1) + 26a*c*d*n*x*x^{**}(2*n)/(120n^{**5} + 274n^{**4} + 225n^{**3} + 85n^{**2} + 15n + 1) + 2a*c*d*x*x^{**}(2*n)/(120n^{**5} + 274n^{**4} + 225n^{**3} + 85n^{**2} + 15n + 1) + 80a*c*e*n^{**4}x*x^{**}(3*n)/(120n^{**5} + 274n^{**4} + 225n^{**3} + 85n^{**2} + 15n + 1) + 156a*c*e*n^{**3}x*x^{**}(3*n)/(120n^{**5} + 274n^{**4} + 225n^{**3} + 85n^{**2} + 15n + 1) + 98a*c*e*n^{**2}x*x^{**}(3*n)/(120n^{**5} + 274n^{**4} + 225n^{**3} + 85n^{**2} + 15n + 1) + 24a*c*e*n*x*x^{**}(3*n)/(120n^{**5} + 274n^{**4} + 225n^{**3} + 85n^{**2} + 15n + 1) + 2a*c*e*x*x^{**}(3*n)/(120n^{**5} + 274n^{**4} + 225n^{**3} + 85n^{**2} + 15n + 1) + 60b^{**2}d*n^{**4}x*x^{**}(2*n)/(120n^{**5} + 274n^{**4} + 225n^{**3} + 85n^{**2} + 15n + 1) + 107b^{**2}d*n^{**3}x*x^{**}(2*n)/(120n^{**5} + 274n^{**4} + 225n^{**3} + 85n^{**2} + 15n + 1) + 59b^{**2}d*n^{**2}x*x^{**}(2*n)/(120n^{**5} + 274n^{**4} + 225n^{**3} + 85n$

```

**2 + 15*n + 1) + 13*b**2*d*n*x*x**(2*n)/(120*n**5 + 274*n**4 + 225*n**3 +
85*n**2 + 15*n + 1) + b**2*d*x*x**(2*n)/(120*n**5 + 274*n**4 + 225*n**3 + 8
5*n**2 + 15*n + 1) + 40*b**2*e*n**4*x*x**(3*n)/(120*n**5 + 274*n**4 + 225*n
**3 + 85*n**2 + 15*n + 1) + 78*b**2*e*n**3*x*x**(3*n)/(120*n**5 + 274*n**4
+ 225*n**3 + 85*n**2 + 15*n + 1) + 49*b**2*e*n**2*x*x**(3*n)/(120*n**5 + 27
4*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 12*b**2*e*n*x*x**(3*n)/(120*n**5
+ 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + b**2*e*x*x**(3*n)/(120*n**5 +
274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 80*b*c*d*n**4*x*x**(3*n)/(120*
n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 156*b*c*d*n**3*x*x**(3*n
)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 98*b*c*d*n**2*x*x
**(3*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 24*b*c*d*n*
x*x**(3*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 2*b*c*d*
x*x**(3*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 60*b*c*e
*n**4*x*x**(4*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 12
2*b*c*e*n**3*x*x**(4*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n +
1) + 82*b*c*e*n**2*x*x**(4*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 1
5*n + 1) + 22*b*c*e*n*x*x**(4*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2
+ 15*n + 1) + 2*b*c*e*x*x**(4*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2
+ 15*n + 1) + 30*c**2*d*n**4*x*x**(4*n)/(120*n**5 + 274*n**4 + 225*n**3 + 8
5*n**2 + 15*n + 1) + 61*c**2*d*n**3*x*x**(4*n)/(120*n**5 + 274*n**4 + 225*n
**3 + 85*n**2 + 15*n + 1) + 41*c**2*d*n**2*x*x**(4*n)/(120*n**5 + 274*n**4
+ 225*n**3 + 85*n**2 + 15*n + 1) + 11*c**2*d*n*x*x**(4*n)/(120*n**5 + 274*n
**4 + 225*n**3 + 85*n**2 + 15*n + 1) + c**2*d*x*x**(4*n)/(120*n**5 + 274*n*
**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 24*c**2*e*n**4*x*x**(5*n)/(120*n**5 +
274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 50*c**2*e*n**3*x*x**(5*n)/(120
*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 35*c**2*e*n**2*x*x**(5*
n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 10*c**2*e*n*x*x*
*(5*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + c**2*e*x*x**
(5*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1), True))

```

---

**Giac [B]** time = 1.13853, size = 1118, normalized size = 8.47

$$120 a^2 d n^5 x + 30 c^2 d n^4 x x^{4n} + 80 b c d n^4 x x^{3n} + 60 b^2 d n^4 x x^{2n} + 120 a c d n^4 x x^{2n} + 240 a b d n^4 x x^n + 24 c^2 n^4 x x^{5n} e + 60 b c n$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e\*x^n)\*(a+b\*x^n+c\*x^(2\*n))^2,x, algorithm="giac")

[Out] (120\*a^2\*d\*n^5\*x + 30\*c^2\*d\*n^4\*x\*x^(4\*n) + 80\*b\*c\*d\*n^4\*x\*x^(3\*n) + 60\*b^2\*d\*n^4\*x\*x^(2\*n) + 120\*a\*c\*d\*n^4\*x\*x^(2\*n) + 240\*a\*b\*d\*n^4\*x\*x^n + 24\*c^2\*n^4\*x\*x^(5\*n)\*e + 60\*b\*c\*n^4\*x\*x^(4\*n)\*e + 40\*b^2\*n^4\*x\*x^(3\*n)\*e + 80\*a\*c\*n^4\*x\*x^(3\*n)\*e + 120\*a\*b\*n^4\*x\*x^(2\*n)\*e + 120\*a^2\*n^4\*x\*x^n\*e + 274\*a^2\*d

$$\begin{aligned}
& n^4 x + 61 c^2 d n^3 x x^{(4n)} + 156 b c d n^3 x x^{(3n)} + 107 b^2 d n^3 x x^{(2n)} + 214 a c d n^3 x x^{(2n)} + 308 a b d n^3 x x^n + 50 c^2 n^3 x x^{(5n)} e + 122 b c n^3 x x^{(4n)} e + 78 b^2 n^3 x x^{(3n)} e + 156 a c n^3 x x^{(3n)} e + 214 a b n^3 x x^{(2n)} e + 154 a^2 n^3 x x^n e + 225 a^2 d n^3 x + 41 c^2 d n^2 x x^{(4n)} + 98 b c d n^2 x x^{(3n)} + 59 b^2 d n^2 x x^{(2n)} + 118 a c d n^2 x x^{(2n)} + 142 a b d n^2 x x^n + 35 c^2 n^2 x x^{(5n)} e + 82 b c n^2 x x^{(4n)} e + 49 b^2 n^2 x x^{(3n)} e + 98 a c n^2 x x^{(3n)} e + 118 a b n^2 x x^{(2n)} e + 71 a^2 n^2 x x^n e + 85 a^2 d n^2 x + 11 c^2 d n x x^{(4n)} + 24 b c d n x x^{(3n)} + 13 b^2 d n x x^{(2n)} + 26 a c d n x x^{(2n)} + 28 a b d n x x^n + 10 c^2 n x x^{(5n)} e + 22 b c n x x^{(4n)} e + 12 b^2 n x x^{(3n)} e + 24 a c n x x^{(3n)} e + 26 a b n x x^{(2n)} e + 14 a^2 n x x^n e + 15 a^2 d n x + c^2 d x x^{(4n)} + 2 b c d x x^{(3n)} + b^2 d x x^{(2n)} + 2 a c d x x^{(2n)} + 2 a b d x x^n + c^2 x x^{(5n)} e + 2 b c x x^{(4n)} e + b^2 x x^{(3n)} e + 2 a c x x^{(3n)} e + 2 a b x x^{(2n)} e + a^2 x x^n e + a^2 d x) / (120 n^5 + 274 n^4 + 225 n^3 + 85 n^2 + 15 n + 1)
\end{aligned}$$



### 3.68 $\int (d + ex^n) (a + bx^n + cx^{2n})^3 dx$

**Optimal.** Leaf size=218

$$\frac{x^{3n+1} (3a^2ce + 3ab^2e + 6abcd + b^3d)}{3n+1} + \frac{a^2x^{n+1}(ae + 3bd)}{n+1} + a^3dx + \frac{x^{4n+1} (6abce + 3ac^2d + 3b^2cd + b^3e)}{4n+1} + \frac{3ax^{2n+1} (abce + 3ac^2d + 3b^2cd + b^3e)}{2n+1}$$

```
[Out] a^3*d*x + (a^2*(3*b*d + a*e)*x^(1 + n))/(1 + n) + (3*a*(b^2*d + a*c*d + a*b
*e)*x^(1 + 2*n))/(1 + 2*n) + ((b^3*d + 6*a*b*c*d + 3*a*b^2*e + 3*a^2*c*e)*x
^(1 + 3*n))/(1 + 3*n) + ((3*b^2*c*d + 3*a*c^2*d + b^3*e + 6*a*b*c*e)*x^(1 +
4*n))/(1 + 4*n) + (3*c*(b*c*d + b^2*e + a*c*e)*x^(1 + 5*n))/(1 + 5*n) + (c
^2*(c*d + 3*b*e)*x^(1 + 6*n))/(1 + 6*n) + (c^3*e*x^(1 + 7*n))/(1 + 7*n)
```

**Rubi [A]** time = 0.20067, antiderivative size = 218, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$ , Rules used = {1432}

$$\frac{x^{3n+1} (3a^2ce + 3ab^2e + 6abcd + b^3d)}{3n+1} + \frac{a^2x^{n+1}(ae + 3bd)}{n+1} + a^3dx + \frac{x^{4n+1} (6abce + 3ac^2d + 3b^2cd + b^3e)}{4n+1} + \frac{3ax^{2n+1} (abce + 3ac^2d + 3b^2cd + b^3e)}{2n+1}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^n)*(a + b*x^n + c*x^(2*n))^3,x]
```

```
[Out] a^3*d*x + (a^2*(3*b*d + a*e)*x^(1 + n))/(1 + n) + (3*a*(b^2*d + a*c*d + a*b
*e)*x^(1 + 2*n))/(1 + 2*n) + ((b^3*d + 6*a*b*c*d + 3*a*b^2*e + 3*a^2*c*e)*x
^(1 + 3*n))/(1 + 3*n) + ((3*b^2*c*d + 3*a*c^2*d + b^3*e + 6*a*b*c*e)*x^(1 +
4*n))/(1 + 4*n) + (3*c*(b*c*d + b^2*e + a*c*e)*x^(1 + 5*n))/(1 + 5*n) + (c
^2*(c*d + 3*b*e)*x^(1 + 6*n))/(1 + 6*n) + (c^3*e*x^(1 + 7*n))/(1 + 7*n)
```

#### Rule 1432

```
Int[((d_) + (e_)*(x_)^(n_))*((a_) + (b_)*(x_)^(n_) + (c_)*(x_)^(2n_))^(p
_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^n)*(a + b*x^n + c*x^(2*n))^p,
x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
0]
```

#### Rubi steps

$$\int (d + ex^n)(a + bx^n + cx^{2n})^3 dx = \int (a^3d + a^2(3bd + ae)x^n + 3a(b^2d + acd + abe)x^{2n} + (b^3d + 6abcd + 3ab^2e + 3a^2ce)x^{3n}) dx$$

$$= a^3dx + \frac{a^2(3bd + ae)x^{1+n}}{1+n} + \frac{3a(b^2d + acd + abe)x^{1+2n}}{1+2n} + \frac{(b^3d + 6abcd + 3ab^2e + 3a^2ce)x^{1+3n}}{1+3n}$$

**Mathematica [A]** time = 0.44151, size = 205, normalized size = 0.94

$$x \left( \frac{x^{3n} (3a^2ce + 3ab^2e + 6abcd + b^3d)}{3n+1} + \frac{a^2x^n (ae + 3bd)}{n+1} + a^3d + \frac{x^{4n} (6abce + 3ac^2d + 3b^2cd + b^3e)}{4n+1} + \frac{3ax^{2n} (abe + acd + a^2ce)}{2n+1} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(d + e\*x^n)\*(a + b\*x^n + c\*x^(2\*n))^3,x]

[Out] x\*(a^3\*d + (a^2\*(3\*b\*d + a\*e)\*x^n)/(1 + n) + (3\*a\*(b^2\*d + a\*c\*d + a\*b\*e)\*x^(2\*n))/(1 + 2\*n) + ((b^3\*d + 6\*a\*b\*c\*d + 3\*a\*b^2\*e + 3\*a^2\*c\*e)\*x^(3\*n))/(1 + 3\*n) + ((3\*b^2\*c\*d + 3\*a\*c^2\*d + b^3\*e + 6\*a\*b\*c\*e)\*x^(4\*n))/(1 + 4\*n) + (3\*c\*(b\*c\*d + b^2\*e + a\*c\*e)\*x^(5\*n))/(1 + 5\*n) + (c^2\*(c\*d + 3\*b\*e)\*x^(6\*n))/(1 + 6\*n) + (c^3\*e\*x^(7\*n))/(1 + 7\*n))

**Maple [A]** time = 0.014, size = 226, normalized size = 1.

$$a^3dx + \frac{(6abce + 3ac^2d + b^3e + 3b^2cd)x(e^{n \ln(x)})^4}{1+4n} + \frac{(3a^2ce + 3ab^2e + 6abcd + b^3d)x(e^{n \ln(x)})^3}{1+3n} + \frac{a^2(ae + 3bd)xe^{n \ln(x)}}{1+n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d+e\*x^n)\*(a+b\*x^n+c\*x^(2\*n))^3,x)

[Out] a^3\*d\*x+(6\*a\*b\*c\*e+3\*a\*c^2\*d+b^3\*e+3\*b^2\*c\*d)/(1+4\*n)\*x\*exp(n\*ln(x))^4+(3\*a^2\*c\*e+3\*a\*b^2\*e+6\*a\*b\*c\*d+b^3\*d)/(1+3\*n)\*x\*exp(n\*ln(x))^3+a^2\*(a\*e+3\*b\*d)/(1+n)\*x\*exp(n\*ln(x))+c^2\*(3\*b\*e+c\*d)/(1+6\*n)\*x\*exp(n\*ln(x))^6+c^3\*e/(1+7\*n)\*x\*exp(n\*ln(x))^7+3\*a\*(a\*b\*e+a\*c\*d+b^2\*d)/(1+2\*n)\*x\*exp(n\*ln(x))^2+3\*c\*(a\*c\*e+b^2\*e+b\*c\*d)/(1+5\*n)\*x\*exp(n\*ln(x))^5

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e\*x^n)\*(a+b\*x^n+c\*x^(2\*n))^3,x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [B]** time = 1.45508, size = 2761, normalized size = 12.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e\*x^n)\*(a+b\*x^n+c\*x^(2\*n))^3,x, algorithm="fricas")

[Out] ((720\*c^3\*e\*n^6 + 1764\*c^3\*e\*n^5 + 1624\*c^3\*e\*n^4 + 735\*c^3\*e\*n^3 + 175\*c^3\*e\*n^2 + 21\*c^3\*e\*n + c^3\*e)\*x\*x^(7\*n) + (840\*(c^3\*d + 3\*b\*c^2\*e)\*n^6 + 2038\*(c^3\*d + 3\*b\*c^2\*e)\*n^5 + 1849\*(c^3\*d + 3\*b\*c^2\*e)\*n^4 + c^3\*d + 3\*b\*c^2\*e + 820\*(c^3\*d + 3\*b\*c^2\*e)\*n^3 + 190\*(c^3\*d + 3\*b\*c^2\*e)\*n^2 + 22\*(c^3\*d + 3\*b\*c^2\*e)\*n)\*x\*x^(6\*n) + 3\*(1008\*(b\*c^2\*d + (b^2\*c + a\*c^2)\*e)\*n^6 + 2412\*(b\*c^2\*d + (b^2\*c + a\*c^2)\*e)\*n^5 + 2144\*(b\*c^2\*d + (b^2\*c + a\*c^2)\*e)\*n^4 + b\*c^2\*d + 925\*(b\*c^2\*d + (b^2\*c + a\*c^2)\*e)\*n^3 + 207\*(b\*c^2\*d + (b^2\*c + a\*c^2)\*e)\*n^2 + (b^2\*c + a\*c^2)\*e + 23\*(b\*c^2\*d + (b^2\*c + a\*c^2)\*e)\*n)\*x\*x^(5\*n) + (1260\*(3\*(b^2\*c + a\*c^2)\*d + (b^3 + 6\*a\*b\*c)\*e)\*n^6 + 2952\*(3\*(b^2\*c + a\*c^2)\*d + (b^3 + 6\*a\*b\*c)\*e)\*n^5 + 2545\*(3\*(b^2\*c + a\*c^2)\*d + (b^3 + 6\*a\*b\*c)\*e)\*n^4 + 1056\*(3\*(b^2\*c + a\*c^2)\*d + (b^3 + 6\*a\*b\*c)\*e)\*n^3 + 226\*(3\*(b^2\*c + a\*c^2)\*d + (b^3 + 6\*a\*b\*c)\*e)\*n^2 + 3\*(b^2\*c + a\*c^2)\*d + (b^3 + 6\*a\*b\*c)\*e + 24\*(3\*(b^2\*c + a\*c^2)\*d + (b^3 + 6\*a\*b\*c)\*e)\*n)\*x\*x^(4\*n) + (1680\*((b^3 + 6\*a\*b\*c)\*d + 3\*(a\*b^2 + a^2\*c)\*e)\*n^6 + 3796\*((b^3 + 6\*a\*b\*c)\*d + 3\*(a\*b^2 + a^2\*c)\*e)\*n^5 + 3112\*((b^3 + 6\*a\*b\*c)\*d + 3\*(a\*b^2 + a^2\*c)\*e)\*n^4 + 1219\*((b^3 + 6\*a\*b\*c)\*d + 3\*(a\*b^2 + a^2\*c)\*e)\*n^3 + 247\*((b^3 + 6\*a\*b\*c)\*d + 3\*(a\*b^2 + a^2\*c)\*e)\*n^2 + (b^3 + 6\*a\*b\*c)\*d + 3\*(a\*b^2 + a^2\*c)\*e + 25\*((b^3 + 6\*a\*b\*c)\*d + 3\*(a\*b^2 + a^2\*c)\*e)\*n)\*x\*x^(3\*n) + 3\*(2520\*(a^2\*b\*e + (a\*b^2 + a^2\*c)\*d)\*n^6 + 5274\*(a^2\*b\*e + (a\*b^2 + a^2\*c)\*d)\*n^5 + 3929\*(a^2\*b\*e + (a\*b^2 + a^2\*c)\*d)\*n^4 + a^2\*b\*e + 1420\*(a^2\*b\*e + (a\*b^2 + a^2\*c)\*d)\*n^3 + 270\*(a^2\*b\*e + (a\*b^2 + a^2\*c)\*d)\*n^2 + (a\*b^2 + a^2\*c)\*d + 26\*(a^2\*b\*e + (a\*b^2 + a^2\*c)\*d)\*n)\*x\*x^(2\*n) + (5040\*(3\*a^2\*b\*d + a^3\*e)\*n^6 + 8028\*(3\*a^2\*b\*d + a^3\*e)\*n^5 + 5104\*(3\*a^2\*b\*d + a^3\*e)\*n^4 + 3\*a^2\*b\*d + a^3\*e + 1665\*(3\*a^2\*b\*d + a^3\*e)\*n^3 + 295\*(3\*a^2\*b\*d + a^3\*e)\*n

$$\frac{x^{2n+2} + 27(3a^2bd + a^3e)n x^{2n+1} + (5040a^3d^7n + 13068a^3d^6n + 13132a^3d^5n + 6769a^3d^4n + 1960a^3d^3n + 322a^3d^2n + 28a^3dn + a^3d)x}{(5040n^7 + 13068n^6 + 13132n^5 + 6769n^4 + 1960n^3 + 322n^2 + 28n + 1)}$$

**Sympy [A]** time = 108.281, size = 9190, normalized size = 42.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e\*x\*\*n)\*(a+b\*x\*\*n+c\*x\*\*(2\*n))\*\*3,x)

[Out] Piecewise((a\*\*3\*d\*x + a\*\*3\*e\*log(x) + 3\*a\*\*2\*b\*d\*log(x) - 3\*a\*\*2\*b\*e/x - 3\*a\*\*2\*c\*d/x - 3\*a\*\*2\*c\*e/(2\*x\*\*2) - 3\*a\*b\*\*2\*d/x - 3\*a\*b\*\*2\*e/(2\*x\*\*2) - 3\*a\*b\*c\*d/x\*\*2 - 2\*a\*b\*c\*e/x\*\*3 - a\*c\*\*2\*d/x\*\*3 - 3\*a\*c\*\*2\*e/(4\*x\*\*4) - b\*\*3\*d/(2\*x\*\*2) - b\*\*3\*e/(3\*x\*\*3) - b\*\*2\*c\*d/x\*\*3 - 3\*b\*\*2\*c\*e/(4\*x\*\*4) - 3\*b\*c\*\*2\*d/(4\*x\*\*4) - 3\*b\*c\*\*2\*e/(5\*x\*\*5) - c\*\*3\*d/(5\*x\*\*5) - c\*\*3\*e/(6\*x\*\*6), Eq(n, -1)), (a\*\*3\*d\*x + 2\*a\*\*3\*e\*sqrt(x) + 6\*a\*\*2\*b\*d\*sqrt(x) + 3\*a\*\*2\*b\*e\*log(x) + 3\*a\*\*2\*c\*d\*log(x) - 6\*a\*\*2\*c\*e/sqrt(x) + 3\*a\*b\*\*2\*d\*log(x) - 6\*a\*b\*\*2\*e/sqrt(x) - 12\*a\*b\*c\*d/sqrt(x) - 6\*a\*b\*c\*e/x - 3\*a\*c\*\*2\*d/x - 2\*a\*c\*\*2\*e/x\*\*(3/2) - 2\*b\*\*3\*d/sqrt(x) - b\*\*3\*e/x - 3\*b\*\*2\*c\*d/x - 2\*b\*\*2\*c\*e/x\*\*(3/2) - 2\*b\*c\*\*2\*d/x\*\*(3/2) - 3\*b\*c\*\*2\*e/(2\*x\*\*2) - c\*\*3\*d/(2\*x\*\*2) - 2\*c\*\*3\*e/(5\*x\*\*(5/2)), Eq(n, -1/2)), (a\*\*3\*d\*x + 3\*a\*\*3\*e\*x\*\*(2/3)/2 + 9\*a\*\*2\*b\*d\*x\*\*(2/3)/2 + 9\*a\*\*2\*b\*e\*x\*\*(1/3) + 9\*a\*\*2\*c\*d\*x\*\*(1/3) + 3\*a\*\*2\*c\*e\*log(x) + 9\*a\*b\*\*2\*d\*x\*\*(1/3) + 3\*a\*b\*\*2\*e\*log(x) + 6\*a\*b\*c\*d\*log(x) - 18\*a\*b\*c\*e/x\*\*(1/3) - 9\*a\*c\*\*2\*d/x\*\*(1/3) - 9\*a\*c\*\*2\*e/(2\*x\*\*(2/3)) + b\*\*3\*d\*log(x) - 3\*b\*\*3\*e/x\*\*(1/3) - 9\*b\*\*2\*c\*d/x\*\*(1/3) - 9\*b\*\*2\*c\*e/(2\*x\*\*(2/3)) - 9\*b\*c\*\*2\*d/(2\*x\*\*(2/3)) - 3\*b\*c\*\*2\*e/x - c\*\*3\*d/x - 3\*c\*\*3\*e/(4\*x\*\*(4/3)), Eq(n, -1/3)), (a\*\*3\*d\*x + 4\*a\*\*3\*e\*x\*\*(3/4)/3 + 4\*a\*\*2\*b\*d\*x\*\*(3/4) + 6\*a\*\*2\*b\*e\*sqrt(x) + 6\*a\*\*2\*c\*d\*sqrt(x) + 12\*a\*\*2\*c\*e\*x\*\*(1/4) + 6\*a\*b\*\*2\*d\*sqrt(x) + 12\*a\*b\*\*2\*e\*x\*\*(1/4) + 24\*a\*b\*c\*d\*x\*\*(1/4) + 6\*a\*b\*c\*e\*log(x) + 3\*a\*c\*\*2\*d\*log(x) - 12\*a\*c\*\*2\*e/x\*\*(1/4) + 4\*b\*\*3\*d\*x\*\*(1/4) + b\*\*3\*e\*log(x) + 3\*b\*\*2\*c\*d\*log(x) - 12\*b\*\*2\*c\*e/x\*\*(1/4) - 12\*b\*c\*\*2\*d/x\*\*(1/4) - 6\*b\*c\*\*2\*e/sqrt(x) - 2\*c\*\*3\*d/sqrt(x) - 4\*c\*\*3\*e/(3\*x\*\*(3/4)), Eq(n, -1/4)), (a\*\*3\*d\*x + 5\*a\*\*3\*e\*x\*\*(4/5)/4 + 15\*a\*\*2\*b\*d\*x\*\*(4/5)/4 + 5\*a\*\*2\*b\*e\*x\*\*(3/5) + 5\*a\*\*2\*c\*d\*x\*\*(3/5) + 15\*a\*\*2\*c\*e\*x\*\*(2/5)/2 + 5\*a\*b\*\*2\*d\*x\*\*(3/5) + 15\*a\*b\*\*2\*e\*x\*\*(2/5)/2 + 15\*a\*b\*c\*d\*x\*\*(2/5) + 30\*a\*b\*c\*e\*x\*\*(1/5) + 15\*a\*c\*\*2\*d\*x\*\*(1/5) + 3\*a\*c\*\*2\*e\*log(x) + 5\*b\*\*3\*d\*x\*\*(2/5)/2 + 5\*b\*\*3\*e\*x\*\*(1/5) + 15\*b\*\*2\*c\*d\*x\*\*(1/5) + 3\*b\*\*2\*c\*e\*log(x) + 3\*b\*c\*\*2\*d\*log(x) - 15\*b\*c\*\*2\*e/x\*\*(1/5) - 5\*c\*\*3\*d/x\*\*(1/5) - 5\*c\*\*3\*e/(2\*x\*\*(2/5)), Eq(n, -1/5)), (a\*\*3\*d\*x + 6\*a\*\*3\*e\*x\*\*(5/6)/5 + 18\*a\*\*2\*b\*d\*x\*\*(5/6)/5 + 9\*a\*\*2\*b\*e\*x\*\*(2/3)/2 + 9\*a\*\*2\*c\*d\*x\*\*(

$$\begin{aligned}
& 2/3)/2 + 6*a**2*c*e*sqrt(x) + 9*a*b**2*d*x**(2/3)/2 + 6*a*b**2*e*sqrt(x) + \\
& 12*a*b*c*d*sqrt(x) + 18*a*b*c*e*x**(1/3) + 9*a*c**2*d*x**(1/3) + 18*a*c**2* \\
& e*x**(1/6) + 2*b**3*d*sqrt(x) + 3*b**3*e*x**(1/3) + 9*b**2*c*d*x**(1/3) + 1 \\
& 8*b**2*c*e*x**(1/6) + 18*b*c**2*d*x**(1/6) + 3*b*c**2*e*log(x) + c**3*d*log \\
& (x) - 6*c**3*e/x**(1/6), Eq(n, -1/6)), (a**3*d*x + 7*a**3*e*x**(6/7)/6 + 7* \\
& a**2*b*d*x**(6/7)/2 + 21*a**2*b*e*x**(5/7)/5 + 21*a**2*c*d*x**(5/7)/5 + 21* \\
& a**2*c*e*x**(4/7)/4 + 21*a*b**2*d*x**(5/7)/5 + 21*a*b**2*e*x**(4/7)/4 + 21* \\
& a*b*c*d*x**(4/7)/2 + 14*a*b*c*e*x**(3/7) + 7*a*c**2*d*x**(3/7) + 21*a*c**2* \\
& e*x**(2/7)/2 + 7*b**3*d*x**(4/7)/4 + 7*b**3*e*x**(3/7)/3 + 7*b**2*c*d*x**(3 \\
& /7) + 21*b**2*c*e*x**(2/7)/2 + 21*b*c**2*d*x**(2/7)/2 + 21*b*c**2*e*x**(1/7 \\
& ) + 7*c**3*d*x**(1/7) + c**3*e*log(x), Eq(n, -1/7)), (5040*a**3*d*n**7*x/(5 \\
& 040*n**7 + 13068*n**6 + 13132*n**5 + 6769*n**4 + 1960*n**3 + 322*n**2 + 28* \\
& n + 1) + 13068*a**3*d*n**6*x/(5040*n**7 + 13068*n**6 + 13132*n**5 + 6769*n* \\
& **4 + 1960*n**3 + 322*n**2 + 28*n + 1) + 13132*a**3*d*n**5*x/(5040*n**7 + 13 \\
& 068*n**6 + 13132*n**5 + 6769*n**4 + 1960*n**3 + 322*n**2 + 28*n + 1) + 6769 \\
& *a**3*d*n**4*x/(5040*n**7 + 13068*n**6 + 13132*n**5 + 6769*n**4 + 1960*n**3 \\
& + 322*n**2 + 28*n + 1) + 1960*a**3*d*n**3*x/(5040*n**7 + 13068*n**6 + 1313 \\
& 2*n**5 + 6769*n**4 + 1960*n**3 + 322*n**2 + 28*n + 1) + 322*a**3*d*n**2*x/( \\
& 5040*n**7 + 13068*n**6 + 13132*n**5 + 6769*n**4 + 1960*n**3 + 322*n**2 + 28 \\
& *n + 1) + 28*a**3*d*n*x/(5040*n**7 + 13068*n**6 + 13132*n**5 + 6769*n**4 + \\
& 1960*n**3 + 322*n**2 + 28*n + 1) + a**3*d*x/(5040*n**7 + 13068*n**6 + 13132 \\
& *n**5 + 6769*n**4 + 1960*n**3 + 322*n**2 + 28*n + 1) + 5040*a**3*e*n**6*x*x \\
& **n/(5040*n**7 + 13068*n**6 + 13132*n**5 + 6769*n**4 + 1960*n**3 + 322*n**2 \\
& + 28*n + 1) + 8028*a**3*e*n**5*x*x**n/(5040*n**7 + 13068*n**6 + 13132*n**5 \\
& + 6769*n**4 + 1960*n**3 + 322*n**2 + 28*n + 1) + 5104*a**3*e*n**4*x*x**n/( \\
& 5040*n**7 + 13068*n**6 + 13132*n**5 + 6769*n**4 + 1960*n**3 + 322*n**2 + 28 \\
& *n + 1) + 1665*a**3*e*n**3*x*x**n/(5040*n**7 + 13068*n**6 + 13132*n**5 + 67 \\
& 69*n**4 + 1960*n**3 + 322*n**2 + 28*n + 1) + 295*a**3*e*n**2*x*x**n/(5040*n \\
& **7 + 13068*n**6 + 13132*n**5 + 6769*n**4 + 1960*n**3 + 322*n**2 + 28*n + 1 \\
& ) + 27*a**3*e*n*x*x**n/(5040*n**7 + 13068*n**6 + 13132*n**5 + 6769*n**4 + 1 \\
& 960*n**3 + 322*n**2 + 28*n + 1) + a**3*e*x*x**n/(5040*n**7 + 13068*n**6 + 1 \\
& 3132*n**5 + 6769*n**4 + 1960*n**3 + 322*n**2 + 28*n + 1) + 15120*a**2*b*d*n \\
& **6*x*x**n/(5040*n**7 + 13068*n**6 + 13132*n**5 + 6769*n**4 + 1960*n**3 + 3 \\
& 22*n**2 + 28*n + 1) + 24084*a**2*b*d*n**5*x*x**n/(5040*n**7 + 13068*n**6 + \\
& 13132*n**5 + 6769*n**4 + 1960*n**3 + 322*n**2 + 28*n + 1) + 15312*a**2*b*d* \\
& n**4*x*x**n/(5040*n**7 + 13068*n**6 + 13132*n**5 + 6769*n**4 + 1960*n**3 + \\
& 322*n**2 + 28*n + 1) + 4995*a**2*b*d*n**3*x*x**n/(5040*n**7 + 13068*n**6 + \\
& 13132*n**5 + 6769*n**4 + 1960*n**3 + 322*n**2 + 28*n + 1) + 885*a**2*b*d*n* \\
& **2*x*x**n/(5040*n**7 + 13068*n**6 + 13132*n**5 + 6769*n**4 + 1960*n**3 + 32 \\
& 2*n**2 + 28*n + 1) + 81*a**2*b*d*n*x*x**n/(5040*n**7 + 13068*n**6 + 13132*n \\
& **5 + 6769*n**4 + 1960*n**3 + 322*n**2 + 28*n + 1) + 3*a**2*b*d*x*x**n/(504 \\
& 0*n**7 + 13068*n**6 + 13132*n**5 + 6769*n**4 + 1960*n**3 + 322*n**2 + 28*n \\
& + 1) + 7560*a**2*b*e*n**6*x*x**(2*n)/(5040*n**7 + 13068*n**6 + 13132*n**5 + \\
& 6769*n**4 + 1960*n**3 + 322*n**2 + 28*n + 1) + 15822*a**2*b*e*n**5*x*x**(2 \\
& *n)/(5040*n**7 + 13068*n**6 + 13132*n**5 + 6769*n**4 + 1960*n**3 + 322*n**2
\end{aligned}$$



$$\begin{aligned}
& 068*n^{**6} + 13132*n^{**5} + 6769*n^{**4} + 1960*n^{**3} + 322*n^{**2} + 28*n + 1) + 741* \\
& a*b^{**2}*e*n^{**2}*x*x^{**}(3*n)/(5040*n^{**7} + 13068*n^{**6} + 13132*n^{**5} + 6769*n^{**4} + \\
& 1960*n^{**3} + 322*n^{**2} + 28*n + 1) + 75*a*b^{**2}*e*n*x*x^{**}(3*n)/(5040*n^{**7} + 1 \\
& 3068*n^{**6} + 13132*n^{**5} + 6769*n^{**4} + 1960*n^{**3} + 322*n^{**2} + 28*n + 1) + 3*a \\
& *b^{**2}*e*x*x^{**}(3*n)/(5040*n^{**7} + 13068*n^{**6} + 13132*n^{**5} + 6769*n^{**4} + 1960* \\
& n^{**3} + 322*n^{**2} + 28*n + 1) + 10080*a*b*c*d*n^{**6}*x*x^{**}(3*n)/(5040*n^{**7} + 13 \\
& 068*n^{**6} + 13132*n^{**5} + 6769*n^{**4} + 1960*n^{**3} + 322*n^{**2} + 28*n + 1) + 2277 \\
& 6*a*b*c*d*n^{**5}*x*x^{**}(3*n)/(5040*n^{**7} + 13068*n^{**6} + 13132*n^{**5} + 6769*n^{**4} \\
& + 1960*n^{**3} + 322*n^{**2} + 28*n + 1) + 18672*a*b*c*d*n^{**4}*x*x^{**}(3*n)/(5040*n* \\
& *7 + 13068*n^{**6} + 13132*n^{**5} + 6769*n^{**4} + 1960*n^{**3} + 322*n^{**2} + 28*n + 1) \\
& + 7314*a*b*c*d*n^{**3}*x*x^{**}(3*n)/(5040*n^{**7} + 13068*n^{**6} + 13132*n^{**5} + 6769 \\
& *n^{**4} + 1960*n^{**3} + 322*n^{**2} + 28*n + 1) + 1482*a*b*c*d*n^{**2}*x*x^{**}(3*n)/(50 \\
& 40*n^{**7} + 13068*n^{**6} + 13132*n^{**5} + 6769*n^{**4} + 1960*n^{**3} + 322*n^{**2} + 28*n \\
& + 1) + 150*a*b*c*d*n*x*x^{**}(3*n)/(5040*n^{**7} + 13068*n^{**6} + 13132*n^{**5} + 676 \\
& 9*n^{**4} + 1960*n^{**3} + 322*n^{**2} + 28*n + 1) + 6*a*b*c*d*x*x^{**}(3*n)/(5040*n^{**7} \\
& + 13068*n^{**6} + 13132*n^{**5} + 6769*n^{**4} + 1960*n^{**3} + 322*n^{**2} + 28*n + 1) + \\
& 7560*a*b*c*e*n^{**6}*x*x^{**}(4*n)/(5040*n^{**7} + 13068*n^{**6} + 13132*n^{**5} + 6769*n \\
& **4 + 1960*n^{**3} + 322*n^{**2} + 28*n + 1) + 17712*a*b*c*e*n^{**5}*x*x^{**}(4*n)/(504 \\
& 0*n^{**7} + 13068*n^{**6} + 13132*n^{**5} + 6769*n^{**4} + 1960*n^{**3} + 322*n^{**2} + 28*n \\
& + 1) + 15270*a*b*c*e*n^{**4}*x*x^{**}(4*n)/(5040*n^{**7} + 13068*n^{**6} + 13132*n^{**5} + \\
& 6769*n^{**4} + 1960*n^{**3} + 322*n^{**2} + 28*n + 1) + 6336*a*b*c*e*n^{**3}*x*x^{**}(4*n \\
& )/(5040*n^{**7} + 13068*n^{**6} + 13132*n^{**5} + 6769*n^{**4} + 1960*n^{**3} + 322*n^{**2} + \\
& 28*n + 1) + 1356*a*b*c*e*n^{**2}*x*x^{**}(4*n)/(5040*n^{**7} + 13068*n^{**6} + 13132*n \\
& **5 + 6769*n^{**4} + 1960*n^{**3} + 322*n^{**2} + 28*n + 1) + 144*a*b*c*e*n*x*x^{**}(4* \\
& n)/(5040*n^{**7} + 13068*n^{**6} + 13132*n^{**5} + 6769*n^{**4} + 1960*n^{**3} + 322*n^{**2} \\
& + 28*n + 1) + 6*a*b*c*e*x*x^{**}(4*n)/(5040*n^{**7} + 13068*n^{**6} + 13132*n^{**5} + 6 \\
& 769*n^{**4} + 1960*n^{**3} + 322*n^{**2} + 28*n + 1) + 3780*a*c^{**2}*d*n^{**6}*x*x^{**}(4*n) \\
& /(5040*n^{**7} + 13068*n^{**6} + 13132*n^{**5} + 6769*n^{**4} + 1960*n^{**3} + 322*n^{**2} + \\
& 28*n + 1) + 8856*a*c^{**2}*d*n^{**5}*x*x^{**}(4*n)/(5040*n^{**7} + 13068*n^{**6} + 13132*n \\
& **5 + 6769*n^{**4} + 1960*n^{**3} + 322*n^{**2} + 28*n + 1) + 7635*a*c^{**2}*d*n^{**4}*x*x \\
& **4*n)/(5040*n^{**7} + 13068*n^{**6} + 13132*n^{**5} + 6769*n^{**4} + 1960*n^{**3} + 322* \\
& n^{**2} + 28*n + 1) + 3168*a*c^{**2}*d*n^{**3}*x*x^{**}(4*n)/(5040*n^{**7} + 13068*n^{**6} + \\
& 13132*n^{**5} + 6769*n^{**4} + 1960*n^{**3} + 322*n^{**2} + 28*n + 1) + 678*a*c^{**2}*d*n* \\
& *2*x*x^{**}(4*n)/(5040*n^{**7} + 13068*n^{**6} + 13132*n^{**5} + 6769*n^{**4} + 1960*n^{**3} \\
& + 322*n^{**2} + 28*n + 1) + 72*a*c^{**2}*d*n*x*x^{**}(4*n)/(5040*n^{**7} + 13068*n^{**6} + \\
& 13132*n^{**5} + 6769*n^{**4} + 1960*n^{**3} + 322*n^{**2} + 28*n + 1) + 3*a*c^{**2}*d*x*x \\
& **4*n)/(5040*n^{**7} + 13068*n^{**6} + 13132*n^{**5} + 6769*n^{**4} + 1960*n^{**3} + 322* \\
& n^{**2} + 28*n + 1) + 3024*a*c^{**2}*e*n^{**6}*x*x^{**}(5*n)/(5040*n^{**7} + 13068*n^{**6} + \\
& 13132*n^{**5} + 6769*n^{**4} + 1960*n^{**3} + 322*n^{**2} + 28*n + 1) + 7236*a*c^{**2}*e*n \\
& **5*x*x^{**}(5*n)/(5040*n^{**7} + 13068*n^{**6} + 13132*n^{**5} + 6769*n^{**4} + 1960*n^{**3} \\
& + 322*n^{**2} + 28*n + 1) + 6432*a*c^{**2}*e*n^{**4}*x*x^{**}(5*n)/(5040*n^{**7} + 13068* \\
& n^{**6} + 13132*n^{**5} + 6769*n^{**4} + 1960*n^{**3} + 322*n^{**2} + 28*n + 1) + 2775*a*c \\
& **2*e*n^{**3}*x*x^{**}(5*n)/(5040*n^{**7} + 13068*n^{**6} + 13132*n^{**5} + 6769*n^{**4} + 19 \\
& 60*n^{**3} + 322*n^{**2} + 28*n + 1) + 621*a*c^{**2}*e*n^{**2}*x*x^{**}(5*n)/(5040*n^{**7} + \\
& 13068*n^{**6} + 13132*n^{**5} + 6769*n^{**4} + 1960*n^{**3} + 322*n^{**2} + 28*n + 1) + 69
\end{aligned}$$

$$\begin{aligned}
& *a*c**2*e*n*x*x**(5*n)/(5040*n**7 + 13068*n**6 + 13132*n**5 + 6769*n**4 + 1 \\
& 960*n**3 + 322*n**2 + 28*n + 1) + 3*a*c**2*e*x*x**(5*n)/(5040*n**7 + 13068* \\
& n**6 + 13132*n**5 + 6769*n**4 + 1960*n**3 + 322*n**2 + 28*n + 1) + 1680*b** \\
& 3*d*n**6*x*x**(3*n)/(5040*n**7 + 13068*n**6 + 13132*n**5 + 6769*n**4 + 1960 \\
& *n**3 + 322*n**2 + 28*n + 1) + 3796*b**3*d*n**5*x*x**(3*n)/(5040*n**7 + 130 \\
& 68*n**6 + 13132*n**5 + 6769*n**4 + 1960*n**3 + 322*n**2 + 28*n + 1) + 3112* \\
& b**3*d*n**4*x*x**(3*n)/(5040*n**7 + 13068*n**6 + 13132*n**5 + 6769*n**4 + 1 \\
& 960*n**3 + 322*n**2 + 28*n + 1) + 1219*b**3*d*n**3*x*x**(3*n)/(5040*n**7 + \\
& 13068*n**6 + 13132*n**5 + 6769*n**4 + 1960*n**3 + 322*n**2 + 28*n + 1) + 24 \\
& 7*b**3*d*n**2*x*x**(3*n)/(5040*n**7 + 13068*n**6 + 13132*n**5 + 6769*n**4 + \\
& 1960*n**3 + 322*n**2 + 28*n + 1) + 25*b**3*d*n*x*x**(3*n)/(5040*n**7 + 130 \\
& 68*n**6 + 13132*n**5 + 6769*n**4 + 1960*n**3 + 322*n**2 + 28*n + 1) + b**3* \\
& d*x*x**(3*n)/(5040*n**7 + 13068*n**6 + 13132*n**5 + 6769*n**4 + 1960*n**3 + \\
& 322*n**2 + 28*n + 1) + 1260*b**3*e*n**6*x*x**(4*n)/(5040*n**7 + 13068*n**6 \\
& + 13132*n**5 + 6769*n**4 + 1960*n**3 + 322*n**2 + 28*n + 1) + 2952*b**3*e* \\
& n**5*x*x**(4*n)/(5040*n**7 + 13068*n**6 + 13132*n**5 + 6769*n**4 + 1960*n** \\
& 3 + 322*n**2 + 28*n + 1) + 2545*b**3*e*n**4*x*x**(4*n)/(5040*n**7 + 13068*n \\
& **6 + 13132*n**5 + 6769*n**4 + 1960*n**3 + 322*n**2 + 28*n + 1) + 1056*b**3 \\
& *e*n**3*x*x**(4*n)/(5040*n**7 + 13068*n**6 + 13132*n**5 + 6769*n**4 + 1960* \\
& n**3 + 322*n**2 + 28*n + 1) + 226*b**3*e*n**2*x*x**(4*n)/(5040*n**7 + 13068 \\
& *n**6 + 13132*n**5 + 6769*n**4 + 1960*n**3 + 322*n**2 + 28*n + 1) + 24*b**3 \\
& *e*n*x*x**(4*n)/(5040*n**7 + 13068*n**6 + 13132*n**5 + 6769*n**4 + 1960*n** \\
& 3 + 322*n**2 + 28*n + 1) + b**3*e*x*x**(4*n)/(5040*n**7 + 13068*n**6 + 1313 \\
& 2*n**5 + 6769*n**4 + 1960*n**3 + 322*n**2 + 28*n + 1) + 3780*b**2*c*d*n**6* \\
& x*x**(4*n)/(5040*n**7 + 13068*n**6 + 13132*n**5 + 6769*n**4 + 1960*n**3 + 3 \\
& 22*n**2 + 28*n + 1) + 8856*b**2*c*d*n**5*x*x**(4*n)/(5040*n**7 + 13068*n**6 \\
& + 13132*n**5 + 6769*n**4 + 1960*n**3 + 322*n**2 + 28*n + 1) + 7635*b**2*c* \\
& d*n**4*x*x**(4*n)/(5040*n**7 + 13068*n**6 + 13132*n**5 + 6769*n**4 + 1960*n \\
& **3 + 322*n**2 + 28*n + 1) + 3168*b**2*c*d*n**3*x*x**(4*n)/(5040*n**7 + 130 \\
& 68*n**6 + 13132*n**5 + 6769*n**4 + 1960*n**3 + 322*n**2 + 28*n + 1) + 678*b \\
& **2*c*d*n**2*x*x**(4*n)/(5040*n**7 + 13068*n**6 + 13132*n**5 + 6769*n**4 + \\
& 1960*n**3 + 322*n**2 + 28*n + 1) + 72*b**2*c*d*n*x*x**(4*n)/(5040*n**7 + 13 \\
& 068*n**6 + 13132*n**5 + 6769*n**4 + 1960*n**3 + 322*n**2 + 28*n + 1) + 3*b* \\
& **2*c*d*x*x**(4*n)/(5040*n**7 + 13068*n**6 + 13132*n**5 + 6769*n**4 + 1960*n \\
& **3 + 322*n**2 + 28*n + 1) + 3024*b**2*c*e*n**6*x*x**(5*n)/(5040*n**7 + 130 \\
& 68*n**6 + 13132*n**5 + 6769*n**4 + 1960*n**3 + 322*n**2 + 28*n + 1) + 7236* \\
& b**2*c*e*n**5*x*x**(5*n)/(5040*n**7 + 13068*n**6 + 13132*n**5 + 6769*n**4 + \\
& 1960*n**3 + 322*n**2 + 28*n + 1) + 6432*b**2*c*e*n**4*x*x**(5*n)/(5040*n** \\
& 7 + 13068*n**6 + 13132*n**5 + 6769*n**4 + 1960*n**3 + 322*n**2 + 28*n + 1) \\
& + 2775*b**2*c*e*n**3*x*x**(5*n)/(5040*n**7 + 13068*n**6 + 13132*n**5 + 6769 \\
& *n**4 + 1960*n**3 + 322*n**2 + 28*n + 1) + 621*b**2*c*e*n**2*x*x**(5*n)/(50 \\
& 40*n**7 + 13068*n**6 + 13132*n**5 + 6769*n**4 + 1960*n**3 + 322*n**2 + 28*n \\
& + 1) + 69*b**2*c*e*n*x*x**(5*n)/(5040*n**7 + 13068*n**6 + 13132*n**5 + 676 \\
& 9*n**4 + 1960*n**3 + 322*n**2 + 28*n + 1) + 3*b**2*c*e*x*x**(5*n)/(5040*n** \\
& 7 + 13068*n**6 + 13132*n**5 + 6769*n**4 + 1960*n**3 + 322*n**2 + 28*n + 1)
\end{aligned}$$



```

+ 3024*b*c**2*d*n**6*x*x**(5*n)/(5040*n**7 + 13068*n**6 + 13132*n**5 + 6769
*n**4 + 1960*n**3 + 322*n**2 + 28*n + 1) + 7236*b*c**2*d*n**5*x*x**(5*n)/(5
040*n**7 + 13068*n**6 + 13132*n**5 + 6769*n**4 + 1960*n**3 + 322*n**2 + 28*
n + 1) + 6432*b*c**2*d*n**4*x*x**(5*n)/(5040*n**7 + 13068*n**6 + 13132*n**5
+ 6769*n**4 + 1960*n**3 + 322*n**2 + 28*n + 1) + 2775*b*c**2*d*n**3*x*x**(
5*n)/(5040*n**7 + 13068*n**6 + 13132*n**5 + 6769*n**4 + 1960*n**3 + 322*n**
2 + 28*n + 1) + 621*b*c**2*d*n**2*x*x**(5*n)/(5040*n**7 + 13068*n**6 + 1313
2*n**5 + 6769*n**4 + 1960*n**3 + 322*n**2 + 28*n + 1) + 69*b*c**2*d*n*x*x**
(5*n)/(5040*n**7 + 13068*n**6 + 13132*n**5 + 6769*n**4 + 1960*n**3 + 322*n*
**2 + 28*n + 1) + 3*b*c**2*d*x*x**(5*n)/(5040*n**7 + 13068*n**6 + 13132*n**5
+ 6769*n**4 + 1960*n**3 + 322*n**2 + 28*n + 1) + 2520*b*c**2*e*n**6*x*x**(
6*n)/(5040*n**7 + 13068*n**6 + 13132*n**5 + 6769*n**4 + 1960*n**3 + 322*n**
2 + 28*n + 1) + 6114*b*c**2*e*n**5*x*x**(6*n)/(5040*n**7 + 13068*n**6 + 131
32*n**5 + 6769*n**4 + 1960*n**3 + 322*n**2 + 28*n + 1) + 5547*b*c**2*e*n**4
*x*x**(6*n)/(5040*n**7 + 13068*n**6 + 13132*n**5 + 6769*n**4 + 1960*n**3 +
322*n**2 + 28*n + 1) + 2460*b*c**2*e*n**3*x*x**(6*n)/(5040*n**7 + 13068*n**
6 + 13132*n**5 + 6769*n**4 + 1960*n**3 + 322*n**2 + 28*n + 1) + 570*b*c**2*
e*n**2*x*x**(6*n)/(5040*n**7 + 13068*n**6 + 13132*n**5 + 6769*n**4 + 1960*n
**3 + 322*n**2 + 28*n + 1) + 66*b*c**2*e*n*x*x**(6*n)/(5040*n**7 + 13068*n*
**6 + 13132*n**5 + 6769*n**4 + 1960*n**3 + 322*n**2 + 28*n + 1) + 3*b*c**2*e
*x*x**(6*n)/(5040*n**7 + 13068*n**6 + 13132*n**5 + 6769*n**4 + 1960*n**3 +
322*n**2 + 28*n + 1) + 840*c**3*d*n**6*x*x**(6*n)/(5040*n**7 + 13068*n**6 +
13132*n**5 + 6769*n**4 + 1960*n**3 + 322*n**2 + 28*n + 1) + 2038*c**3*d*n*
**5*x*x**(6*n)/(5040*n**7 + 13068*n**6 + 13132*n**5 + 6769*n**4 + 1960*n**3
+ 322*n**2 + 28*n + 1) + 1849*c**3*d*n**4*x*x**(6*n)/(5040*n**7 + 13068*n**
6 + 13132*n**5 + 6769*n**4 + 1960*n**3 + 322*n**2 + 28*n + 1) + 820*c**3*d*
n**3*x*x**(6*n)/(5040*n**7 + 13068*n**6 + 13132*n**5 + 6769*n**4 + 1960*n**
3 + 322*n**2 + 28*n + 1) + 190*c**3*d*n**2*x*x**(6*n)/(5040*n**7 + 13068*n*
**6 + 13132*n**5 + 6769*n**4 + 1960*n**3 + 322*n**2 + 28*n + 1) + 22*c**3*d*
n*x*x**(6*n)/(5040*n**7 + 13068*n**6 + 13132*n**5 + 6769*n**4 + 1960*n**3 +
322*n**2 + 28*n + 1) + c**3*d*x*x**(6*n)/(5040*n**7 + 13068*n**6 + 13132*n
**5 + 6769*n**4 + 1960*n**3 + 322*n**2 + 28*n + 1) + 720*c**3*e*n**6*x*x**(
7*n)/(5040*n**7 + 13068*n**6 + 13132*n**5 + 6769*n**4 + 1960*n**3 + 322*n**
2 + 28*n + 1) + 1764*c**3*e*n**5*x*x**(7*n)/(5040*n**7 + 13068*n**6 + 13132
*n**5 + 6769*n**4 + 1960*n**3 + 322*n**2 + 28*n + 1) + 1624*c**3*e*n**4*x*x
**(7*n)/(5040*n**7 + 13068*n**6 + 13132*n**5 + 6769*n**4 + 1960*n**3 + 322*
n**2 + 28*n + 1) + 735*c**3*e*n**3*x*x**(7*n)/(5040*n**7 + 13068*n**6 + 131
32*n**5 + 6769*n**4 + 1960*n**3 + 322*n**2 + 28*n + 1) + 175*c**3*e*n**2*x*
x**(7*n)/(5040*n**7 + 13068*n**6 + 13132*n**5 + 6769*n**4 + 1960*n**3 + 322
*n**2 + 28*n + 1) + 21*c**3*e*n*x*x**(7*n)/(5040*n**7 + 13068*n**6 + 13132*
n**5 + 6769*n**4 + 1960*n**3 + 322*n**2 + 28*n + 1) + c**3*e*x*x**(7*n)/(50
40*n**7 + 13068*n**6 + 13132*n**5 + 6769*n**4 + 1960*n**3 + 322*n**2 + 28*n
+ 1), True))

```

**Giac [B]** time = 1.23794, size = 2881, normalized size = 13.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e\*x^n)\*(a+b\*x^n+c\*x^(2\*n))^3,x, algorithm="giac")

[Out] (5040\*a^3\*d\*n^7\*x + 840\*c^3\*d\*n^6\*x\*x^(6\*n) + 3024\*b\*c^2\*d\*n^6\*x\*x^(5\*n) + 3780\*b^2\*c\*d\*n^6\*x\*x^(4\*n) + 3780\*a\*c^2\*d\*n^6\*x\*x^(4\*n) + 1680\*b^3\*d\*n^6\*x\*x^(3\*n) + 10080\*a\*b\*c\*d\*n^6\*x\*x^(3\*n) + 7560\*a\*b^2\*d\*n^6\*x\*x^(2\*n) + 7560\*a^2\*c\*d\*n^6\*x\*x^(2\*n) + 15120\*a^2\*b\*d\*n^6\*x\*x^n + 720\*c^3\*n^6\*x\*x^(7\*n))\*e + 2520\*b\*c^2\*n^6\*x\*x^(6\*n)\*e + 3024\*b^2\*c\*n^6\*x\*x^(5\*n)\*e + 3024\*a\*c^2\*n^6\*x\*x^(5\*n)\*e + 1260\*b^3\*n^6\*x\*x^(4\*n)\*e + 7560\*a\*b\*c\*n^6\*x\*x^(4\*n)\*e + 5040\*a\*b^2\*n^6\*x\*x^(3\*n)\*e + 5040\*a^2\*c\*n^6\*x\*x^(3\*n)\*e + 7560\*a^2\*b\*n^6\*x\*x^(2\*n)\*e + 5040\*a^3\*n^6\*x\*x^n\*e + 13068\*a^3\*d\*n^6\*x + 2038\*c^3\*d\*n^5\*x\*x^(6\*n) + 7236\*b\*c^2\*d\*n^5\*x\*x^(5\*n) + 8856\*b^2\*c\*d\*n^5\*x\*x^(4\*n) + 8856\*a\*c^2\*d\*n^5\*x\*x^(4\*n) + 3796\*b^3\*d\*n^5\*x\*x^(3\*n) + 22776\*a\*b\*c\*d\*n^5\*x\*x^(3\*n) + 15822\*a\*b^2\*d\*n^5\*x\*x^(2\*n) + 15822\*a^2\*c\*d\*n^5\*x\*x^(2\*n) + 24084\*a^2\*b\*d\*n^5\*x\*x^n + 1764\*c^3\*n^5\*x\*x^(7\*n)\*e + 6114\*b\*c^2\*n^5\*x\*x^(6\*n)\*e + 7236\*b^2\*c\*n^5\*x\*x^(5\*n)\*e + 7236\*a\*c^2\*n^5\*x\*x^(5\*n)\*e + 2952\*b^3\*n^5\*x\*x^(4\*n)\*e + 17712\*a\*b\*c\*n^5\*x\*x^(4\*n)\*e + 11388\*a\*b^2\*n^5\*x\*x^(3\*n)\*e + 11388\*a^2\*c\*n^5\*x\*x^(3\*n)\*e + 15822\*a^2\*b\*n^5\*x\*x^(2\*n)\*e + 8028\*a^3\*n^5\*x\*x^n\*e + 13132\*a^3\*d\*n^5\*x + 1849\*c^3\*d\*n^4\*x\*x^(6\*n) + 6432\*b\*c^2\*d\*n^4\*x\*x^(5\*n) + 7635\*b^2\*c\*d\*n^4\*x\*x^(4\*n) + 7635\*a\*c^2\*d\*n^4\*x\*x^(4\*n) + 3112\*b^3\*d\*n^4\*x\*x^(3\*n) + 18672\*a\*b\*c\*d\*n^4\*x\*x^(3\*n) + 11787\*a\*b^2\*d\*n^4\*x\*x^(2\*n) + 11787\*a^2\*c\*d\*n^4\*x\*x^(2\*n) + 15312\*a^2\*b\*d\*n^4\*x\*x^n + 1624\*c^3\*n^4\*x\*x^(7\*n)\*e + 5547\*b\*c^2\*n^4\*x\*x^(6\*n)\*e + 6432\*b^2\*c\*n^4\*x\*x^(5\*n)\*e + 6432\*a\*c^2\*n^4\*x\*x^(5\*n)\*e + 2545\*b^3\*n^4\*x\*x^(4\*n)\*e + 15270\*a\*b\*c\*n^4\*x\*x^(4\*n)\*e + 9336\*a\*b^2\*n^4\*x\*x^(3\*n)\*e + 9336\*a^2\*c\*n^4\*x\*x^(3\*n)\*e + 11787\*a^2\*b\*n^4\*x\*x^(2\*n)\*e + 5104\*a^3\*n^4\*x\*x^n\*e + 6769\*a^3\*d\*n^4\*x + 820\*c^3\*d\*n^3\*x\*x^(6\*n) + 2775\*b\*c^2\*d\*n^3\*x\*x^(5\*n) + 3168\*b^2\*c\*d\*n^3\*x\*x^(4\*n) + 3168\*a\*c^2\*d\*n^3\*x\*x^(4\*n) + 1219\*b^3\*d\*n^3\*x\*x^(3\*n) + 7314\*a\*b\*c\*d\*n^3\*x\*x^(3\*n) + 4260\*a\*b^2\*d\*n^3\*x\*x^(2\*n) + 4260\*a^2\*c\*d\*n^3\*x\*x^(2\*n) + 4995\*a^2\*b\*d\*n^3\*x\*x^n + 735\*c^3\*n^3\*x\*x^(7\*n)\*e + 2460\*b\*c^2\*n^3\*x\*x^(6\*n)\*e + 2775\*b^2\*c\*n^3\*x\*x^(5\*n)\*e + 2775\*a\*c^2\*n^3\*x\*x^(5\*n)\*e + 1056\*b^3\*n^3\*x\*x^(4\*n)\*e + 6336\*a\*b\*c\*n^3\*x\*x^(4\*n)\*e + 3657\*a\*b^2\*n^3\*x\*x^(3\*n)\*e + 3657\*a^2\*c\*n^3\*x\*x^(3\*n)\*e + 4260\*a^2\*b\*n^3\*x\*x^(2\*n)\*e + 1665\*a^3\*n^3\*x\*x^n\*e + 1960\*a^3\*d\*n^3\*x + 190\*c^3\*d\*n^2\*x\*x^(6\*n) + 621\*b\*c^2\*d\*n^2\*x\*x^(5\*n) + 678\*b^2\*c\*d\*n^2\*x\*x^(4\*n) + 678\*a\*c^2\*d\*n^2\*x\*x^(4\*n) + 247\*b^3\*d\*n^2\*x\*x^(3\*n) + 1482\*a\*b\*c\*d\*n^2\*x\*x^(3\*n) + 810\*a\*b^2\*d\*n^2\*x\*x^(2\*n) + 810\*a^2\*c\*d\*n^2\*x\*x^(2\*n) + 885\*a^2\*b\*d\*n^2\*x\*x^n + 175\*c^3\*n^2\*x\*x^(7\*n)\*e + 570\*b\*c^2\*n^2\*x\*x^(6\*n)\*e + 621\*b^2\*c\*n^2\*x\*x^(5\*n)\*e + 621\*a\*c^2\*n^2\*x\*x^(5\*n)\*e + 226\*b^3\*n^2\*x\*x^(4\*n)\*e + 13

$$\begin{aligned}
& 56*a*b*c*n^2*x*x^{(4*n)}*e + 741*a*b^2*n^2*x*x^{(3*n)}*e + 741*a^2*c*n^2*x*x^{(3*n)}*e + 810*a^2*b*n^2*x*x^{(2*n)}*e + 295*a^3*n^2*x*x^n*e + 322*a^3*d*n^2*x + \\
& 22*c^3*d*n*x*x^{(6*n)} + 69*b*c^2*d*n*x*x^{(5*n)} + 72*b^2*c*d*n*x*x^{(4*n)} + 72*a*c^2*d*n*x*x^{(4*n)} + 25*b^3*d*n*x*x^{(3*n)} + 150*a*b*c*d*n*x*x^{(3*n)} + 78 \\
& *a*b^2*d*n*x*x^{(2*n)} + 78*a^2*c*d*n*x*x^{(2*n)} + 81*a^2*b*d*n*x*x^n + 21*c^3*n*x*x^{(7*n)}*e + 66*b*c^2*n*x*x^{(6*n)}*e + 69*b^2*c*n*x*x^{(5*n)}*e + 69*a*c^2 \\
& *n*x*x^{(5*n)}*e + 24*b^3*n*x*x^{(4*n)}*e + 144*a*b*c*n*x*x^{(4*n)}*e + 75*a*b^2*n*x*x^{(3*n)}*e + 75*a^2*c*n*x*x^{(3*n)}*e + 78*a^2*b*n*x*x^{(2*n)}*e + 27*a^3*n* \\
& x*x^n*e + 28*a^3*d*n*x + c^3*d*x*x^{(6*n)} + 3*b*c^2*d*x*x^{(5*n)} + 3*b^2*c*d*x*x^{(4*n)} + 3*a*c^2*d*x*x^{(4*n)} + b^3*d*x*x^{(3*n)} + 6*a*b*c*d*x*x^{(3*n)} + 3 \\
& *a*b^2*d*x*x^{(2*n)} + 3*a^2*c*d*x*x^{(2*n)} + 3*a^2*b*d*x*x^n + c^3*x*x^{(7*n)}*e + 3*b*c^2*x*x^{(6*n)}*e + 3*b^2*c*x*x^{(5*n)}*e + 3*a*c^2*x*x^{(5*n)}*e + b^3*x \\
& *x^{(4*n)}*e + 6*a*b*c*x*x^{(4*n)}*e + 3*a*b^2*x*x^{(3*n)}*e + 3*a^2*c*x*x^{(3*n)}*e + 3*a^2*b*x*x^{(2*n)}*e + a^3*x*x^n*e + a^3*d*x)/(5040*n^7 + 13068*n^6 + 13 \\
& 132*n^5 + 6769*n^4 + 1960*n^3 + 322*n^2 + 28*n + 1)
\end{aligned}$$

$$3.69 \quad \int \frac{(d+ex^n)^3}{a+bx^n+cx^{2n}} dx$$

**Optimal.** Leaf size=308

$$\frac{x \left( \frac{(2cd-be)(-ce(3ae+bd)+b^2e^2+c^2d^2)}{\sqrt{b^2-4ac}} - ace^3 + b^2e^3 - 3bcde^2 + 3c^2d^2e \right) {}_2F_1 \left( 1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{2cx^n}{b-\sqrt{b^2-4ac}} \right) + x \left( -\frac{(2cd-be)(-ce(3ae+bd)+b^2e^2+c^2d^2)}{\sqrt{b^2-4ac}} - ace^3 + b^2e^3 - 3bcde^2 + 3c^2d^2e \right) {}_2F_1 \left( 1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{2cx^n}{b+\sqrt{b^2-4ac}} \right)}{c^2 (b - \sqrt{b^2 - 4ac})}$$

[Out] (e<sup>2</sup>\*(3\*c\*d - b\*e)\*x)/c<sup>2</sup> + (e<sup>3</sup>\*x<sup>(1 + n)</sup>)/(c\*(1 + n)) + ((3\*c<sup>2</sup>\*d<sup>2</sup>\*e - 3\*b\*c\*d\*e<sup>2</sup> + b<sup>2</sup>\*e<sup>3</sup> - a\*c\*e<sup>3</sup> + ((2\*c\*d - b\*e)\*(c<sup>2</sup>\*d<sup>2</sup> + b<sup>2</sup>\*e<sup>2</sup> - c\*e\*(b\*d + 3\*a\*e)))/Sqrt[b<sup>2</sup> - 4\*a\*c])\*x\*Hypergeometric2F1[1, n<sup>(-1)</sup>, 1 + n<sup>(-1)</sup>, (-2\*c\*x<sup>n</sup>)/(b - Sqrt[b<sup>2</sup> - 4\*a\*c])]/(c<sup>2</sup>\*(b - Sqrt[b<sup>2</sup> - 4\*a\*c])) + ((3\*c<sup>2</sup>\*d<sup>2</sup>\*e - 3\*b\*c\*d\*e<sup>2</sup> + b<sup>2</sup>\*e<sup>3</sup> - a\*c\*e<sup>3</sup> - ((2\*c\*d - b\*e)\*(c<sup>2</sup>\*d<sup>2</sup> + b<sup>2</sup>\*e<sup>2</sup> - c\*e\*(b\*d + 3\*a\*e)))/Sqrt[b<sup>2</sup> - 4\*a\*c])\*x\*Hypergeometric2F1[1, n<sup>(-1)</sup>, 1 + n<sup>(-1)</sup>, (-2\*c\*x<sup>n</sup>)/(b + Sqrt[b<sup>2</sup> - 4\*a\*c])]/(c<sup>2</sup>\*(b + Sqrt[b<sup>2</sup> - 4\*a\*c]))

**Rubi [A]** time = 0.69909, antiderivative size = 308, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$ , Rules used = {1424, 1422, 245}

$$\frac{x \left( \frac{(2cd-be)(-ce(3ae+bd)+b^2e^2+c^2d^2)}{\sqrt{b^2-4ac}} - ace^3 + b^2e^3 - 3bcde^2 + 3c^2d^2e \right) {}_2F_1 \left( 1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{2cx^n}{b-\sqrt{b^2-4ac}} \right) + x \left( -\frac{(2cd-be)(-ce(3ae+bd)+b^2e^2+c^2d^2)}{\sqrt{b^2-4ac}} - ace^3 + b^2e^3 - 3bcde^2 + 3c^2d^2e \right) {}_2F_1 \left( 1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{2cx^n}{b+\sqrt{b^2-4ac}} \right)}{c^2 (b - \sqrt{b^2 - 4ac})}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x^n)^3/(a + b\*x^n + c\*x^(2\*n)),x]

[Out] (e<sup>2</sup>\*(3\*c\*d - b\*e)\*x)/c<sup>2</sup> + (e<sup>3</sup>\*x<sup>(1 + n)</sup>)/(c\*(1 + n)) + ((3\*c<sup>2</sup>\*d<sup>2</sup>\*e - 3\*b\*c\*d\*e<sup>2</sup> + b<sup>2</sup>\*e<sup>3</sup> - a\*c\*e<sup>3</sup> + ((2\*c\*d - b\*e)\*(c<sup>2</sup>\*d<sup>2</sup> + b<sup>2</sup>\*e<sup>2</sup> - c\*e\*(b\*d + 3\*a\*e)))/Sqrt[b<sup>2</sup> - 4\*a\*c])\*x\*Hypergeometric2F1[1, n<sup>(-1)</sup>, 1 + n<sup>(-1)</sup>, (-2\*c\*x<sup>n</sup>)/(b - Sqrt[b<sup>2</sup> - 4\*a\*c])]/(c<sup>2</sup>\*(b - Sqrt[b<sup>2</sup> - 4\*a\*c])) + ((3\*c<sup>2</sup>\*d<sup>2</sup>\*e - 3\*b\*c\*d\*e<sup>2</sup> + b<sup>2</sup>\*e<sup>3</sup> - a\*c\*e<sup>3</sup> - ((2\*c\*d - b\*e)\*(c<sup>2</sup>\*d<sup>2</sup> + b<sup>2</sup>\*e<sup>2</sup> - c\*e\*(b\*d + 3\*a\*e)))/Sqrt[b<sup>2</sup> - 4\*a\*c])\*x\*Hypergeometric2F1[1, n<sup>(-1)</sup>, 1 + n<sup>(-1)</sup>, (-2\*c\*x<sup>n</sup>)/(b + Sqrt[b<sup>2</sup> - 4\*a\*c])]/(c<sup>2</sup>\*(b + Sqrt[b<sup>2</sup> - 4\*a\*c]))

Rule 1424

```
Int[((d_) + (e_)*(x_)^(n_))^(q_)/((a_) + (b_)*(x_)^(n_) + (c_)*(x_)^(n2_)), x_Symbol] := Int[ExpandIntegrand[(d + e*x^n)^q/(a + b*x^n + c*x^(2*n)), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[q]
```

### Rule 1422

```
Int[((d_) + (e_)*(x_)^(n_))/((a_) + (b_)*(x_)^(n_) + (c_)*(x_)^(n2_)), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a*c] || !IGtQ[n/2, 0])
```

### Rule 245

```
Int[((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Simp[a^p*x*Hypergeometric2F1[-p, 1/n, 1/n + 1, -(b*x^n)/a], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] || GtQ[a, 0])
```

### Rubi steps

$$\begin{aligned} \int \frac{(d + ex^n)^3}{a + bx^n + cx^{2n}} dx &= \int \left( \frac{e^2(3cd - be)}{c^2} + \frac{e^3x^n}{c} + \frac{c^2d^3 - 3acde^2 + abe^3 + (3c^2d^2e - 3bcde^2 + b^2e^3 - ace^3)x^n}{c^2(a + bx^n + cx^{2n})} \right) dx \\ &= \frac{e^2(3cd - be)x}{c^2} + \frac{e^3x^{1+n}}{c(1+n)} + \frac{\int \frac{c^2d^3 - 3acde^2 + abe^3 + (3c^2d^2e - 3bcde^2 + b^2e^3 - ace^3)x^n}{a + bx^n + cx^{2n}} dx}{c^2} \\ &= \frac{e^2(3cd - be)x}{c^2} + \frac{e^3x^{1+n}}{c(1+n)} + \frac{\left( 3c^2d^2e - 3bcde^2 + b^2e^3 - ace^3 - \frac{(2cd - be)(c^2d^2 + b^2e^2 - ce(bd + 3ae))}{\sqrt{b^2 - 4ac}} \right) \int \frac{\frac{b}{2} + \frac{1}{2} \sqrt{b^2 - 4ac}}{2c^2}}{2c^2} \\ &= \frac{e^2(3cd - be)x}{c^2} + \frac{e^3x^{1+n}}{c(1+n)} + \frac{\left( 3c^2d^2e - 3bcde^2 + b^2e^3 - ace^3 + \frac{(2cd - be)(c^2d^2 + b^2e^2 - ce(bd + 3ae))}{\sqrt{b^2 - 4ac}} \right) x {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; \frac{2cx^n}{\sqrt{b^2 - 4ac} - b}\right)}{c^2 \left( b - \sqrt{b^2 - 4ac} \right)} \end{aligned}$$

**Mathematica [A]** time = 0.87564, size = 295, normalized size = 0.96

$$x \left( \frac{\left( \frac{(2cd - be)(-ce(3ae + bd) + b^2e^2 + c^2d^2)}{\sqrt{b^2 - 4ac}} - ace^3 + b^2e^3 - 3bcde^2 + 3c^2d^2e \right) {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; \frac{2cx^n}{\sqrt{b^2 - 4ac} - b}\right)}{b - \sqrt{b^2 - 4ac}} + \frac{\left( \frac{(be - 2cd)(-ce(3ae + bd) + b^2e^2 + c^2d^2)}{\sqrt{b^2 - 4ac}} - ace^3 + b^2e^3 - 3bcde^2 + 3c^2d^2e \right) {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; \frac{2cx^n}{\sqrt{b^2 - 4ac} + b}\right)}{\sqrt{b^2 - 4ac} + b} \right) / c^2$$

Antiderivative was successfully verified.

[In] Integrate[(d + e\*x^n)^3/(a + b\*x^n + c\*x^(2\*n)),x]

[Out] (x\*(e^2\*(3\*c\*d - b\*e) + (c\*e^3\*x^n)/(1 + n) + ((3\*c^2\*d^2\*e - 3\*b\*c\*d\*e^2 + b^2\*e^3 - a\*c\*e^3 + ((2\*c\*d - b\*e)\*(c^2\*d^2 + b^2\*e^2 - c\*e\*(b\*d + 3\*a\*e)))/Sqrt[b^2 - 4\*a\*c])\*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (2\*c\*x^n)/(-b + Sqrt[b^2 - 4\*a\*c])])/(b - Sqrt[b^2 - 4\*a\*c]) + ((3\*c^2\*d^2\*e - 3\*b\*c\*d\*e^2 + b^2\*e^3 - a\*c\*e^3 + ((-2\*c\*d + b\*e)\*(c^2\*d^2 + b^2\*e^2 - c\*e\*(b\*d + 3\*a\*e)))/Sqrt[b^2 - 4\*a\*c])\*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2\*c\*x^n)/(b + Sqrt[b^2 - 4\*a\*c])])/(b + Sqrt[b^2 - 4\*a\*c]))/c^2

**Maple [F]** time = 0.052, size = 0, normalized size = 0.

$$\int \frac{(d + ex^n)^3}{a + bx^n + cx^{2n}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d+e\*x^n)^3/(a+b\*x^n+c\*x^(2\*n)),x)

[Out] int((d+e\*x^n)^3/(a+b\*x^n+c\*x^(2\*n)),x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{ce^3xx^n + (3cde^2(n+1) - be^3(n+1))x}{c^2(n+1)} - \int \frac{c^2d^3 - (3cde^2 - be^3)a + (3c^2d^2e - 3bcde^2 + b^2e^3 - ace^3)x^n}{c^3x^{2n} + bc^2x^n + ac^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e\*x^n)^3/(a+b\*x^n+c\*x^(2\*n)),x, algorithm="maxima")

[Out] (c\*e^3\*x\*x^n + (3\*c\*d\*e^2\*(n + 1) - b\*e^3\*(n + 1))\*x)/(c^2\*(n + 1)) - integrate(-(c^2\*d^3 - (3\*c\*d\*e^2 - b\*e^3)\*a + (3\*c^2\*d^2\*e - 3\*b\*c\*d\*e^2 + b^2\*e^3 - a\*c\*e^3)\*x^n)/(c^3\*x^(2\*n) + b\*c^2\*x^n + a\*c^2), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{e^3 x^{3n} + 3 d e^2 x^{2n} + 3 d^2 e x^n + d^3}{c x^{2n} + b x^n + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e\*x^n)^3/(a+b\*x^n+c\*x^(2\*n)),x, algorithm="fricas")

[Out] integral((e^3\*x^(3\*n) + 3\*d\*e^2\*x^(2\*n) + 3\*d^2\*e\*x^n + d^3)/(c\*x^(2\*n) + b\*x^n + a), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e\*x\*\*n)\*\*3/(a+b\*x\*\*n+c\*x\*\*(2\*n)),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(e x^n + d)^3}{c x^{2n} + b x^n + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e\*x^n)^3/(a+b\*x^n+c\*x^(2\*n)),x, algorithm="giac")

[Out] integrate((e\*x^n + d)^3/(c\*x^(2\*n) + b\*x^n + a), x)

$$3.70 \quad \int \frac{(d+ex^n)^2}{a+bx^n+cx^{2n}} dx$$

**Optimal.** Leaf size=224

$$\frac{x \left( \frac{-2ce(ae+bd)+b^2e^2+2c^2d^2}{\sqrt{b^2-4ac}} - be^2 + 2cde \right) {}_2F_1 \left( 1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{2cx^n}{b-\sqrt{b^2-4ac}} \right)}{c \left( b - \sqrt{b^2 - 4ac} \right)} + \frac{x \left( \frac{-2ce(ae+bd)+b^2e^2+2c^2d^2}{\sqrt{b^2-4ac}} - be^2 + 2cde \right) {}_2F_1 \left( 1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{2cx^n}{b+\sqrt{b^2-4ac}} \right)}{c \left( \sqrt{b^2 - 4ac} + b \right)}$$

```
[Out] (e^2*x)/c + ((2*c*d*e - b*e^2 + (2*c^2*d^2 + b^2*e^2 - 2*c*e*(b*d + a*e))/Sqrt[b^2 - 4*a*c])*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])])/(c*(b - Sqrt[b^2 - 4*a*c])) + ((2*c*d*e - b*e^2 - (2*c^2*d^2 + b^2*e^2 - 2*c*e*(b*d + a*e))/Sqrt[b^2 - 4*a*c])*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(c*(b + Sqrt[b^2 - 4*a*c]))
```

**Rubi [A]** time = 0.475921, antiderivative size = 224, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$ , Rules used = {1424, 1422, 245}

$$\frac{x \left( \frac{-2ce(ae+bd)+b^2e^2+2c^2d^2}{\sqrt{b^2-4ac}} - be^2 + 2cde \right) {}_2F_1 \left( 1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{2cx^n}{b-\sqrt{b^2-4ac}} \right)}{c \left( b - \sqrt{b^2 - 4ac} \right)} + \frac{x \left( \frac{-2ce(ae+bd)+b^2e^2+2c^2d^2}{\sqrt{b^2-4ac}} - be^2 + 2cde \right) {}_2F_1 \left( 1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{2cx^n}{b+\sqrt{b^2-4ac}} \right)}{c \left( \sqrt{b^2 - 4ac} + b \right)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^n)^2/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] (e^2*x)/c + ((2*c*d*e - b*e^2 + (2*c^2*d^2 + b^2*e^2 - 2*c*e*(b*d + a*e))/Sqrt[b^2 - 4*a*c])*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])])/(c*(b - Sqrt[b^2 - 4*a*c])) + ((2*c*d*e - b*e^2 - (2*c^2*d^2 + b^2*e^2 - 2*c*e*(b*d + a*e))/Sqrt[b^2 - 4*a*c])*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(c*(b + Sqrt[b^2 - 4*a*c]))
```

**Rule 1424**

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x_Symbol] := Int[ExpandIntegrand[(d + e*x^n)^q/(a + b*x^n + c*x^(2*n)), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[q]
```



Rule 1422

```
Int[((d_) + (e_)*(x_)^(n_))/((a_) + (b_)*(x_)^(n_) + (c_)*(x_)^(2n_)), x
_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] || !IGtQ[n/2, 0])
```

Rule 245

```
Int[((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{(d + ex^n)^2}{a + bx^n + cx^{2n}} dx &= \int \left( \frac{e^2}{c} + \frac{cd^2 - ae^2 + (2cde - be^2)x^n}{c(a + bx^n + cx^{2n})} \right) dx \\ &= \frac{e^2 x}{c} + \frac{\int \frac{cd^2 - ae^2 + (2cde - be^2)x^n}{a + bx^n + cx^{2n}} dx}{c} \\ &= \frac{e^2 x}{c} + \frac{\left(2cde - be^2 - \frac{2c^2 d^2 + b^2 e^2 - 2ce(bd + ae)}{\sqrt{b^2 - 4ac}}\right) \int \frac{1}{\frac{b}{2} + \frac{1}{2}\sqrt{b^2 - 4ac} + cx^n} dx}{2c} + \frac{\left(2cde - be^2 + \frac{2c^2 d^2 + b^2 e^2 - 2ce(bd + ae)}{\sqrt{b^2 - 4ac}}\right)}{2c} \\ &= \frac{e^2 x}{c} + \frac{\left(2cde - be^2 + \frac{2c^2 d^2 + b^2 e^2 - 2ce(bd + ae)}{\sqrt{b^2 - 4ac}}\right) x {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{2cx^n}{b - \sqrt{b^2 - 4ac}}\right)}{c(b - \sqrt{b^2 - 4ac})} + \frac{\left(2cde - be^2 - \frac{2c^2 d^2 + b^2 e^2 - 2ce(bd + ae)}{\sqrt{b^2 - 4ac}}\right)}{c(b + \sqrt{b^2 - 4ac})} \end{aligned}$$

**Mathematica [A]** time = 0.531575, size = 216, normalized size = 0.96

$$x \left( \frac{\left( \frac{-2ce(ae+bd)+b^2e^2+2c^2d^2}{\sqrt{b^2-4ac}} - be^2 + 2cde \right) {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{2cx^n}{\sqrt{b^2-4ac}-b}\right)}{b - \sqrt{b^2-4ac}} + \frac{\left( \frac{-2ce(ae+bd)+b^2e^2+2c^2d^2}{\sqrt{b^2-4ac}} - be^2 + 2cde \right) {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{2cx^n}{b + \sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}+b} + e^2 \right)$$

c

Antiderivative was successfully verified.

[In] Integrate[(d + e\*x^n)^2/(a + b\*x^n + c\*x^(2\*n)), x]

```
[Out] (x*(e^2 + ((2*c*d*e - b*e^2 + (2*c^2*d^2 + b^2*e^2 - 2*c*e*(b*d + a*e))/Sqrt[b^2 - 4*a*c])*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])]))/(b - Sqrt[b^2 - 4*a*c]) + ((2*c*d*e - b*e^2 - (2*c^2*d^2 + b^2*e^2 - 2*c*e*(b*d + a*e))/Sqrt[b^2 - 4*a*c])*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]))/(b + Sqrt[b^2 - 4*a*c])))/c
```

**Maple [F]** time = 0.036, size = 0, normalized size = 0.

$$\int \frac{(d + ex^n)^2}{a + bx^n + cx^{2n}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+e*x^n)^2/(a+b*x^n+c*x^(2*n)),x)
```

```
[Out] int((d+e*x^n)^2/(a+b*x^n+c*x^(2*n)),x)
```

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{e^2x}{c} - \int -\frac{cd^2 - ae^2 + (2cde - be^2)x^n}{c^2x^{2n} + bcx^n + ac} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x^n)^2/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

```
[Out] e^2*x/c - integrate(-(c*d^2 - a*e^2 + (2*c*d*e - b*e^2)*x^n)/(c^2*x^(2*n) + b*c*x^n + a*c), x)
```

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{e^2x^{2n} + 2dex^n + d^2}{cx^{2n} + bx^n + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e\*x^n)^2/(a+b\*x^n+c\*x^(2\*n)),x, algorithm="fricas")

[Out] integral((e^2\*x^(2\*n) + 2\*d\*e\*x^n + d^2)/(c\*x^(2\*n) + b\*x^n + a), x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(d + ex^n)^2}{a + bx^n + cx^{2n}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e\*x\*\*n)\*\*2/(a+b\*x\*\*n+c\*x\*\*(2\*n)),x)

[Out] Integral((d + e\*x\*\*n)\*\*2/(a + b\*x\*\*n + c\*x\*\*(2\*n)), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^n + d)^2}{cx^{2n} + bx^n + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e\*x^n)^2/(a+b\*x^n+c\*x^(2\*n)),x, algorithm="giac")

[Out] integrate((e\*x^n + d)^2/(c\*x^(2\*n) + b\*x^n + a), x)

### 3.71 $\int \frac{d+ex^n}{a+bx^n+cx^{2n}} dx$

**Optimal.** Leaf size=154

$$\frac{x \left( \frac{2cd-be}{\sqrt{b^2-4ac}} + e \right) {}_2F_1 \left( 1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{2cx^n}{b-\sqrt{b^2-4ac}} \right)}{b - \sqrt{b^2 - 4ac}} + \frac{x \left( e - \frac{2cd-be}{\sqrt{b^2-4ac}} \right) {}_2F_1 \left( 1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{2cx^n}{b+\sqrt{b^2-4ac}} \right)}{\sqrt{b^2 - 4ac} + b}$$

[Out] ((e + (2\*c\*d - b\*e)/Sqrt[b^2 - 4\*a\*c])\*x\*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2\*c\*x^n)/(b - Sqrt[b^2 - 4\*a\*c])])/(b - Sqrt[b^2 - 4\*a\*c]) + ((e - (2\*c\*d - b\*e)/Sqrt[b^2 - 4\*a\*c])\*x\*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2\*c\*x^n)/(b + Sqrt[b^2 - 4\*a\*c])])/(b + Sqrt[b^2 - 4\*a\*c])

**Rubi [A]** time = 0.120234, antiderivative size = 154, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$ , Rules used = {1422, 245}

$$\frac{x \left( \frac{2cd-be}{\sqrt{b^2-4ac}} + e \right) {}_2F_1 \left( 1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{2cx^n}{b-\sqrt{b^2-4ac}} \right)}{b - \sqrt{b^2 - 4ac}} + \frac{x \left( e - \frac{2cd-be}{\sqrt{b^2-4ac}} \right) {}_2F_1 \left( 1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{2cx^n}{b+\sqrt{b^2-4ac}} \right)}{\sqrt{b^2 - 4ac} + b}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x^n)/(a + b\*x^n + c\*x^(2\*n)),x]

[Out] ((e + (2\*c\*d - b\*e)/Sqrt[b^2 - 4\*a\*c])\*x\*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2\*c\*x^n)/(b - Sqrt[b^2 - 4\*a\*c])])/(b - Sqrt[b^2 - 4\*a\*c]) + ((e - (2\*c\*d - b\*e)/Sqrt[b^2 - 4\*a\*c])\*x\*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2\*c\*x^n)/(b + Sqrt[b^2 - 4\*a\*c])])/(b + Sqrt[b^2 - 4\*a\*c])

#### Rule 1422

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] || !IGtQ[n/2, 0])
```

#### Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
```

, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] || GtQ[a, 0])

### Rubi steps

$$\int \frac{d + ex^n}{a + bx^n + cx^{2n}} dx = \frac{1}{2} \left( e - \frac{2cd - be}{\sqrt{b^2 - 4ac}} \right) \int \frac{1}{\frac{b}{2} + \frac{1}{2}\sqrt{b^2 - 4ac} + cx^n} dx + \frac{1}{2} \left( e + \frac{2cd - be}{\sqrt{b^2 - 4ac}} \right) \int \frac{1}{\frac{b}{2} - \frac{1}{2}\sqrt{b^2 - 4ac} + cx^n} dx$$

$$= \frac{\left( e + \frac{2cd - be}{\sqrt{b^2 - 4ac}} \right) x {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{2cx^n}{b - \sqrt{b^2 - 4ac}}\right)}{b - \sqrt{b^2 - 4ac}} + \frac{\left( e - \frac{2cd - be}{\sqrt{b^2 - 4ac}} \right) x {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{2cx^n}{b + \sqrt{b^2 - 4ac}}\right)}{b + \sqrt{b^2 - 4ac}}$$

**Mathematica [A]** time = 0.0754941, size = 134, normalized size = 0.87

$$\frac{x \left( \left( d\sqrt{b^2 - 4ac} - 2ae + bd \right) {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; \frac{2cx^n}{\sqrt{b^2 - 4ac} - b}\right) + \left( d\sqrt{b^2 - 4ac} + 2ae - bd \right) {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{2cx^n}{b + \sqrt{b^2 - 4ac}}\right) \right)}{2a\sqrt{b^2 - 4ac}}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e\*x^n)/(a + b\*x^n + c\*x^(2\*n)), x]

[Out] (x\*((b\*d + Sqrt[b^2 - 4\*a\*c]\*d - 2\*a\*e)\*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (2\*c\*x^n)/(-b + Sqrt[b^2 - 4\*a\*c])]) + (-b\*d) + Sqrt[b^2 - 4\*a\*c]\*d + 2\*a\*e)\*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2\*c\*x^n)/(b + Sqrt[b^2 - 4\*a\*c])])/(2\*a\*Sqrt[b^2 - 4\*a\*c])

**Maple [F]** time = 0.021, size = 0, normalized size = 0.

$$\int \frac{d + ex^n}{a + bx^n + cx^{2n}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d+e\*x^n)/(a+b\*x^n+c\*x^(2\*n)), x)

[Out] int((d+e\*x^n)/(a+b\*x^n+c\*x^(2\*n)), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{ex^n + d}{cx^{2n} + bx^n + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e\*x^n)/(a+b\*x^n+c\*x^(2\*n)),x, algorithm="maxima")

[Out] integrate((e\*x^n + d)/(c\*x^(2\*n) + b\*x^n + a), x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{ex^n + d}{cx^{2n} + bx^n + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e\*x^n)/(a+b\*x^n+c\*x^(2\*n)),x, algorithm="fricas")

[Out] integral((e\*x^n + d)/(c\*x^(2\*n) + b\*x^n + a), x)

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{d + ex^n}{a + bx^n + cx^{2n}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e\*x\*\*n)/(a+b\*x\*\*n+c\*x\*\*(2\*n)),x)

[Out] Integral((d + e\*x\*\*n)/(a + b\*x\*\*n + c\*x\*\*(2\*n)), x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{ex^n + d}{cx^{2n} + bx^n + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x^n)/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
```

```
[Out] integrate((e*x^n + d)/(c*x^(2*n) + b*x^n + a), x)
```

$$3.72 \quad \int \frac{1}{(d+ex^n)(a+bx^n+cx^{2n})} dx$$

**Optimal.** Leaf size=243

$$\frac{cx \left( 2cd - e \left( \sqrt{b^2 - 4ac} + b \right) \right) {}_2F_1 \left( 1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{2cx^n}{b - \sqrt{b^2 - 4ac}} \right)}{\left( -b\sqrt{b^2 - 4ac} - 4ac + b^2 \right) (ae^2 - bde + cd^2)} - \frac{cx \left( \frac{2cd - be}{\sqrt{b^2 - 4ac}} + e \right) {}_2F_1 \left( 1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{2cx^n}{b + \sqrt{b^2 - 4ac}} \right)}{\left( \sqrt{b^2 - 4ac} + b \right) (ae^2 - bde + cd^2)} + \frac{e^2 x {}_2F_1 \left( 1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{2cx^n}{b + \sqrt{b^2 - 4ac}} \right)}{d (ae^2 - bde + cd^2)}$$

[Out] -((c\*(2\*c\*d - (b + Sqrt[b^2 - 4\*a\*c]))\*e)\*x\*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2\*c\*x^n)/(b - Sqrt[b^2 - 4\*a\*c])])/(b^2 - 4\*a\*c - b\*Sqrt[b^2 - 4\*a\*c])\*(c\*d^2 - b\*d\*e + a\*e^2)) - (c\*(e + (2\*c\*d - b\*e)/Sqrt[b^2 - 4\*a\*c])\*x\*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2\*c\*x^n)/(b + Sqrt[b^2 - 4\*a\*c])])/(b + Sqrt[b^2 - 4\*a\*c])\*(c\*d^2 - b\*d\*e + a\*e^2)) + (e^2\*x\*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -(e\*x^n)/d])/(d\*(c\*d^2 - b\*d\*e + a\*e^2))

**Rubi [A]** time = 0.469391, antiderivative size = 243, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$ , Rules used = {1424, 245, 1422}

$$\frac{cx \left( 2cd - e \left( \sqrt{b^2 - 4ac} + b \right) \right) {}_2F_1 \left( 1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{2cx^n}{b - \sqrt{b^2 - 4ac}} \right)}{\left( -b\sqrt{b^2 - 4ac} - 4ac + b^2 \right) (ae^2 - bde + cd^2)} - \frac{cx \left( \frac{2cd - be}{\sqrt{b^2 - 4ac}} + e \right) {}_2F_1 \left( 1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{2cx^n}{b + \sqrt{b^2 - 4ac}} \right)}{\left( \sqrt{b^2 - 4ac} + b \right) (ae^2 - bde + cd^2)} + \frac{e^2 x {}_2F_1 \left( 1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{2cx^n}{b + \sqrt{b^2 - 4ac}} \right)}{d (ae^2 - bde + cd^2)}$$

Antiderivative was successfully verified.

[In] Int[1/((d + e\*x^n)\*(a + b\*x^n + c\*x^(2\*n))),x]

[Out] -((c\*(2\*c\*d - (b + Sqrt[b^2 - 4\*a\*c]))\*e)\*x\*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2\*c\*x^n)/(b - Sqrt[b^2 - 4\*a\*c])])/(b^2 - 4\*a\*c - b\*Sqrt[b^2 - 4\*a\*c])\*(c\*d^2 - b\*d\*e + a\*e^2)) - (c\*(e + (2\*c\*d - b\*e)/Sqrt[b^2 - 4\*a\*c])\*x\*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2\*c\*x^n)/(b + Sqrt[b^2 - 4\*a\*c])])/(b + Sqrt[b^2 - 4\*a\*c])\*(c\*d^2 - b\*d\*e + a\*e^2)) + (e^2\*x\*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -(e\*x^n)/d])/(d\*(c\*d^2 - b\*d\*e + a\*e^2))

**Rule 1424**

Int[((d\_) + (e\_.)\*(x\_)^(n\_))^(q\_)/((a\_) + (b\_.)\*(x\_)^(n\_) + (c\_.)\*(x\_)^(n2\_)), x\_Symbol] :> Int[ExpandIntegrand[(d + e\*x^n)^q/(a + b\*x^n + c\*x^(2\*n)), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2\*n] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && IntegerQ[q]



Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 1422

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(2n_)), x
_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] || !IGtQ[n/2, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{1}{(d + ex^n)(a + bx^n + cx^{2n})} dx &= \int \left( \frac{e^2}{(cd^2 - bde + ae^2)(d + ex^n)} + \frac{cd - be - cex^n}{(cd^2 - bde + ae^2)(a + bx^n + cx^{2n})} \right) dx \\ &= \frac{\int \frac{cd - be - cex^n}{a + bx^n + cx^{2n}} dx}{cd^2 - bde + ae^2} + \frac{e^2 \int \frac{1}{d + ex^n} dx}{cd^2 - bde + ae^2} \\ &= \frac{e^2 x {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{ex^n}{d}\right)}{d(cd^2 - bde + ae^2)} - \frac{\left(c\left(e - \frac{2cd - be}{\sqrt{b^2 - 4ac}}\right)\right) \int \frac{1}{\frac{b}{2} - \frac{1}{2}\sqrt{b^2 - 4ac} + cx^n} dx}{2(cd^2 - bde + ae^2)} - \frac{c\left(e + \frac{2cd - be}{\sqrt{b^2 - 4ac}}\right)}{2(cd^2 - bde + ae^2)} \\ &= -\frac{c\left(e - \frac{2cd - be}{\sqrt{b^2 - 4ac}}\right) x {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{2cx^n}{b - \sqrt{b^2 - 4ac}}\right)}{(b - \sqrt{b^2 - 4ac})(cd^2 - bde + ae^2)} - \frac{c\left(e + \frac{2cd - be}{\sqrt{b^2 - 4ac}}\right) x {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{2cx^n}{b + \sqrt{b^2 - 4ac}}\right)}{(b + \sqrt{b^2 - 4ac})(cd^2 - bde + ae^2)} \end{aligned}$$

**Mathematica [A]** time = 0.447375, size = 200, normalized size = 0.82

$$\frac{x \left( -\frac{c\left(\frac{be - 2cd}{\sqrt{b^2 - 4ac}} + e\right) {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{2cx^n}{\sqrt{b^2 - 4ac} - b}\right)}{b - \sqrt{b^2 - 4ac}} - \frac{c\left(\frac{2cd - be}{\sqrt{b^2 - 4ac}} + e\right) {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{2cx^n}{b + \sqrt{b^2 - 4ac}}\right)}{\sqrt{b^2 - 4ac} + b} + \frac{e^2 {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{ex^n}{d}\right)}{d} \right)}{e(ae - bd) + cd^2}$$

Antiderivative was successfully verified.

[In] Integrate[1/((d + e\*x^n)\*(a + b\*x^n + c\*x^(2\*n))), x]

```
[Out] (x*(-((c*(e + (-2*c*d + b*e)/Sqrt[b^2 - 4*a*c])*Hypergeometric2F1[1, n^(-1)
, 1 + n^(-1), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])]))/(b - Sqrt[b^2 - 4*a*c]))
- (c*(e + (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*Hypergeometric2F1[1, n^(-1), 1
+ n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]))/(b + Sqrt[b^2 - 4*a*c]) + (e
^2*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((e*x^n)/d)]/d))/(c*d^2 + e*(
-(b*d) + a*e))
```

**Maple [F]** time = 0.06, size = 0, normalized size = 0.

$$\int \frac{1}{(d + ex^n)(a + bx^n + cx^{2n})} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d+e*x^n)/(a+b*x^n+c*x^(2*n)),x)
```

```
[Out] int(1/(d+e*x^n)/(a+b*x^n+c*x^(2*n)),x)
```

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(cx^{2n} + bx^n + a)(ex^n + d)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x^n)/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^(2*n) + b*x^n + a)*(e*x^n + d)), x)
```

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{bex^{2n} + ad + (cex^n + cd)x^{2n} + (bd + ae)x^n}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x^n)/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

[Out] `integral(1/(b*e*x^(2*n) + a*d + (c*e*x^n + c*d)*x^(2*n) + (b*d + a*e)*x^n), x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(d+e*x**n)/(a+b*x**n+c*x**(2*n)), x)`

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(cx^{2n} + bx^n + a)(ex^n + d)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(d+e*x^n)/(a+b*x^n+c*x^(2*n)), x, algorithm="giac")`

[Out] `integrate(1/((c*x^(2*n) + b*x^n + a)*(e*x^n + d)), x)`

$$3.73 \quad \int \frac{1}{(d+ex^n)^2(a+bx^n+cx^{2n})} dx$$

**Optimal.** Leaf size=368

$$\frac{cx \left( -2ce \left( d\sqrt{b^2 - 4ac} + ae + bd \right) + be^2 \left( \sqrt{b^2 - 4ac} + b \right) + 2c^2d^2 \right) {}_2F_1 \left( 1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{2cx^n}{b - \sqrt{b^2 - 4ac}} \right) - cx \left( -2ce \left( -d\sqrt{b^2 - 4ac} + ae + bd \right) + be^2 \left( \sqrt{b^2 - 4ac} + b \right) + 2c^2d^2 \right) {}_2F_1 \left( 1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{2cx^n}{b + \sqrt{b^2 - 4ac}} \right)}{\left( -b\sqrt{b^2 - 4ac} - 4ac + b^2 \right) \left( ae^2 - bde + cd^2 \right)^2}$$

[Out] -((c\*(2\*c^2\*d^2 + b\*(b + Sqrt[b^2 - 4\*a\*c]))\*e^2 - 2\*c\*e\*(b\*d + Sqrt[b^2 - 4\*a\*c]\*d + a\*e))\*x\*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2\*c\*x^n)/(b - Sqrt[b^2 - 4\*a\*c])])/((b^2 - 4\*a\*c - b\*Sqrt[b^2 - 4\*a\*c])\*(c\*d^2 - b\*d\*e + a\*e^2)^2) - (c\*(2\*c^2\*d^2 + b\*(b - Sqrt[b^2 - 4\*a\*c]))\*e^2 - 2\*c\*e\*(b\*d - Sqrt[b^2 - 4\*a\*c]\*d + a\*e))\*x\*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2\*c\*x^n)/(b + Sqrt[b^2 - 4\*a\*c])])/((b^2 - 4\*a\*c + b\*Sqrt[b^2 - 4\*a\*c])\*(c\*d^2 - b\*d\*e + a\*e^2)^2) + (e^2\*(2\*c\*d - b\*e))\*x\*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((e\*x^n)/d)]/(d\*(c\*d^2 - b\*d\*e + a\*e^2)^2) + (e^2\*x\*Hypergeometric2F1[2, n^(-1), 1 + n^(-1), -((e\*x^n)/d)]/(d^2\*(c\*d^2 - b\*d\*e + a\*e^2)))

**Rubi [A]** time = 0.706893, antiderivative size = 368, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 3, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$ , Rules used = {1424, 245, 1422}

$$\frac{cx \left( -2ce \left( d\sqrt{b^2 - 4ac} + ae + bd \right) + be^2 \left( \sqrt{b^2 - 4ac} + b \right) + 2c^2d^2 \right) {}_2F_1 \left( 1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{2cx^n}{b - \sqrt{b^2 - 4ac}} \right) - cx \left( -2ce \left( -d\sqrt{b^2 - 4ac} + ae + bd \right) + be^2 \left( \sqrt{b^2 - 4ac} + b \right) + 2c^2d^2 \right) {}_2F_1 \left( 1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{2cx^n}{b + \sqrt{b^2 - 4ac}} \right)}{\left( -b\sqrt{b^2 - 4ac} - 4ac + b^2 \right) \left( ae^2 - bde + cd^2 \right)^2}$$

Antiderivative was successfully verified.

[In] Int[1/((d + e\*x^n)^2\*(a + b\*x^n + c\*x^(2\*n))),x]

[Out] -((c\*(2\*c^2\*d^2 + b\*(b + Sqrt[b^2 - 4\*a\*c]))\*e^2 - 2\*c\*e\*(b\*d + Sqrt[b^2 - 4\*a\*c]\*d + a\*e))\*x\*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2\*c\*x^n)/(b - Sqrt[b^2 - 4\*a\*c])])/((b^2 - 4\*a\*c - b\*Sqrt[b^2 - 4\*a\*c])\*(c\*d^2 - b\*d\*e + a\*e^2)^2) - (c\*(2\*c^2\*d^2 + b\*(b - Sqrt[b^2 - 4\*a\*c]))\*e^2 - 2\*c\*e\*(b\*d - Sqrt[b^2 - 4\*a\*c]\*d + a\*e))\*x\*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2\*c\*x^n)/(b + Sqrt[b^2 - 4\*a\*c])])/((b^2 - 4\*a\*c + b\*Sqrt[b^2 - 4\*a\*c])\*(c\*d^2 - b\*d\*e + a\*e^2)^2) + (e^2\*(2\*c\*d - b\*e))\*x\*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((e\*x^n)/d)]/(d\*(c\*d^2 - b\*d\*e + a\*e^2)^2) + (e^2\*x\*Hypergeometric2F1[2, n^(-1), 1 + n^(-1), -((e\*x^n)/d)]/(d^2\*(c\*d^2 - b\*d\*e + a\*e^2)))

Rule 1424

```
Int[((d_) + (e_)*(x_)^(n_))^(q_)/((a_) + (b_)*(x_)^(n_) + (c_)*(x_)^(n2_)), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^n)^q/(a + b*x^n + c*x^(2*n)), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[q]
```

Rule 245

```
Int[((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] || GtQ[a, 0])
```

Rule 1422

```
Int[((d_) + (e_)*(x_)^(n_))/((a_) + (b_)*(x_)^(n_) + (c_)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a*c] || !IGtQ[n/2, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{1}{(d + ex^n)^2 (a + bx^n + cx^{2n})} dx &= \int \left( \frac{e^2}{(cd^2 - bde + ae^2)(d + ex^n)^2} - \frac{e^2(-2cd + be)}{(cd^2 - bde + ae^2)^2 (d + ex^n)} + \frac{c^2d^2 - 2bcde + b^2e}{(cd^2 - bde + ae^2)^2} \right) dx \\ &= \frac{\int \frac{c^2d^2 - 2bcde + b^2e^2 - ace^2 - (2c^2de - bce^2)x^n}{a + bx^n + cx^{2n}} dx}{(cd^2 - bde + ae^2)^2} + \frac{(e^2(2cd - be)) \int \frac{1}{d + ex^n} dx}{(cd^2 - bde + ae^2)^2} + \frac{e^2 \int \frac{1}{(d + ex^n)^2} dx}{cd^2 - bde + ae^2} \\ &= \frac{e^2(2cd - be)x {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{ex^n}{d}\right)}{d(cd^2 - bde + ae^2)^2} + \frac{e^2x {}_2F_1\left(2, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{ex^n}{d}\right)}{d^2(cd^2 - bde + ae^2)} - \frac{c(2c^2d^2 + b(b + \sqrt{b^2 - 4ac}))}{(cd^2 - bde + ae^2)^2} \\ &= \frac{c(2c^2d^2 + b(b + \sqrt{b^2 - 4ac}))e^2 - 2ce(bd + \sqrt{b^2 - 4acd} + ae)}{\sqrt{b^2 - 4ac}(b - \sqrt{b^2 - 4ac})(cd^2 - bde + ae^2)^2} x {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{ex^n}{d}\right) \end{aligned}$$

**Mathematica [A]** time = 0.940668, size = 327, normalized size = 0.89

$$x \left( \frac{c \left( -2ce \left( d\sqrt{b^2-4ac+ae+bd} \right) + be^2 \left( \sqrt{b^2-4ac+b} \right) + 2c^2 d^2 \right) {}_2F_1 \left( 1, \frac{1}{n}; 1 + \frac{1}{n}; \frac{2cx^n}{\sqrt{b^2-4ac-b}} \right)}{b\sqrt{b^2-4ac+4ac-b^2}} + \frac{c \left( 2ce \left( -d\sqrt{b^2-4ac+ae+bd} \right) + be^2 \left( \sqrt{b^2-4ac-b} \right) - 2c^2 d^2 \right) {}_2F_1 \left( 1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{2}{b+\sqrt{b^2-4ac-b}} \right)}{b\sqrt{b^2-4ac-4ac+b^2}} \right) \frac{1}{(e(ae-bd) + cd^2)^2}$$

Antiderivative was successfully verified.

[In] Integrate[1/((d + e\*x^n)^2\*(a + b\*x^n + c\*x^(2\*n))),x]

[Out] (x\*((c\*(2\*c^2\*d^2 + b\*(b + Sqrt[b^2 - 4\*a\*c]))\*e^2 - 2\*c\*e\*(b\*d + Sqrt[b^2 - 4\*a\*c]\*d + a\*e))\*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (2\*c\*x^n)/(-b + Sqrt[b^2 - 4\*a\*c])])/(-b^2 + 4\*a\*c + b\*Sqrt[b^2 - 4\*a\*c]) + (c\*(-2\*c^2\*d^2 + b\*(-b + Sqrt[b^2 - 4\*a\*c]))\*e^2 + 2\*c\*e\*(b\*d - Sqrt[b^2 - 4\*a\*c]\*d + a\*e))\*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2\*c\*x^n)/(b + Sqrt[b^2 - 4\*a\*c])])/(-b^2 - 4\*a\*c + b\*Sqrt[b^2 - 4\*a\*c]) + (e^2\*(2\*c\*d - b\*e)\*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -(e\*x^n)/d])/d + (e^2\*(c\*d^2 + e\*(-(b\*d) + a\*e))\*Hypergeometric2F1[2, n^(-1), 1 + n^(-1), -(e\*x^n)/d])/d^2)/(c\*d^2 + e\*(-(b\*d) + a\*e))^2

**Maple [F]** time = 0.136, size = 0, normalized size = 0.

$$\int \frac{1}{(d + ex^n)^2 (a + bx^n + cx^{2n})} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(d+e\*x^n)^2/(a+b\*x^n+c\*x^(2\*n)),x)

[Out] int(1/(d+e\*x^n)^2/(a+b\*x^n+c\*x^(2\*n)),x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{e^2 x}{cd^4 n - bd^3 en + ad^2 e^2 n + (cd^3 en - bd^2 e^2 n + ade^3 n)x^n} + (cd^2 e^2 (3n - 1) - bde^3 (2n - 1) + ae^4 (n - 1)) \int \frac{1}{c^2 d^6 n - 2bcd^5 en}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d+e\*x^n)^2/(a+b\*x^n+c\*x^(2\*n)),x, algorithm="maxima")

[Out] e^2\*x/(c\*d^4\*n - b\*d^3\*e\*n + a\*d^2\*e^2\*n + (c\*d^3\*e\*n - b\*d^2\*e^2\*n + a\*d\*e^3\*n)\*x^n) + (c\*d^2\*e^2\*(3\*n - 1) - b\*d\*e^3\*(2\*n - 1) + a\*e^4\*(n - 1))\*integrate(1/(c^2\*d^6\*n - 2\*b\*c\*d^5\*e\*n + b^2\*d^4\*e^2\*n + a^2\*d^2\*e^4\*n + 2\*(c\*d^4\*e^2\*n - b\*d^3\*e^3\*n)\*a + (c^2\*d^5\*e\*n - 2\*b\*c\*d^4\*e^2\*n + b^2\*d^3\*e^3\*n + a^2\*d\*e^5\*n + 2\*(c\*d^3\*e^3\*n - b\*d^2\*e^4\*n)\*a)\*x^n), x) + integrate((c^2\*d^2 - 2\*b\*c\*d\*e + b^2\*e^2 - a\*c\*e^2 - (2\*c^2\*d\*e - b\*c\*e^2)\*x^n)/(a^3\*e^4 + 2\*(c\*d^2\*e^2 - b\*d\*e^3)\*a^2 + (c^2\*d^4 - 2\*b\*c\*d^3\*e + b^2\*d^2\*e^2)\*a + (c^3\*d^4 - 2\*b\*c^2\*d^3\*e + b^2\*c\*d^2\*e^2 + a^2\*c\*e^4 + 2\*(c^2\*d^2\*e^2 - b\*c\*d\*e^3)\*a)\*x^(2\*n) + (b\*c^2\*d^4 - 2\*b^2\*c\*d^3\*e + b^3\*d^2\*e^2 + a^2\*b\*e^4 + 2\*(b\*c\*d^2\*e^2 - b^2\*d\*e^3)\*a)\*x^n), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{be^2x^{3n} + ad^2 + (ce^2x^{2n} + 2cdex^n + cd^2)x^{2n} + (2bde + ae^2)x^{2n} + (bd^2 + 2ade)x^n}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d+e\*x^n)^2/(a+b\*x^n+c\*x^(2\*n)),x, algorithm="fricas")

[Out] integral(1/(b\*e^2\*x^(3\*n) + a\*d^2 + (c\*e^2\*x^(2\*n) + 2\*c\*d\*e\*x^n + c\*d^2)\*x^(2\*n) + (2\*b\*d\*e + a\*e^2)\*x^(2\*n) + (b\*d^2 + 2\*a\*d\*e)\*x^n), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d+e\*x\*\*n)\*\*2/(a+b\*x\*\*n+c\*x\*\*(2\*n)),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(cx^{2n} + bx^n + a)(ex^n + d)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x^n)^2/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^(2*n) + b*x^n + a)*(e*x^n + d)^2), x)
```



$$3.74 \quad \int \frac{1}{(d+ex^n)^3(a+bx^n+cx^{2n})} dx$$

**Optimal.** Leaf size=552

$$\frac{e^2x(-ce(ae+3bd)+b^2e^2+3c^2d^2) {}_2F_1\left(1, \frac{1}{n}; 1+\frac{1}{n}; -\frac{ex^n}{d}\right)}{d(ae^2-bde+cd^2)^3} - \frac{cx(-3c^2de(d\sqrt{b^2-4ac}+2ae+bd)+ce^2(3b(d\sqrt{b^2-4ac}(-b\sqrt{b^2-4ac}))))}{d(ae^2-bde+cd^2)^3}$$

[Out] -((c\*(2\*c^3\*d^3 - b^2\*(b + Sqrt[b^2 - 4\*a\*c])\*e^3 - 3\*c^2\*d\*e\*(b\*d + Sqrt[b^2 - 4\*a\*c]\*d + 2\*a\*e) + c\*e^2\*(3\*b^2\*d + a\*Sqrt[b^2 - 4\*a\*c]\*e + 3\*b\*(Sqrt[b^2 - 4\*a\*c]\*d + a\*e))) \* x \* Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2\*c\*x^n)/(b - Sqrt[b^2 - 4\*a\*c])]) / ((b^2 - 4\*a\*c - b\*Sqrt[b^2 - 4\*a\*c]) \* (c\*d^2 - b\*d\*e + a\*e^2)^3) - (c\*(2\*c^3\*d^3 - b^2\*(b - Sqrt[b^2 - 4\*a\*c])\*e^3 - 3\*c^2\*d\*e\*(b\*d - Sqrt[b^2 - 4\*a\*c]\*d + 2\*a\*e) + c\*e^2\*(3\*b^2\*d - 3\*b\*Sqrt[b^2 - 4\*a\*c]\*d + 3\*a\*b\*e - a\*Sqrt[b^2 - 4\*a\*c]\*e)) \* x \* Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2\*c\*x^n)/(b + Sqrt[b^2 - 4\*a\*c])]) / ((b^2 - 4\*a\*c + b\*Sqrt[b^2 - 4\*a\*c]) \* (c\*d^2 - b\*d\*e + a\*e^2)^3) + (e^2\*(3\*c^2\*d^2 + b^2\*e^2 - c\*e\*(3\*b\*d + a\*e)) \* x \* Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -(e\*x^n)/d]) / (d \* (c\*d^2 - b\*d\*e + a\*e^2)^3) + (e^2\*(2\*c\*d - b\*e) \* x \* Hypergeometric2F1[2, n^(-1), 1 + n^(-1), -(e\*x^n)/d]) / (d^2 \* (c\*d^2 - b\*d\*e + a\*e^2)^2) + (e^2 \* x \* Hypergeometric2F1[3, n^(-1), 1 + n^(-1), -(e\*x^n)/d]) / (d^3 \* (c\*d^2 - b\*d\*e + a\*e^2))

**Rubi [A]** time = 1.02335, antiderivative size = 552, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 3, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$ , Rules used = {1424, 245, 1422}

$$\frac{e^2x(-ce(ae+3bd)+b^2e^2+3c^2d^2) {}_2F_1\left(1, \frac{1}{n}; 1+\frac{1}{n}; -\frac{ex^n}{d}\right)}{d(ae^2-bde+cd^2)^3} - \frac{cx(-3c^2de(d\sqrt{b^2-4ac}+2ae+bd)+ce^2(3b(d\sqrt{b^2-4ac}(-b\sqrt{b^2-4ac}))))}{d(ae^2-bde+cd^2)^3}$$

Antiderivative was successfully verified.

[In] Int[1/((d + e\*x^n)^3\*(a + b\*x^n + c\*x^(2\*n))), x]

[Out] -((c\*(2\*c^3\*d^3 - b^2\*(b + Sqrt[b^2 - 4\*a\*c])\*e^3 - 3\*c^2\*d\*e\*(b\*d + Sqrt[b^2 - 4\*a\*c]\*d + 2\*a\*e) + c\*e^2\*(3\*b^2\*d + a\*Sqrt[b^2 - 4\*a\*c]\*e + 3\*b\*(Sqrt[b^2 - 4\*a\*c]\*d + a\*e))) \* x \* Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2\*c\*x^n)/(b - Sqrt[b^2 - 4\*a\*c])]) / ((b^2 - 4\*a\*c - b\*Sqrt[b^2 - 4\*a\*c]) \* (c\*d^2 - b\*d\*e + a\*e^2)^3) - (c\*(2\*c^3\*d^3 - b^2\*(b - Sqrt[b^2 - 4\*a\*c])\*e^3 - 3\*c^2\*d\*e\*(b\*d - Sqrt[b^2 - 4\*a\*c]\*d + 2\*a\*e) + c\*e^2\*(3\*b^2\*d - 3\*b\*Sqrt[b^2 - 4\*a\*c]\*d + 3\*a\*b\*e - a\*Sqrt[b^2 - 4\*a\*c]\*e)) \* x \* Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2\*c\*x^n)/(b + Sqrt[b^2 - 4\*a\*c])]) / ((b^2 - 4\*a\*c + b\*Sqrt[b^2 - 4\*a\*c]) \* (c\*d^2 - b\*d\*e + a\*e^2)^3) + (e^2\*(3\*c^2\*d^2 + b^2\*e^2 - c\*e\*(3\*b\*d + a\*e)) \* x \* Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -(e\*x^n)/d]) / (d \* (c\*d^2 - b\*d\*e + a\*e^2)^3) + (e^2\*(2\*c\*d - b\*e) \* x \* Hypergeometric2F1[2, n^(-1), 1 + n^(-1), -(e\*x^n)/d]) / (d^2 \* (c\*d^2 - b\*d\*e + a\*e^2)^2) + (e^2 \* x \* Hypergeometric2F1[3, n^(-1), 1 + n^(-1), -(e\*x^n)/d]) / (d^3 \* (c\*d^2 - b\*d\*e + a\*e^2))

$$\begin{aligned} &^2*d*e*(b*d - \text{Sqrt}[b^2 - 4*a*c]*d + 2*a*e) + c*e^2*(3*b^2*d - 3*b*\text{Sqrt}[b^2 \\ &- 4*a*c]*d + 3*a*b*e - a*\text{Sqrt}[b^2 - 4*a*c]*e))*x*\text{Hypergeometric2F1}[1, n^{(-1)} \\ &), 1 + n^{(-1)}, (-2*c*x^n)/(b + \text{Sqrt}[b^2 - 4*a*c])]/((b^2 - 4*a*c + b*\text{Sqrt}[ \\ &b^2 - 4*a*c])*(c*d^2 - b*d*e + a*e^2)^3) + (e^2*(3*c^2*d^2 + b^2*e^2 - c*e* \\ &(3*b*d + a*e))*x*\text{Hypergeometric2F1}[1, n^{(-1)}, 1 + n^{(-1)}, -((e*x^n)/d)]/(d \\ &*(c*d^2 - b*d*e + a*e^2)^3) + (e^2*(2*c*d - b*e))*x*\text{Hypergeometric2F1}[2, n^{( \\ &-1)}, 1 + n^{(-1)}, -((e*x^n)/d)]/(d^2*(c*d^2 - b*d*e + a*e^2)^2) + (e^2*x*Hy \\ &pergeometric2F1[3, n^{(-1)}, 1 + n^{(-1)}, -((e*x^n)/d)]/(d^3*(c*d^2 - b*d*e + \\ &a*e^2)) \end{aligned}$$

### Rule 1424

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x_Symbol] := Int[ExpandIntegrand[(d + e*x^n)^q/(a + b*x^n + c*x^(2*n)), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[q]
```

### Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[a^p*x*Hypergeometric2F1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] || GtQ[a, 0])
```

### Rule 1422

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a*c] || !IGtQ[n/2, 0])
```

### Rubi steps

$$\begin{aligned}
\int \frac{1}{(d + ex^n)^3 (a + bx^n + cx^{2n})} dx &= \int \left( \frac{e^2}{(cd^2 - bde + ae^2)(d + ex^n)^3} - \frac{e^2(-2cd + be)}{(cd^2 - bde + ae^2)^2 (d + ex^n)^2} + \frac{e^2(3c^2d^2 + b^2e^2)}{(cd^2 - bde + ae^2)^3} \right) dx \\
&= \frac{\int \frac{c^3d^3 - 3bc^2d^2e + 3b^2cde^2 - 3ac^2de^2 - b^3e^3 + 2abce^3 - (3c^3d^2e - 3bc^2de^2 + b^2ce^3 - ac^2e^3)x^n}{a + bx^n + cx^{2n}} dx}{(cd^2 - bde + ae^2)^3} + \frac{e^2(2cd - be)}{(cd^2 - bde + ae^2)^2} \\
&= \frac{e^2(3c^2d^2 + b^2e^2 - ce(3bd + ae)) x {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{ex^n}{d}\right)}{d(cd^2 - bde + ae^2)^3} + \frac{e^2(2cd - be) x {}_2F_1\left(2, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{ex^n}{d}\right)}{d^2(cd^2 - bde + ae^2)^2} \\
&= \frac{c\left(2c^3d^3 - b^2\left(b + \sqrt{b^2 - 4ac}\right)e^3 - 3c^2de\left(bd + \sqrt{b^2 - 4ac}d + 2ae\right) + ce^2\left(3b^2d + a\sqrt{b^2 - 4ac}\right)\right)}{\sqrt{b^2 - 4ac}\left(b - \sqrt{b^2 - 4ac}\right)\left(cd^2 - bde + ae^2\right)^3}
\end{aligned}$$

**Mathematica [A]** time = 1.87317, size = 509, normalized size = 0.92

$$x \left( \frac{e^2(-ce(ae+3bd)+b^2e^2+3c^2d^2) {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{ex^n}{d}\right)}{d} + \frac{c\left(3c^2de\left(d\sqrt{b^2-4ac}+2ae+bd\right) - ce^2\left(3b\left(d\sqrt{b^2-4ac}+ae\right) + ae\sqrt{b^2-4ac}+3b^2d\right) + b^2e^3\left(\sqrt{b^2-4ac}+b\right) - 2c^3d^3\right)}{-b\sqrt{b^2-4ac}-4ac+b^2} \right)$$

Antiderivative was successfully verified.

[In] Integrate[1/((d + e\*x^n)^3\*(a + b\*x^n + c\*x^(2\*n))),x]

[Out] (x\*((c\*(-2\*c^3\*d^3 + b^2\*(b + Sqrt[b^2 - 4\*a\*c])\*e^3 + 3\*c^2\*d\*e\*(b\*d + Sqrt[b^2 - 4\*a\*c]\*d + 2\*a\*e) - c\*e^2\*(3\*b^2\*d + a\*Sqrt[b^2 - 4\*a\*c]\*e + 3\*b\*(Sqrt[b^2 - 4\*a\*c]\*d + a\*e)))\*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (2\*c\*x^n)/(-b + Sqrt[b^2 - 4\*a\*c])])/(b^2 - 4\*a\*c - b\*Sqrt[b^2 - 4\*a\*c]) - (c\*(2\*c^3\*d^3 + b^2\*(-b + Sqrt[b^2 - 4\*a\*c])\*e^3 + 3\*c^2\*d\*e\*(-(b\*d) + Sqrt[b^2 - 4\*a\*c]\*d - 2\*a\*e) + c\*e^2\*(3\*b^2\*d - 3\*b\*Sqrt[b^2 - 4\*a\*c]\*d + 3\*a\*b\*e - a\*Sqrt[b^2 - 4\*a\*c]\*e))\*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2\*c\*x^n)/(b + Sqrt[b^2 - 4\*a\*c])])/(b^2 - 4\*a\*c + b\*Sqrt[b^2 - 4\*a\*c]) + (e^2\*(3\*c^2\*d^2 + b^2\*e^2 - c\*e\*(3\*b\*d + a\*e))\*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((e\*x^n)/d)]/d + (e^2\*(2\*c\*d - b\*e)\*(c\*d^2 + e\*(-(b\*d) + a\*e))\*Hypergeometric2F1[2, n^(-1), 1 + n^(-1), -((e\*x^n)/d)]/d^2 + (e^2\*(c\*d^2 + e\*(-(b\*d) + a\*e))^2\*Hypergeometric2F1[3, n^(-1), 1 + n^(-1), -((e\*x^n)/d)]/d^3))/(c\*d^2 + e\*(-(b\*d) + a\*e))^3

**Maple [F]** time = 0.163, size = 0, normalized size = 0.

$$\int \frac{1}{(d + ex^n)^3 (a + bx^n + cx^{2n})} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(d+e\*x^n)^3/(a+b\*x^n+c\*x^(2\*n)),x)

[Out] int(1/(d+e\*x^n)^3/(a+b\*x^n+c\*x^(2\*n)),x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d+e\*x^n)^3/(a+b\*x^n+c\*x^(2\*n)),x, algorithm="maxima")

[Out] ((12\*n^2 - 7\*n + 1)\*c^2\*d^4\*e^2 - 2\*(8\*n^2 - 6\*n + 1)\*b\*c\*d^3\*e^3 + (6\*n^2 - 5\*n + 1)\*b^2\*d^2\*e^4 + (2\*n^2 - 3\*n + 1)\*a^2\*e^6 + 2\*((3\*n^2 - 5\*n + 1)\*c\*d^2\*e^4 - (3\*n^2 - 4\*n + 1)\*b\*d\*e^5)\*a)\*integrate(1/2/(c^3\*d^9\*n^2 - 3\*b\*c^2\*d^8\*e\*n^2 + 3\*b^2\*c\*d^7\*e^2\*n^2 - b^3\*d^6\*e^3\*n^2 + a^3\*d^3\*e^6\*n^2 + 3\*(c\*d^5\*e^4\*n^2 - b\*d^4\*e^5\*n^2)\*a^2 + 3\*(c^2\*d^7\*e^2\*n^2 - 2\*b\*c\*d^6\*e^3\*n^2 + b^2\*d^5\*e^4\*n^2)\*a + (c^3\*d^8\*e\*n^2 - 3\*b\*c^2\*d^7\*e^2\*n^2 + 3\*b^2\*c\*d^6\*e^3\*n^2 - b^3\*d^5\*e^4\*n^2 + a^3\*d^2\*e^7\*n^2 + 3\*(c\*d^4\*e^5\*n^2 - b\*d^3\*e^6\*n^2)\*a^2 + 3\*(c^2\*d^6\*e^3\*n^2 - 2\*b\*c\*d^5\*e^4\*n^2 + b^2\*d^4\*e^5\*n^2)\*a)\*x^n), x) + 1/2\*((c\*d^2\*e^3\*(6\*n - 1) - b\*d\*e^4\*(4\*n - 1) + a\*e^5\*(2\*n - 1))\*x\*x^n + (c\*d^3\*e^2\*(7\*n - 1) - b\*d^2\*e^3\*(5\*n - 1) + a\*d\*e^4\*(3\*n - 1))\*x)/(c^2\*d^8\*n^2 - 2\*b\*c\*d^7\*e\*n^2 + b^2\*d^6\*e^2\*n^2 + a^2\*d^4\*e^4\*n^2 + 2\*(c\*d^6\*e^2\*n^2 - b\*d^5\*e^3\*n^2)\*a + (c^2\*d^6\*e^2\*n^2 - 2\*b\*c\*d^5\*e^3\*n^2 + b^2\*d^4\*e^4\*n^2 + a^2\*d^2\*e^6\*n^2 + 2\*(c\*d^4\*e^4\*n^2 - b\*d^3\*e^5\*n^2)\*a)\*x^(2\*n) + 2\*(c^2\*d^7\*e\*n^2 - 2\*b\*c\*d^6\*e^2\*n^2 + b^2\*d^5\*e^3\*n^2 + a^2\*d^3\*e^5\*n^2 + 2\*(c\*d^5\*e^3\*n^2 - b\*d^4\*e^4\*n^2)\*a)\*x^n) + integrate((c^3\*d^3 - 3\*b\*c^2\*d^2\*e + 3\*b^2\*c\*d\*e^2 - b^3\*e^3 - (3\*c^2\*d\*e^2 - 2\*b\*c\*e^3)\*a - (3\*c^3\*d^2\*e - 3\*b\*c^2\*d\*e^2 + b^2\*c\*e^3 - a\*c^2\*e^3)\*x^n)/(a^4\*e^6 + 3\*(c\*d^2\*e^4 - b\*d\*e^5)\*a^3 + 3\*(c^2\*d^4\*e^2 - 2\*b\*c\*d^3\*e^3 + b^2\*d^2\*e^4)\*a^2 + (c^3\*d^6 - 3\*b\*c^2\*d^5\*e + 3\*b^2\*c\*d^4\*e^2 - b^3\*d^3\*e^3)\*a + (c^4\*d^6 - 3\*b\*c^3\*d^5\*e + 3\*b^2\*c^2\*d^4\*e^2 - b^3\*c\*d^3\*e^3 + a^3\*c\*e^6 + 3\*(c^2\*d^2\*e^4 - b\*c\*d\*e^5)\*a^2 + 3\*(c^3\*d^4\*e^2 - 2\*b\*c^2\*d^3\*e^3 + b^2\*c\*d^2\*e^4)\*a)\*x^(2\*n) + (b\*c^3\*d^6 - 3\*b^2\*c^2\*d^5\*e + 3\*b^3\*c\*d^4\*e^2 - b^4\*d^3\*e^3 + a^3\*b\*e^6 +

$3*(b*c*d^2*e^4 - b^2*d*e^5)*a^2 + 3*(b*c^2*d^4*e^2 - 2*b^2*c*d^3*e^3 + b^3*d^2*e^4)*a*x^n), x)$

**Fricas [F]** time = 0., size = 0, normalized size = 0.

integral  $\left( \frac{1}{be^3x^{4n} + ad^3 + (3bde^2 + ae^3)x^{3n} + (ce^3x^{3n} + 3cde^2x^{2n} + 3cd^2ex^n + cd^3)x^{2n} + 3(bd^2e + ade^2)x^{2n} + (bd^3 +$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d+e\*x^n)^3/(a+b\*x^n+c\*x^(2\*n)),x, algorithm="fricas")

[Out] integral(1/(b\*e^3\*x^(4\*n) + a\*d^3 + (3\*b\*d\*e^2 + a\*e^3)\*x^(3\*n) + (c\*e^3\*x^(3\*n) + 3\*c\*d\*e^2\*x^(2\*n) + 3\*c\*d^2\*e\*x^n + c\*d^3)\*x^(2\*n) + 3\*(b\*d^2\*e + a\*d\*e^2)\*x^(2\*n) + (b\*d^3 + 3\*a\*d^2\*e)\*x^n), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d+e\*x\*\*n)\*\*3/(a+b\*x\*\*n+c\*x\*\*(2\*n)),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(cx^{2n} + bx^n + a)(ex^n + d)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d+e\*x^n)^3/(a+b\*x^n+c\*x^(2\*n)),x, algorithm="giac")

[Out] integrate(1/((c\*x^(2\*n) + b\*x^n + a)\*(e\*x^n + d)^3), x)

$$3.75 \quad \int \frac{(d+ex^n)^3}{(a+bx^n+cx^{2n})^2} dx$$

**Optimal.** Leaf size=750

$$x \left( \frac{b^2cd(3ae^2(1-3n)-cd^2(1-n))-ab^3e^3(1-3n)+2abce(ae^2(2-5n)+3cd^2n)+4ac^2d(1-2n)(cd^2-3ae^2)}{\sqrt{b^2-4ac}} + (1-n)(ab^2e^3 - bcd(3ae^2 + cd^2)) + 2ace(3cd^2 - ab^2e^3) \right) / acn(b^2 - 4ac) \left( b - \sqrt{b^2 - 4ac} \right)$$

[Out] (x\*(b^2\*c\*d^3 - 2\*a\*c\*d\*(c\*d^2 - 3\*a\*e^2) - a\*b\*e\*(3\*c\*d^2 + a\*e^2) - (a\*b^2\*e^3 + 2\*a\*c\*e\*(3\*c\*d^2 - a\*e^2) - b\*c\*d\*(c\*d^2 + 3\*a\*e^2))\*x^n))/(a\*c\*(b^2 - 4\*a\*c)\*n\*(a + b\*x^n + c\*x^(2\*n))) + (e^2\*(e + (6\*c\*d - 3\*b\*e)/Sqrt[b^2 - 4\*a\*c]))\*x\*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2\*c\*x^n)/(b - Sqrt[b^2 - 4\*a\*c])]/(c\*(b - Sqrt[b^2 - 4\*a\*c])) + (((a\*b^2\*e^3 + 2\*a\*c\*e\*(3\*c\*d^2 - a\*e^2) - b\*c\*d\*(c\*d^2 + 3\*a\*e^2))\*(1 - n) + (b^2\*c\*d\*(3\*a\*e^2\*(1 - 3\*n) - c\*d^2\*(1 - n)) - a\*b^3\*e^3\*(1 - 3\*n) + 4\*a\*c^2\*d\*(c\*d^2 - 3\*a\*e^2)\*(1 - 2\*n) + 2\*a\*b\*c\*e\*(a\*e^2\*(2 - 5\*n) + 3\*c\*d^2\*n))/Sqrt[b^2 - 4\*a\*c])\*x\*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2\*c\*x^n)/(b - Sqrt[b^2 - 4\*a\*c])]/(a\*c\*(b^2 - 4\*a\*c)\*(b - Sqrt[b^2 - 4\*a\*c])\*n) + (e^2\*(e - (3\*(2\*c\*d - b\*e))/Sqrt[b^2 - 4\*a\*c]))\*x\*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2\*c\*x^n)/(b + Sqrt[b^2 - 4\*a\*c])]/(c\*(b + Sqrt[b^2 - 4\*a\*c])) + (((a\*b^2\*e^3 + 2\*a\*c\*e\*(3\*c\*d^2 - a\*e^2) - b\*c\*d\*(c\*d^2 + 3\*a\*e^2))\*(1 - n) - (b^2\*c\*d\*(3\*a\*e^2\*(1 - 3\*n) - c\*d^2\*(1 - n)) - a\*b^3\*e^3\*(1 - 3\*n) + 4\*a\*c^2\*d\*(c\*d^2 - 3\*a\*e^2)\*(1 - 2\*n) + 2\*a\*b\*c\*e\*(a\*e^2\*(2 - 5\*n) + 3\*c\*d^2\*n))/Sqrt[b^2 - 4\*a\*c])\*x\*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2\*c\*x^n)/(b + Sqrt[b^2 - 4\*a\*c])]/(a\*c\*(b^2 - 4\*a\*c)\*(b + Sqrt[b^2 - 4\*a\*c])\*n)

**Rubi [A]** time = 2.93827, antiderivative size = 750, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 4, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$ , Rules used = {1436, 1430, 1422, 245}

$$x \left( \frac{b^2cd(3ae^2(1-3n)-cd^2(1-n))-ab^3e^3(1-3n)+2abce(ae^2(2-5n)+3cd^2n)+4ac^2d(1-2n)(cd^2-3ae^2)}{\sqrt{b^2-4ac}} + (1-n)(ab^2e^3 - bcd(3ae^2 + cd^2)) + 2ace(3cd^2 - ab^2e^3) \right) / acn(b^2 - 4ac) \left( b - \sqrt{b^2 - 4ac} \right)$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x^n)^3/(a + b\*x^n + c\*x^(2\*n))^2,x]

[Out] (x\*(b^2\*c\*d^3 - 2\*a\*c\*d\*(c\*d^2 - 3\*a\*e^2) - a\*b\*e\*(3\*c\*d^2 + a\*e^2) - (a\*b^2\*e^3 + 2\*a\*c\*e\*(3\*c\*d^2 - a\*e^2) - b\*c\*d\*(c\*d^2 + 3\*a\*e^2))\*x^n))/(a\*c\*(b^2 - 4\*a\*c)\*n\*(a + b\*x^n + c\*x^(2\*n))) + (e^2\*(e + (6\*c\*d - 3\*b\*e)/Sqrt[b^2 - 4\*a\*c]))\*x\*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2\*c\*x^n)/(b - Sqrt[b^2 - 4\*a\*c])]/(c\*(b - Sqrt[b^2 - 4\*a\*c])) + (((a\*b^2\*e^3 + 2\*a\*c\*e\*(3\*c\*d^2 - a\*e^2) - b\*c\*d\*(c\*d^2 + 3\*a\*e^2))\*(1 - n) + (b^2\*c\*d\*(3\*a\*e^2\*(1 - 3\*n) - c\*d^2\*(1 - n)) - a\*b^3\*e^3\*(1 - 3\*n) + 4\*a\*c^2\*d\*(c\*d^2 - 3\*a\*e^2)\*(1 - 2\*n) + 2\*a\*b\*c\*e\*(a\*e^2\*(2 - 5\*n) + 3\*c\*d^2\*n))/Sqrt[b^2 - 4\*a\*c])\*x\*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2\*c\*x^n)/(b - Sqrt[b^2 - 4\*a\*c])]/(a\*c\*(b^2 - 4\*a\*c)\*(b - Sqrt[b^2 - 4\*a\*c])\*n) + (e^2\*(e - (3\*(2\*c\*d - b\*e))/Sqrt[b^2 - 4\*a\*c]))\*x\*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2\*c\*x^n)/(b + Sqrt[b^2 - 4\*a\*c])]/(c\*(b + Sqrt[b^2 - 4\*a\*c])) + (((a\*b^2\*e^3 + 2\*a\*c\*e\*(3\*c\*d^2 - a\*e^2) - b\*c\*d\*(c\*d^2 + 3\*a\*e^2))\*(1 - n) - (b^2\*c\*d\*(3\*a\*e^2\*(1 - 3\*n) - c\*d^2\*(1 - n)) - a\*b^3\*e^3\*(1 - 3\*n) + 4\*a\*c^2\*d\*(c\*d^2 - 3\*a\*e^2)\*(1 - 2\*n) + 2\*a\*b\*c\*e\*(a\*e^2\*(2 - 5\*n) + 3\*c\*d^2\*n))/Sqrt[b^2 - 4\*a\*c])\*x\*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2\*c\*x^n)/(b + Sqrt[b^2 - 4\*a\*c])]/(a\*c\*(b^2 - 4\*a\*c)\*(b + Sqrt[b^2 - 4\*a\*c])\*n)

```

2 - 4*a*c)*n*(a + b*x^n + c*x^(2*n)) + (e^2*(e + (6*c*d - 3*b*e)/Sqrt[b^2
- 4*a*c])*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2*c*x^n)/(b - Sqrt[b
^2 - 4*a*c]))/(c*(b - Sqrt[b^2 - 4*a*c])) + (((a*b^2*e^3 + 2*a*c*e*(3*c*d^
2 - a*e^2) - b*c*d*(c*d^2 + 3*a*e^2))*(1 - n) + (b^2*c*d*(3*a*e^2*(1 - 3*n)
- c*d^2*(1 - n)) - a*b^3*e^3*(1 - 3*n) + 4*a*c^2*d*(c*d^2 - 3*a*e^2)*(1 -
2*n) + 2*a*b*c*e*(a*e^2*(2 - 5*n) + 3*c*d^2*n))/Sqrt[b^2 - 4*a*c])*x*Hyperg
eometric2F1[1, n^(-1), 1 + n^(-1), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))]/(a*
c*(b^2 - 4*a*c)*(b - Sqrt[b^2 - 4*a*c])*n) + (e^2*(e - (3*(2*c*d - b*e))/Sq
rt[b^2 - 4*a*c])*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2*c*x^n)/(b +
Sqrt[b^2 - 4*a*c]))]/(c*(b + Sqrt[b^2 - 4*a*c])) + (((a*b^2*e^3 + 2*a*c*e*
(3*c*d^2 - a*e^2) - b*c*d*(c*d^2 + 3*a*e^2))*(1 - n) - (b^2*c*d*(3*a*e^2*(1
- 3*n) - c*d^2*(1 - n)) - a*b^3*e^3*(1 - 3*n) + 4*a*c^2*d*(c*d^2 - 3*a*e^2
)*(1 - 2*n) + 2*a*b*c*e*(a*e^2*(2 - 5*n) + 3*c*d^2*n))/Sqrt[b^2 - 4*a*c])*x
*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]
)])/((a*c*(b^2 - 4*a*c)*(b + Sqrt[b^2 - 4*a*c])*n)

```

### Rule 1436

```

Int[((d_) + (e_)*(x_)^(n_))^(q_)*((a_) + (b_)*(x_)^(n_) + (c_)*(x_)^(n2_
))^p_, x_Symbol] :> Int[ExpandIntegrand[(d + e*x^n)^q*(a + b*x^n + c*x^(2
*n))^p, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && ((IntegersQ[p, q] &&
!IntegerQ[n]) || IGtQ[p, 0] || (IGtQ[q, 0] && !IntegerQ[n]))

```

### Rule 1430

```

Int[((d_) + (e_)*(x_)^(n_))*((a_) + (b_)*(x_)^(n_) + (c_)*(x_)^(n2_))^p
_, x_Symbol] :> -Simp[(x*(d*b^2 - a*b*e - 2*a*c*d + (b*d - 2*a*e)*c*x^n)*(
a + b*x^n + c*x^(2*n))^p + 1)/(a*n*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(a
*n*(p + 1)*(b^2 - 4*a*c)), Int[Simp[(n*p + n + 1)*d*b^2 - a*b*e - 2*a*c*d*(
2*n*p + 2*n + 1) + (2*n*p + 3*n + 1)*(d*b - 2*a*e)*c*x^n, x]*(a + b*x^n + c
*x^(2*n))^p + 1, x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] &
& NeQ[b^2 - 4*a*c, 0] && ILtQ[p, -1]

```

### Rule 1422

```

Int[((d_) + (e_)*(x_)^(n_))/((a_) + (b_)*(x_)^(n_) + (c_)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] || !IGtQ[n/2, 0])

```

### Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

### Rubi steps

$$\begin{aligned}
\int \frac{(d + ex^n)^3}{(a + bx^n + cx^{2n})^2} dx &= \int \left( \frac{c^2 d^3 - 3acde^2 + abe^3 + (3c^2 d^2 e - 3bcde^2 + b^2 e^3 - ace^3) x^n}{c^2 (a + bx^n + cx^{2n})^2} + \frac{e^2 (3cd - be + cex^n)}{c^2 (a + bx^n + cx^{2n})} \right) dx \\
&= \frac{\int \frac{c^2 d^3 - 3acde^2 + abe^3 + (3c^2 d^2 e - 3bcde^2 + b^2 e^3 - ace^3) x^n}{(a + bx^n + cx^{2n})^2} dx}{c^2} + \frac{e^2 \int \frac{3cd - be + cex^n}{a + bx^n + cx^{2n}} dx}{c^2} \\
&= \frac{x (b^2 cd^3 - 2acd (cd^2 - 3ae^2) - abe (3cd^2 + ae^2) - (ab^2 e^3 + 2ace (3cd^2 - ae^2) - bcd (cd^2 + 3ae^2))}{ac (b^2 - 4ac) n (a + bx^n + cx^{2n})} \\
&= \frac{x (b^2 cd^3 - 2acd (cd^2 - 3ae^2) - abe (3cd^2 + ae^2) - (ab^2 e^3 + 2ace (3cd^2 - ae^2) - bcd (cd^2 + 3ae^2))}{ac (b^2 - 4ac) n (a + bx^n + cx^{2n})} \\
&= \frac{x (b^2 cd^3 - 2acd (cd^2 - 3ae^2) - abe (3cd^2 + ae^2) - (ab^2 e^3 + 2ace (3cd^2 - ae^2) - bcd (cd^2 + 3ae^2))}{ac (b^2 - 4ac) n (a + bx^n + cx^{2n})}
\end{aligned}$$

**Mathematica [B]** time = 4.67146, size = 2462, normalized size = 3.28

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^n)^3/(a + b*x^n + c*x^(2*n))^2, x]
```

```
[Out] -((x*((b^2 - 4*a*c)*(-b + Sqrt[b^2 - 4*a*c]))*(b + Sqrt[b^2 - 4*a*c]))*(1 + n)
)*(b^2*(-(c*d^3) + a*e^3*x^n) + b*(a^2*e^3 - c^2*d^3*x^n + 3*a*c*d*e*(d - e
*x^n)) - 2*a*c*(a*e^2*(3*d + e*x^n) - c*d^2*(d + 3*e*x^n))) + 2*b*c^3*Sqrt[
b^2 - 4*a*c]*d^3*x^n*(a + x^n*(b + c*x^n))*(-(b + Sqrt[b^2 - 4*a*c])*Hyper
geometric2F1[1, 1 + n^(-1), 2 + n^(-1), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])]
) + (b - Sqrt[b^2 - 4*a*c])*Hypergeometric2F1[1, 1 + n^(-1), 2 + n^(-1), (-
2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])] - 12*a*c^3*Sqrt[b^2 - 4*a*c]*d^2*e*x^n*(
```





ypergeometric2F1[1, n^(-1), 1 + n^(-1), (2\*c\*x^n)/(-b + Sqrt[b^2 - 4\*a\*c])] + (b - Sqrt[b^2 - 4\*a\*c])\*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2\*c\*x^n)/(b + Sqrt[b^2 - 4\*a\*c])] - 2\*b^2\*c^2\*Sqrt[b^2 - 4\*a\*c]\*d^3\*n\*(1 + n)\*(a + x^n\*(b + c\*x^n))\*(-(b + Sqrt[b^2 - 4\*a\*c])\*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (2\*c\*x^n)/(-b + Sqrt[b^2 - 4\*a\*c])]) + (b - Sqrt[b^2 - 4\*a\*c])\*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2\*c\*x^n)/(b + Sqrt[b^2 - 4\*a\*c])] + 8\*a\*c^3\*Sqrt[b^2 - 4\*a\*c]\*d^3\*n\*(1 + n)\*(a + x^n\*(b + c\*x^n))\*(-(b + Sqrt[b^2 - 4\*a\*c])\*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (2\*c\*x^n)/(-b + Sqrt[b^2 - 4\*a\*c])]) + (b - Sqrt[b^2 - 4\*a\*c])\*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2\*c\*x^n)/(b + Sqrt[b^2 - 4\*a\*c])])]/(a\*c\*(b^2 - 4\*a\*c)^2\*(-b + Sqrt[b^2 - 4\*a\*c])\*(b + Sqrt[b^2 - 4\*a\*c])\*n\*(1 + n)\*(a + x^n\*(b + c\*x^n))))

**Maple [F]** time = 0.063, size = 0, normalized size = 0.

$$\int \frac{(d + ex^n)^3}{(a + bx^n + cx^{2n})^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d+e\*x^n)^3/(a+b\*x^n+c\*x^(2\*n))^2,x)

[Out] int((d+e\*x^n)^3/(a+b\*x^n+c\*x^(2\*n))^2,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{(bc^2d^3 + 2a^2ce^3 - (6c^2d^2e - 3bcde^2 + b^2e^3)a)xx^n + (b^2cd^3 + (6cde^2 - be^3)a^2 - (2c^2d^3 + 3bcd^2e)a)x}{a^2b^2cn - 4a^3c^2n + (ab^2c^2n - 4a^2c^3n)x^{2n} + (ab^3cn - 4a^2bc^2n)x^n} + \int \frac{b^2cd^3(n-1)}{a^2b^2cn - 4a^3c^2n + (ab^2c^2n - 4a^2c^3n)x^{2n} + (ab^3cn - 4a^2bc^2n)x^n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e\*x^n)^3/(a+b\*x^n+c\*x^(2\*n))^2,x, algorithm="maxima")

[Out] ((b\*c^2\*d^3 + 2\*a^2\*c\*e^3 - (6\*c^2\*d^2\*e - 3\*b\*c\*d\*e^2 + b^2\*e^3)\*a)\*x\*x^n + (b^2\*c\*d^3 + (6\*c\*d\*e^2 - b\*e^3)\*a^2 - (2\*c^2\*d^3 + 3\*b\*c\*d^2\*e)\*a)\*x)/(a^2\*b^2\*c\*n - 4\*a^3\*c^2\*n + (a\*b^2\*c^2\*n - 4\*a^2\*c^3\*n)\*x^(2\*n) + (a\*b^3\*c\*n - 4\*a^2\*b\*c^2\*n)\*x^n) + integrate((b^2\*c\*d^3\*(n - 1) - (6\*c\*d\*e^2 - b\*e^3)\*a^2 - (2\*c^2\*d^3\*(2\*n - 1) - 3\*b\*c\*d^2\*e)\*a - (2\*a^2\*c\*e^3\*(n + 1) - b\*c^2

$$\frac{d^3(n-1) + (6c^2d^2e^{n-1} - 3b^2cd^2e^{n-1} - b^2e^3)a)x^n}{(a^2b^2c^n - 4a^3c^2n + (ab^2c^2n - 4a^2c^3n)x^{2n} + (ab^3c^n - 4a^2b^2c^2n)x^n), x}$$

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{e^3x^{3n} + 3de^2x^{2n} + 3d^2ex^n + d^3}{c^2x^{4n} + b^2x^{2n} + 2abx^n + a^2 + 2(bc x^n + ac)x^{2n}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e\*x^n)^3/(a+b\*x^n+c\*x^(2\*n))^2,x, algorithm="fricas")

[Out] integral((e^3\*x^(3\*n) + 3\*d\*e^2\*x^(2\*n) + 3\*d^2\*e\*x^n + d^3)/(c^2\*x^(4\*n) + b^2\*x^(2\*n) + 2\*a\*b\*x^n + a^2 + 2\*(b\*c\*x^n + a\*c)\*x^(2\*n)), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e\*x\*\*n)\*\*3/(a+b\*x\*\*n+c\*x\*\*(2\*n))\*\*2,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^n + d)^3}{(cx^{2n} + bx^n + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e\*x^n)^3/(a+b\*x^n+c\*x^(2\*n))^2,x, algorithm="giac")

[Out] integrate((e\*x^n + d)^3/(c\*x^(2\*n) + b\*x^n + a)^2, x)

$$3.76 \quad \int \frac{(d+ex^n)^2}{(a+bx^n+cx^{2n})^2} dx$$

**Optimal.** Leaf size=543

$$\frac{x \left( (1-n)(abe^2 - 4acde + bcd^2) - \frac{b^2(ae^2(1-3n) - cd^2(1-n)) + 4abcden + 4ac(1-2n)(cd^2 - ae^2)}{\sqrt{b^2 - 4ac}} \right) {}_2F_1 \left( 1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{2cx^n}{b - \sqrt{b^2 - 4ac}} \right) x \left( \frac{b^2(ae^2(1-3n) - cd^2(1-n)) + 4abcden + 4ac(1-2n)(cd^2 - ae^2)}{\sqrt{b^2 - 4ac}} \right)}{an(b^2 - 4ac)(b - \sqrt{b^2 - 4ac})}$$

[Out] (x\*(b^2\*d^2 - 2\*a\*b\*d\*e - 2\*a\*(c\*d^2 - a\*e^2) + (b\*c\*d^2 - 4\*a\*c\*d\*e + a\*b\*e^2)\*x^n))/(a\*(b^2 - 4\*a\*c)\*n\*(a + b\*x^n + c\*x^(2\*n))) - (2\*e^2\*x\*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2\*c\*x^n)/(b - Sqrt[b^2 - 4\*a\*c])]/(b^2 - 4\*a\*c - b\*Sqrt[b^2 - 4\*a\*c]) - (((b\*c\*d^2 - 4\*a\*c\*d\*e + a\*b\*e^2)\*(1 - n) - (b^2\*(a\*e^2\*(1 - 3\*n) - c\*d^2\*(1 - n)) + 4\*a\*c\*(c\*d^2 - a\*e^2)\*(1 - 2\*n) + 4\*a\*b\*c\*d\*e\*n)/Sqrt[b^2 - 4\*a\*c])\*x\*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2\*c\*x^n)/(b - Sqrt[b^2 - 4\*a\*c])]/(a\*(b^2 - 4\*a\*c)\*(b - Sqrt[b^2 - 4\*a\*c]))\*n - (2\*e^2\*x\*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2\*c\*x^n)/(b + Sqrt[b^2 - 4\*a\*c])]/(b^2 - 4\*a\*c + b\*Sqrt[b^2 - 4\*a\*c]) - (((b\*c\*d^2 - 4\*a\*c\*d\*e + a\*b\*e^2)\*(1 - n) + (b^2\*(a\*e^2\*(1 - 3\*n) - c\*d^2\*(1 - n)) + 4\*a\*c\*(c\*d^2 - a\*e^2)\*(1 - 2\*n) + 4\*a\*b\*c\*d\*e\*n)/Sqrt[b^2 - 4\*a\*c])\*x\*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2\*c\*x^n)/(b + Sqrt[b^2 - 4\*a\*c])]/(a\*(b^2 - 4\*a\*c)\*(b + Sqrt[b^2 - 4\*a\*c]))\*n)

**Rubi [A]** time = 1.81832, antiderivative size = 543, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$ , Rules used = {1436, 1430, 1422, 245, 1347}

$$\frac{x \left( (1-n)(abe^2 - 4acde + bcd^2) - \frac{b^2(ae^2(1-3n) - cd^2(1-n)) + 4abcden + 4ac(1-2n)(cd^2 - ae^2)}{\sqrt{b^2 - 4ac}} \right) {}_2F_1 \left( 1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{2cx^n}{b - \sqrt{b^2 - 4ac}} \right) x \left( \frac{b^2(ae^2(1-3n) - cd^2(1-n)) + 4abcden + 4ac(1-2n)(cd^2 - ae^2)}{\sqrt{b^2 - 4ac}} \right)}{an(b^2 - 4ac)(b - \sqrt{b^2 - 4ac})}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x^n)^2/(a + b\*x^n + c\*x^(2\*n))^2,x]

[Out] (x\*(b^2\*d^2 - 2\*a\*b\*d\*e - 2\*a\*(c\*d^2 - a\*e^2) + (b\*c\*d^2 - 4\*a\*c\*d\*e + a\*b\*e^2)\*x^n))/(a\*(b^2 - 4\*a\*c)\*n\*(a + b\*x^n + c\*x^(2\*n))) - (2\*e^2\*x\*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2\*c\*x^n)/(b - Sqrt[b^2 - 4\*a\*c])]/(b^2 - 4\*a\*c - b\*Sqrt[b^2 - 4\*a\*c]) - (((b\*c\*d^2 - 4\*a\*c\*d\*e + a\*b\*e^2)\*(1 - n) - (b^2\*(a\*e^2\*(1 - 3\*n) - c\*d^2\*(1 - n)) + 4\*a\*c\*(c\*d^2 - a\*e^2)\*(1 - 2\*n) + 4\*a\*b\*c\*d\*e\*n)/Sqrt[b^2 - 4\*a\*c])\*x\*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2\*c\*x^n)/(b - Sqrt[b^2 - 4\*a\*c])]/(a\*(b^2 - 4\*a\*c)\*(b - Sqrt[b^2 - 4\*a\*c]))\*n - (2\*e^2\*x\*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2\*c\*x^n)/(b + Sqrt[b^2 - 4\*a\*c])]/(b^2 - 4\*a\*c + b\*Sqrt[b^2 - 4\*a\*c]) - (((b\*c\*d^2 - 4\*a\*c\*d\*e + a\*b\*e^2)\*(1 - n) + (b^2\*(a\*e^2\*(1 - 3\*n) - c\*d^2\*(1 - n)) + 4\*a\*c\*(c\*d^2 - a\*e^2)\*(1 - 2\*n) + 4\*a\*b\*c\*d\*e\*n)/Sqrt[b^2 - 4\*a\*c])\*x\*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2\*c\*x^n)/(b + Sqrt[b^2 - 4\*a\*c])]/(a\*(b^2 - 4\*a\*c)\*(b + Sqrt[b^2 - 4\*a\*c]))\*n)

$$\frac{4*a*b*c*d*e*n}{\text{Sqrt}[b^2 - 4*a*c]}*x*\text{Hypergeometric2F1}[1, n^{(-1)}, 1 + n^{(-1)}, (-2*c*x^n)/(b - \text{Sqrt}[b^2 - 4*a*c])]/(a*(b^2 - 4*a*c)*(b - \text{Sqrt}[b^2 - 4*a*c])*n) - \frac{2*e^2*x*\text{Hypergeometric2F1}[1, n^{(-1)}, 1 + n^{(-1)}, (-2*c*x^n)/(b + \text{Sqrt}[b^2 - 4*a*c])]}{(b^2 - 4*a*c + b*\text{Sqrt}[b^2 - 4*a*c])} - \frac{((b*c*d^2 - 4*a*c*d*e + a*b*e^2)*(1 - n) + (b^2*(a*e^2*(1 - 3*n) - c*d^2*(1 - n)) + 4*a*c*(c*d^2 - a*e^2)*(1 - 2*n) + 4*a*b*c*d*e*n)}{\text{Sqrt}[b^2 - 4*a*c]}*x*\text{Hypergeometric2F1}[1, n^{(-1)}, 1 + n^{(-1)}, (-2*c*x^n)/(b + \text{Sqrt}[b^2 - 4*a*c])]/(a*(b^2 - 4*a*c)*(b + \text{Sqrt}[b^2 - 4*a*c])*n)$$

### Rule 1436

```
Int[((d_) + (e_)*(x_)^(n_))^(q_)*((a_) + (b_)*(x_)^(n_) + (c_)*(x_)^(n2_))^(p_), x_Symbol] := Int[ExpandIntegrand[(d + e*x^n)^q*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && ((IntegersQ[p, q] && !IntegerQ[n]) || IGtQ[p, 0] || (IGtQ[q, 0] && !IntegerQ[n]))
```

### Rule 1430

```
Int[((d_) + (e_)*(x_)^(n_))*((a_) + (b_)*(x_)^(n_) + (c_)*(x_)^(n2_))^(p_), x_Symbol] := -Simp[(x*(d*b^2 - a*b*e - 2*a*c*d + (b*d - 2*a*e)*c*x^n)*(a + b*x^n + c*x^(2*n))^(p + 1))/(a*n*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(a*n*(p + 1)*(b^2 - 4*a*c)), Int[Simp[(n*p + n + 1)*d*b^2 - a*b*e - 2*a*c*d*(2*n*p + 2*n + 1) + (2*n*p + 3*n + 1)*(d*b - 2*a*e)*c*x^n, x]*(a + b*x^n + c*x^(2*n))^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && ILtQ[p, -1]
```

### Rule 1422

```
Int[((d_) + (e_)*(x_)^(n_))/((a_) + (b_)*(x_)^(n_) + (c_)*(x_)^(n2_)), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a*c] || !IGtQ[n/2, 0])
```

### Rule 245

```
Int[((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Simp[a^p*x*Hypergeometric2F1[-p, 1/n, 1/n + 1, -(b*x^n)/a], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] || GtQ[a, 0])
```

### Rule 1347

```
Int[((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(n_+1), x_Symbol] := With[{q
= Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^n), x], x] - Dist[
c/q, Int[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*
n] && NeQ[b^2 - 4*a*c, 0]
```

### Rubi steps

$$\begin{aligned}
\int \frac{(d + ex^n)^2}{(a + bx^n + cx^{2n})^2} dx &= \int \left( \frac{cd^2 - ae^2 + (2cde - be^2)x^n}{c(a + bx^n + cx^{2n})^2} + \frac{e^2}{c(a + bx^n + cx^{2n})} \right) dx \\
&= \frac{\int \frac{cd^2 - ae^2 + (2cde - be^2)x^n}{(a + bx^n + cx^{2n})^2} dx}{c} + \frac{e^2 \int \frac{1}{a + bx^n + cx^{2n}} dx}{c} \\
&= \frac{x(b^2d^2 - 2abde - 2a(cd^2 - ae^2) + (bcd^2 - 4acde + abe^2)x^n)}{a(b^2 - 4ac)n(a + bx^n + cx^{2n})} + \frac{e^2 \int \frac{1}{\frac{b}{2} - \frac{1}{2}\sqrt{b^2 - 4ac} + cx^n} dx}{\sqrt{b^2 - 4ac}} - \frac{e^2 \int}{\sqrt{b^2 - 4ac}} \\
&= \frac{x(b^2d^2 - 2abde - 2a(cd^2 - ae^2) + (bcd^2 - 4acde + abe^2)x^n)}{a(b^2 - 4ac)n(a + bx^n + cx^{2n})} - \frac{2e^2x {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{2cx^n}{b - \sqrt{b^2 - 4ac}}\right)}{b^2 - 4ac - b\sqrt{b^2 - 4ac}} \\
&= \frac{x(b^2d^2 - 2abde - 2a(cd^2 - ae^2) + (bcd^2 - 4acde + abe^2)x^n)}{a(b^2 - 4ac)n(a + bx^n + cx^{2n})} - \frac{2e^2x {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{2cx^n}{b - \sqrt{b^2 - 4ac}}\right)}{b^2 - 4ac - b\sqrt{b^2 - 4ac}}
\end{aligned}$$

**Mathematica [B]** time = 2.78323, size = 1835, normalized size = 3.38

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^n)^2/(a + b*x^n + c*x^(2*n))^2, x]
```

```
[Out] -((x*(-((b^2 - 4*a*c)*(-b + Sqrt[b^2 - 4*a*c]))*(b + Sqrt[b^2 - 4*a*c]))*(1 +
n)*(b^2*d^2 + 2*a^2*e^2 + b*c*d^2*x^n + a*b*e*(-2*d + e*x^n) - 2*a*c*d*(d
+ 2*e*x^n))) + 2*b*c^2*Sqrt[b^2 - 4*a*c]*d^2*x^n*(a + x^n*(b + c*x^n))*(-(
b + Sqrt[b^2 - 4*a*c])*Hypergeometric2F1[1, 1 + n^(-1), 2 + n^(-1), (2*c*x^n
)/(-b + Sqrt[b^2 - 4*a*c])]) + (b - Sqrt[b^2 - 4*a*c])*Hypergeometric2F1[1
, 1 + n^(-1), 2 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]) - 8*a*c^2*Sq
rt[b^2 - 4*a*c]*d*e*x^n*(a + x^n*(b + c*x^n))*(-(b + Sqrt[b^2 - 4*a*c])*Hy
pergeometric2F1[1, 1 + n^(-1), 2 + n^(-1), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c
```

$$\begin{aligned}
& ])) + (b - \text{Sqrt}[b^2 - 4*a*c]) * \text{Hypergeometric2F1}[1, 1 + n^{(-1)}, 2 + n^{(-1)}, \\
& \quad (-2*c*x^n)/(b + \text{Sqrt}[b^2 - 4*a*c])] + 2*a*b*c*\text{Sqrt}[b^2 - 4*a*c]*e^2*x^n*( \\
& a + x^n*(b + c*x^n))*(-((b + \text{Sqrt}[b^2 - 4*a*c]) * \text{Hypergeometric2F1}[1, 1 + n^{(-1)}, 2 + n^{(-1)}, \\
& \quad (2*c*x^n)/(-b + \text{Sqrt}[b^2 - 4*a*c])]) + (b - \text{Sqrt}[b^2 - 4* \\
& a*c]) * \text{Hypergeometric2F1}[1, 1 + n^{(-1)}, 2 + n^{(-1)}, (-2*c*x^n)/(b + \text{Sqrt}[b^2 \\
& \quad - 4*a*c])]) - 2*b*c^2*\text{Sqrt}[b^2 - 4*a*c]*d^2*n*x^n*(a + x^n*(b + c*x^n))*(- \\
& ((b + \text{Sqrt}[b^2 - 4*a*c]) * \text{Hypergeometric2F1}[1, 1 + n^{(-1)}, 2 + n^{(-1)}, (2*c* \\
& x^n)/(-b + \text{Sqrt}[b^2 - 4*a*c])]) + (b - \text{Sqrt}[b^2 - 4*a*c]) * \text{Hypergeometric2F1} \\
& [1, 1 + n^{(-1)}, 2 + n^{(-1)}, (-2*c*x^n)/(b + \text{Sqrt}[b^2 - 4*a*c])]) + 8*a*c^2* \\
& \text{Sqrt}[b^2 - 4*a*c]*d*e*n*x^n*(a + x^n*(b + c*x^n))*(-((b + \text{Sqrt}[b^2 - 4*a*c] \\
& ) * \text{Hypergeometric2F1}[1, 1 + n^{(-1)}, 2 + n^{(-1)}, (2*c*x^n)/(-b + \text{Sqrt}[b^2 - 4 \\
& *a*c])]) + (b - \text{Sqrt}[b^2 - 4*a*c]) * \text{Hypergeometric2F1}[1, 1 + n^{(-1)}, 2 + n^{(-1)}, \\
& \quad (-2*c*x^n)/(b + \text{Sqrt}[b^2 - 4*a*c])]) - 2*a*b*c*\text{Sqrt}[b^2 - 4*a*c]*e^2*n \\
& *x^n*(a + x^n*(b + c*x^n))*(-((b + \text{Sqrt}[b^2 - 4*a*c]) * \text{Hypergeometric2F1}[1, \\
& 1 + n^{(-1)}, 2 + n^{(-1)}, (2*c*x^n)/(-b + \text{Sqrt}[b^2 - 4*a*c])]) + (b - \text{Sqrt}[b^ \\
& 2 - 4*a*c]) * \text{Hypergeometric2F1}[1, 1 + n^{(-1)}, 2 + n^{(-1)}, (-2*c*x^n)/(b + \text{Sq} \\
& \text{rt}[b^2 - 4*a*c])]) + 2*b^2*c*\text{Sqrt}[b^2 - 4*a*c]*d^2*(1 + n)*(a + x^n*(b + c* \\
& x^n))*(-((b + \text{Sqrt}[b^2 - 4*a*c]) * \text{Hypergeometric2F1}[1, n^{(-1)}, 1 + n^{(-1)}, ( \\
& 2*c*x^n)/(-b + \text{Sqrt}[b^2 - 4*a*c])]) + (b - \text{Sqrt}[b^2 - 4*a*c]) * \text{Hypergeometri} \\
& c2F1[1, n^{(-1)}, 1 + n^{(-1)}, (-2*c*x^n)/(b + \text{Sqrt}[b^2 - 4*a*c])]) - 4*a*c^2* \\
& \text{Sqrt}[b^2 - 4*a*c]*d^2*(1 + n)*(a + x^n*(b + c*x^n))*(-((b + \text{Sqrt}[b^2 - 4*a* \\
& c]) * \text{Hypergeometric2F1}[1, n^{(-1)}, 1 + n^{(-1)}, (2*c*x^n)/(-b + \text{Sqrt}[b^2 - 4*a \\
& *c])]) + (b - \text{Sqrt}[b^2 - 4*a*c]) * \text{Hypergeometric2F1}[1, n^{(-1)}, 1 + n^{(-1)}, ( \\
& -2*c*x^n)/(b + \text{Sqrt}[b^2 - 4*a*c])]) - 4*a*b*c*\text{Sqrt}[b^2 - 4*a*c]*d*e*(1 + n) \\
& *(a + x^n*(b + c*x^n))*(-((b + \text{Sqrt}[b^2 - 4*a*c]) * \text{Hypergeometric2F1}[1, n^{(- \\
& 1)}, 1 + n^{(-1)}, (2*c*x^n)/(-b + \text{Sqrt}[b^2 - 4*a*c])]) + (b - \text{Sqrt}[b^2 - 4*a* \\
& c]) * \text{Hypergeometric2F1}[1, n^{(-1)}, 1 + n^{(-1)}, (-2*c*x^n)/(b + \text{Sqrt}[b^2 - 4*a \\
& *c])]) + 4*a^2*c*\text{Sqrt}[b^2 - 4*a*c]*e^2*(1 + n)*(a + x^n*(b + c*x^n))*(-((b \\
& + \text{Sqrt}[b^2 - 4*a*c]) * \text{Hypergeometric2F1}[1, n^{(-1)}, 1 + n^{(-1)}, (2*c*x^n)/(-b \\
& + \text{Sqrt}[b^2 - 4*a*c])]) + (b - \text{Sqrt}[b^2 - 4*a*c]) * \text{Hypergeometric2F1}[1, n^{(- \\
& 1)}, 1 + n^{(-1)}, (-2*c*x^n)/(b + \text{Sqrt}[b^2 - 4*a*c])]) - 2*b^2*c*\text{Sqrt}[b^2 - 4 \\
& *a*c]*d^2*n*(1 + n)*(a + x^n*(b + c*x^n))*(-((b + \text{Sqrt}[b^2 - 4*a*c]) * \text{Hyperg} \\
& eometric2F1[1, n^{(-1)}, 1 + n^{(-1)}, (2*c*x^n)/(-b + \text{Sqrt}[b^2 - 4*a*c])]) + ( \\
& b - \text{Sqrt}[b^2 - 4*a*c]) * \text{Hypergeometric2F1}[1, n^{(-1)}, 1 + n^{(-1)}, (-2*c*x^n)/ \\
& (b + \text{Sqrt}[b^2 - 4*a*c])]) + 8*a*c^2*\text{Sqrt}[b^2 - 4*a*c]*d^2*n*(1 + n)*(a + x^ \\
& n*(b + c*x^n))*(-((b + \text{Sqrt}[b^2 - 4*a*c]) * \text{Hypergeometric2F1}[1, n^{(-1)}, 1 + \\
& n^{(-1)}, (2*c*x^n)/(-b + \text{Sqrt}[b^2 - 4*a*c])]) + (b - \text{Sqrt}[b^2 - 4*a*c]) * \text{Hype} \\
& rgeometric2F1[1, n^{(-1)}, 1 + n^{(-1)}, (-2*c*x^n)/(b + \text{Sqrt}[b^2 - 4*a*c])])]) \\
& / (a*(b^2 - 4*a*c)^2*(-b + \text{Sqrt}[b^2 - 4*a*c])*(b + \text{Sqrt}[b^2 - 4*a*c])*n*(1 + \\
& n)*(a + x^n*(b + c*x^n)))
\end{aligned}$$


---

**Maple [F]** time = 0.059, size = 0, normalized size = 0.

$$\int \frac{(d + ex^n)^2}{(a + bx^n + cx^{2n})^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d+e\*x^n)^2/(a+b\*x^n+c\*x^(2\*n))^2,x)

[Out] int((d+e\*x^n)^2/(a+b\*x^n+c\*x^(2\*n))^2,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{(bcd^2 - (4cde - be^2)a)xx^n + (b^2d^2 + 2a^2e^2 - 2(cd^2 + bde)a)x}{a^2b^2n - 4a^3cn + (ab^2cn - 4a^2c^2n)x^{2n} + (ab^3n - 4a^2bcn)x^n} - \int \frac{b^2d^2(n-1) - 2a^2e^2 - 2(cd^2(2n-1) - bde)a + (bcd^2 - (4cde - be^2)a)x}{a^2b^2n - 4a^3cn + (ab^2cn - 4a^2c^2n)x^{2n} + (ab^3n - 4a^2bcn)x^n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e\*x^n)^2/(a+b\*x^n+c\*x^(2\*n))^2,x, algorithm="maxima")

[Out] ((b\*c\*d^2 - (4\*c\*d\*e - b\*e^2)\*a)\*x\*x^n + (b^2\*d^2 + 2\*a^2\*e^2 - 2\*(c\*d^2 + b\*d\*e)\*a)\*x)/(a^2\*b^2\*n - 4\*a^3\*c\*n + (a\*b^2\*c\*n - 4\*a^2\*c^2\*n)\*x^(2\*n) + (a\*b^3\*n - 4\*a^2\*b\*c\*n)\*x^n) - integrate(-(b^2\*d^2\*(n-1) - 2\*a^2\*e^2 - 2\*(c\*d^2\*(2\*n-1) - b\*d\*e)\*a + (b\*c\*d^2\*(n-1) - (4\*c\*d\*e\*(n-1) - b\*e^2\*(n-1))\*a)\*x^n)/(a^2\*b^2\*n - 4\*a^3\*c\*n + (a\*b^2\*c\*n - 4\*a^2\*c^2\*n)\*x^(2\*n) + (a\*b^3\*n - 4\*a^2\*b\*c\*n)\*x^n), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{e^2x^{2n} + 2dex^n + d^2}{c^2x^{4n} + b^2x^{2n} + 2abx^n + a^2 + 2(bc x^n + ac)x^{2n}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e\*x^n)^2/(a+b\*x^n+c\*x^(2\*n))^2,x, algorithm="fricas")

[Out] integral((e^2\*x^(2\*n) + 2\*d\*e\*x^n + d^2)/(c^2\*x^(4\*n) + b^2\*x^(2\*n) + 2\*a\*b\*x^n + a^2 + 2\*(b\*c\*x^n + a\*c)\*x^(2\*n)), x)



---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e\*x\*\*n)\*\*2/(a+b\*x\*\*n+c\*x\*\*(2\*n))\*\*2,x)

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^n + d)^2}{(cx^{2n} + bx^n + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e\*x^n)^2/(a+b\*x^n+c\*x^(2\*n))^2,x, algorithm="giac")

[Out] integrate((e\*x^n + d)^2/(c\*x^(2\*n) + b\*x^n + a)^2, x)

$$3.77 \quad \int \frac{d+ex^n}{(a+bx^n+cx^{2n})^2} dx$$

**Optimal.** Leaf size=362

$$\frac{cx \left( -b \left( d(1-n)\sqrt{b^2-4ac} - 2aen \right) + 2a \left( e(1-n)\sqrt{b^2-4ac} + 2cd(1-2n) \right) + b^2(-d-dn) \right) {}_2F_1 \left( 1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{2cx^n}{b-\sqrt{b^2-4ac}} \right)}{an(b^2-4ac) \left( -b\sqrt{b^2-4ac} - 4ac + b^2 \right)}$$

[Out] (x\*(b^2\*d - 2\*a\*c\*d - a\*b\*e + c\*(b\*d - 2\*a\*e)\*x^n))/(a\*(b^2 - 4\*a\*c)\*n\*(a + b\*x^n + c\*x^(2\*n))) - (c\*(2\*a\*(2\*c\*d\*(1 - 2\*n) + Sqrt[b^2 - 4\*a\*c]\*e\*(1 - n)) - b^2\*(d - d\*n) - b\*(Sqrt[b^2 - 4\*a\*c]\*d\*(1 - n) - 2\*a\*e\*n))\*x\*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2\*c\*x^n)/(b - Sqrt[b^2 - 4\*a\*c])])/(a\*(b^2 - 4\*a\*c)\*(b^2 - 4\*a\*c - b\*Sqrt[b^2 - 4\*a\*c])\*n) - (c\*(2\*a\*(c\*d\*(2 - 4\*n) - Sqrt[b^2 - 4\*a\*c]\*e\*(1 - n)) - b^2\*d\*(1 - n) + b\*(Sqrt[b^2 - 4\*a\*c]\*d\*(1 - n) + 2\*a\*e\*n))\*x\*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2\*c\*x^n)/(b + Sqrt[b^2 - 4\*a\*c])])/(a\*(b^2 - 4\*a\*c)\*(b^2 - 4\*a\*c + b\*Sqrt[b^2 - 4\*a\*c])\*n)

**Rubi [A]** time = 0.63012, antiderivative size = 328, normalized size of antiderivative = 0.91, number of steps used = 4, number of rules used = 3, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {1430, 1422, 245}

$$\frac{cx \left( -(1-n)\sqrt{b^2-4ac}(bd-2ae) + 2aben + 2acd(2-4n) + b^2(-d)(1-n) \right) {}_2F_1 \left( 1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{2cx^n}{b-\sqrt{b^2-4ac}} \right) - cx \left( (1-n)\sqrt{b^2-4ac} \right)}{an(b^2-4ac) \left( -b\sqrt{b^2-4ac} - 4ac + b^2 \right)}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x^n)/(a + b\*x^n + c\*x^(2\*n))^2, x]

[Out] (x\*(b^2\*d - 2\*a\*c\*d - a\*b\*e + c\*(b\*d - 2\*a\*e)\*x^n))/(a\*(b^2 - 4\*a\*c)\*n\*(a + b\*x^n + c\*x^(2\*n))) - (c\*(2\*a\*c\*d\*(2 - 4\*n) - b^2\*d\*(1 - n) - Sqrt[b^2 - 4\*a\*c]\*(b\*d - 2\*a\*e)\*(1 - n) + 2\*a\*b\*e\*n))\*x\*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2\*c\*x^n)/(b - Sqrt[b^2 - 4\*a\*c])])/(a\*(b^2 - 4\*a\*c)\*(b^2 - 4\*a\*c - b\*Sqrt[b^2 - 4\*a\*c])\*n) - (c\*(4\*a\*c\*d\*(1 - 2\*n) - b^2\*d\*(1 - n) + Sqrt[b^2 - 4\*a\*c]\*(b\*d - 2\*a\*e)\*(1 - n) + 2\*a\*b\*e\*n))\*x\*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2\*c\*x^n)/(b + Sqrt[b^2 - 4\*a\*c])])/(a\*(b^2 - 4\*a\*c)\*(b^2 - 4\*a\*c + b\*Sqrt[b^2 - 4\*a\*c])\*n)

Rule 1430

```
Int[((d_) + (e_)*(x_)^(n_))*((a_) + (b_)*(x_)^(n_) + (c_)*(x_)^(n2_))^(p_), x_Symbol] := -Simp[(x*(d*b^2 - a*b*e - 2*a*c*d + (b*d - 2*a*e)*c*x^n)*(a + b*x^n + c*x^(2*n))^(p + 1))/(a*n*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(a*n*(p + 1)*(b^2 - 4*a*c)), Int[Simp[(n*p + n + 1)*d*b^2 - a*b*e - 2*a*c*d*(2*n*p + 2*n + 1) + (2*n*p + 3*n + 1)*(d*b - 2*a*e)*c*x^n, x]*(a + b*x^n + c*x^(2*n))^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && ILtQ[p, -1]
```

### Rule 1422

```
Int[((d_) + (e_)*(x_)^(n_))/((a_) + (b_)*(x_)^(n_) + (c_)*(x_)^(n2_)), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a*c] || !IGtQ[n/2, 0])
```

### Rule 245

```
Int[((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Simp[a^p*x*Hypergeometric2F1[-p, 1/n, 1/n + 1, -(b*x^n)/a], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] || GtQ[a, 0])
```

### Rubi steps

$$\begin{aligned} \int \frac{d + ex^n}{(a + bx^n + cx^{2n})^2} dx &= \frac{x(b^2d - 2acd - abe + c(bd - 2ae)x^n)}{a(b^2 - 4ac)n(a + bx^n + cx^{2n})} - \frac{\int \frac{-abe - 2acd(1-2n) + b^2(d-dn) + c(bd - 2ae)(1-n)x^n}{a + bx^n + cx^{2n}} dx}{a(b^2 - 4ac)n} \\ &= \frac{x(b^2d - 2acd - abe + c(bd - 2ae)x^n)}{a(b^2 - 4ac)n(a + bx^n + cx^{2n})} - \frac{\left(c(2acd(2 - 4n) - b^2d(1 - n) + \sqrt{b^2 - 4ac}(bd - 2ae))\right)}{2a(b^2 - 4ac)^{3/2}} \\ &= \frac{x(b^2d - 2acd - abe + c(bd - 2ae)x^n)}{a(b^2 - 4ac)n(a + bx^n + cx^{2n})} - \frac{c\left((bd - 2ae)(1 - n) - \frac{4acd(1-2n) + 2aben - b^2(d-dn)}{\sqrt{b^2 - 4ac}}\right)}{a(b^2 - 4ac)\left(b - \sqrt{b^2 - 4ac}\right)n} x {}_2F_1 \left( \dots \right) \end{aligned}$$

**Mathematica [B]** time = 1.76775, size = 1381, normalized size = 3.81

result too large to display

Antiderivative was successfully verified.



**Maple [F]** time = 0.053, size = 0, normalized size = 0.

$$\int \frac{d + ex^n}{(a + bx^n + cx^{2n})^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d+e\*x^n)/(a+b\*x^n+c\*x^(2\*n))^2,x)

[Out] int((d+e\*x^n)/(a+b\*x^n+c\*x^(2\*n))^2,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{(bcd - 2ace)xx^n + (b^2d - (2cd + be)a)x}{a^2b^2n - 4a^3cn + (ab^2cn - 4a^2c^2n)x^{2n} + (ab^3n - 4a^2bcn)x^n} + \int \frac{b^2d(n-1) - (2cd(2n-1) - be)a + (bcd(n-1) - 2a}{a^2b^2n - 4a^3cn + (ab^2cn - 4a^2c^2n)x^{2n} + (ab^3n - 4a^2bcn)x^n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e\*x^n)/(a+b\*x^n+c\*x^(2\*n))^2,x, algorithm="maxima")

[Out] ((b\*c\*d - 2\*a\*c\*e)\*x\*x^n + (b^2\*d - (2\*c\*d + b\*e)\*a)\*x)/(a^2\*b^2\*n - 4\*a^3\*c\*n + (a\*b^2\*c\*n - 4\*a^2\*c^2\*n)\*x^(2\*n) + (a\*b^3\*n - 4\*a^2\*b\*c\*n)\*x^n) + integrate((b^2\*d\*(n-1) - (2\*c\*d\*(2\*n-1) - b\*e)\*a + (b\*c\*d\*(n-1) - 2\*a\*c\*e\*(n-1))\*x^n)/(a^2\*b^2\*n - 4\*a^3\*c\*n + (a\*b^2\*c\*n - 4\*a^2\*c^2\*n)\*x^(2\*n) + (a\*b^3\*n - 4\*a^2\*b\*c\*n)\*x^n), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{ex^n + d}{c^2x^{4n} + b^2x^{2n} + 2abx^n + a^2 + 2(bc x^n + ac)x^{2n}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e\*x^n)/(a+b\*x^n+c\*x^(2\*n))^2,x, algorithm="fricas")

[Out] integral((e\*x^n + d)/(c^2\*x^(4\*n) + b^2\*x^(2\*n) + 2\*a\*b\*x^n + a^2 + 2\*(b\*c\*x^n + a\*c)\*x^(2\*n)), x)

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e\*x\*\*n)/(a+b\*x\*\*n+c\*x\*\*(2\*n))\*\*2,x)

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{ex^n + d}{(cx^{2n} + bx^n + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e\*x^n)/(a+b\*x^n+c\*x^(2\*n))^2,x, algorithm="giac")

[Out] integrate((e\*x^n + d)/(c\*x^(2\*n) + b\*x^n + a)^2, x)

$$3.78 \quad \int \frac{1}{(d+ex^n)(a+bx^n+cx^{2n})^2} dx$$

**Optimal.** Leaf size=726

$$\frac{cx \left( \frac{2abce(2-3n)-4ac^2d(1-2n)+b^2cd(1-n)+b^3(-e)(1-n)}{\sqrt{b^2-4ac}} + (1-n)(2ace + b^2(-e) + bcd) \right) {}_2F_1 \left( 1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{2cx^n}{b-\sqrt{b^2-4ac}} \right) + \frac{x(cx^n(2a+bx^n+cx^{2n}))}{an(b^2-4ac)(b-\sqrt{b^2-4ac})(ae^2-bde+cd^2)}}{an(b^2-4ac)(b-\sqrt{b^2-4ac})(ae^2-bde+cd^2)}$$

```
[Out] (x*(b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e + c*(b*c*d - b^2*e + 2*a*c*e)*x^n)/(a*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)*n*(a + b*x^n + c*x^(2*n))) - (c*e^2*(2*c*d - (b + Sqrt[b^2 - 4*a*c]))*e)*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]/((b^2 - 4*a*c - b*Sqrt[b^2 - 4*a*c])*(c*d^2 - b*d*e + a*e^2)^2) - (c*((2*a*b*c*e*(2 - 3*n) - 4*a*c^2*d*(1 - 2*n) + b^2*c*d*(1 - n) - b^3*e*(1 - n))/Sqrt[b^2 - 4*a*c] + (b*c*d - b^2*e + 2*a*c*e)*(1 - n))*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]/(a*(b^2 - 4*a*c)*(b - Sqrt[b^2 - 4*a*c])*(c*d^2 - b*d*e + a*e^2)*n) - (c*e^2*(2*c*d - (b - Sqrt[b^2 - 4*a*c]))*e)*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]/((b^2 - 4*a*c + b*Sqrt[b^2 - 4*a*c])*(c*d^2 - b*d*e + a*e^2)^2) + (c*(b*c*(2*a*e*(2 - 3*n) - Sqrt[b^2 - 4*a*c]*d*(1 - n)) - 2*a*c*(2*c*d*(1 - 2*n) + Sqrt[b^2 - 4*a*c]*e*(1 - n)) - b^3*e*(1 - n) + b^2*(c*d + Sqrt[b^2 - 4*a*c]*e)*(1 - n))*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]/(a*(b^2 - 4*a*c)*(b^2 - 4*a*c + b*Sqrt[b^2 - 4*a*c])*(c*d^2 - b*d*e + a*e^2)*n) + (e^4*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -(e*x^n)/d])/((d*(c*d^2 - b*d*e + a*e^2)^2))
```

**Rubi [A]** time = 1.92679, antiderivative size = 726, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 4, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$ , Rules used = {1436, 245, 1430, 1422}

$$\frac{cx \left( \frac{2abce(2-3n)-4ac^2d(1-2n)+b^2cd(1-n)+b^3(-e)(1-n)}{\sqrt{b^2-4ac}} + (1-n)(2ace + b^2(-e) + bcd) \right) {}_2F_1 \left( 1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{2cx^n}{b-\sqrt{b^2-4ac}} \right) + \frac{x(cx^n(2a+bx^n+cx^{2n}))}{an(b^2-4ac)(b-\sqrt{b^2-4ac})(ae^2-bde+cd^2)}}{an(b^2-4ac)(b-\sqrt{b^2-4ac})(ae^2-bde+cd^2)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x^n)*(a + b*x^n + c*x^(2*n))^2), x]
```

```
[Out] (x*(b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e + c*(b*c*d - b^2*e + 2*a*c*e)*x^n)/(a*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)*n*(a + b*x^n + c*x^(2*n))) -
```

```
(c*e^2*(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])])/(b^2 - 4*a*c - b*Sqrt[b^2 - 4*a*c])*(c*d^2 - b*d*e + a*e^2)^2) - (c*((2*a*b*c*e*(2 - 3*n) - 4*a*c^2*d*(1 - 2*n) + b^2*c*d*(1 - n) - b^3*e*(1 - n))/Sqrt[b^2 - 4*a*c] + (b*c*d - b^2*e + 2*a*c*e)*(1 - n))*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])])/(a*(b^2 - 4*a*c)*(b - Sqrt[b^2 - 4*a*c])*(c*d^2 - b*d*e + a*e^2)*n) - (c*e^2*(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e)*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(b^2 - 4*a*c + b*Sqrt[b^2 - 4*a*c])*(c*d^2 - b*d*e + a*e^2)^2) + (c*(b*c*(2*a*e*(2 - 3*n) - Sqrt[b^2 - 4*a*c]*d*(1 - n)) - 2*a*c*(2*c*d*(1 - 2*n) + Sqrt[b^2 - 4*a*c]*e*(1 - n)) - b^3*e*(1 - n) + b^2*(c*d + Sqrt[b^2 - 4*a*c]*e)*(1 - n))*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(a*(b^2 - 4*a*c)*(b^2 - 4*a*c + b*Sqrt[b^2 - 4*a*c])*(c*d^2 - b*d*e + a*e^2)*n) + (e^4*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -(e*x^n)/d])/(d*(c*d^2 - b*d*e + a*e^2)^2)
```

### Rule 1436

```
Int[((d_) + (e_)*(x_)^(n_))^(q_)*((a_) + (b_)*(x_)^(n_) + (c_)*(x_)^(n2_))^(p_), x_Symbol] := Int[ExpandIntegrand[(d + e*x^n)^q*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && ((IntegersQ[p, q] && !IntegerQ[n]) || IGtQ[p, 0] || (IGtQ[q, 0] && !IntegerQ[n]))
```

### Rule 245

```
Int[((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Simp[a^p*x*Hypergeometric2F1[-p, 1/n, 1/n + 1, -(b*x^n)/a], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] || GtQ[a, 0])
```

### Rule 1430

```
Int[((d_) + (e_)*(x_)^(n_))*((a_) + (b_)*(x_)^(n_) + (c_)*(x_)^(n2_))^(p_), x_Symbol] := -Simp[(x*(d*b^2 - a*b*e - 2*a*c*d + (b*d - 2*a*e)*c*x^n)*(a + b*x^n + c*x^(2*n))^(p + 1))/(a*n*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(a*n*(p + 1)*(b^2 - 4*a*c)), Int[Simp[(n*p + n + 1)*d*b^2 - a*b*e - 2*a*c*d*(2*n*p + 2*n + 1) + (2*n*p + 3*n + 1)*(d*b - 2*a*e)*c*x^n, x]*(a + b*x^n + c*x^(2*n))^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && ILtQ[p, -1]
```

### Rule 1422

```
Int[((d_) + (e_)*(x_)^(n_))/((a_) + (b_)*(x_)^(n_) + (c_)*(x_)^(n2_)), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
```



Int[1/(b/2 - q/2 + c\*x^n), x], x] + Dist[e/2 - (2\*c\*d - b\*e)/(2\*q), Int[1/(b/2 + q/2 + c\*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2\*n] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && (PosQ[b^2 - 4\*a\*c] || !IGtQ[n/2, 0])

### Rubi steps

$$\begin{aligned} \int \frac{1}{(d + ex^n)(a + bx^n + cx^{2n})^2} dx &= \int \left( \frac{e^4}{(cd^2 - bde + ae^2)^2 (d + ex^n)} + \frac{cd - be - cex^n}{(cd^2 - bde + ae^2)(a + bx^n + cx^{2n})^2} - \frac{e^2}{(cd^2 - bde + ae^2)^2} \right) dx \\ &= -\frac{e^2 \int \frac{-cd + be + cex^n}{a + bx^n + cx^{2n}} dx}{(cd^2 - bde + ae^2)^2} + \frac{e^4 \int \frac{1}{d + ex^n} dx}{(cd^2 - bde + ae^2)^2} + \frac{\int \frac{cd - be - cex^n}{(a + bx^n + cx^{2n})^2} dx}{cd^2 - bde + ae^2} \\ &= \frac{x(b^2cd - 2ac^2d - b^3e + 3abce + c(bcd - b^2e + 2ace)x^n)}{a(b^2 - 4ac)(cd^2 - bde + ae^2)n(a + bx^n + cx^{2n})} + \frac{e^4 x {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{ex^n}{d}\right)}{d(cd^2 - bde + ae^2)^2} \\ &= \frac{x(b^2cd - 2ac^2d - b^3e + 3abce + c(bcd - b^2e + 2ace)x^n)}{a(b^2 - 4ac)(cd^2 - bde + ae^2)n(a + bx^n + cx^{2n})} - \frac{ce^2 \left(e - \frac{2cd - be}{\sqrt{b^2 - 4ac}}\right) x {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{ex^n}{b - \sqrt{b^2 - 4ac}}\right)}{(b - \sqrt{b^2 - 4ac})(cd^2 - bde + ae^2)^2} \\ &= \frac{x(b^2cd - 2ac^2d - b^3e + 3abce + c(bcd - b^2e + 2ace)x^n)}{a(b^2 - 4ac)(cd^2 - bde + ae^2)n(a + bx^n + cx^{2n})} - \frac{ce^2 \left(e - \frac{2cd - be}{\sqrt{b^2 - 4ac}}\right) x {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{ex^n}{b - \sqrt{b^2 - 4ac}}\right)}{(b - \sqrt{b^2 - 4ac})(cd^2 - bde + ae^2)^2} \end{aligned}$$

**Mathematica [B]** time = 7.27299, size = 8045, normalized size = 11.08

Result too large to show

Antiderivative was successfully verified.

[In] Integrate[1/((d + e\*x^n)\*(a + b\*x^n + c\*x^(2\*n))^2), x]

[Out] Result too large to show

**Maple [F]** time = 0.174, size = 0, normalized size = 0.

$$\int \frac{1}{(d + ex^n)(a + bx^n + cx^{2n})^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(d+e*x^n)/(a+b*x^n+c*x^(2*n))^2,x)`

[Out] `int(1/(d+e*x^n)/(a+b*x^n+c*x^(2*n))^2,x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(d+e*x^n)/(a+b*x^n+c*x^(2*n))^2,x, algorithm="maxima")`

[Out] `e^4*integrate(1/(c^2*d^5 - 2*b*c*d^4*e + b^2*d^3*e^2 + a^2*d*e^4 + 2*(c*d^3*e^2 - b*d^2*e^3)*a + (c^2*d^4*e - 2*b*c*d^3*e^2 + b^2*d^2*e^3 + a^2*e^5 + 2*(c*d^2*e^3 - b*d*e^4)*a)*x^n), x) - ((b*c^2*d - b^2*c*e + 2*a*c^2*e)*x*x^n + (b^2*c*d - b^3*e - (2*c^2*d - 3*b*c*e)*a)*x)/(4*a^4*c*e^2*n + (4*c^2*d^2*n - 4*b*c*d*e*n - b^2*e^2*n)*a^3 - (b^2*c*d^2*n - b^3*d*e*n)*a^2 + (4*a^3*c^2*e^2*n + (4*c^3*d^2*n - 4*b*c^2*d*e*n - b^2*c*e^2*n)*a^2 - (b^2*c^2*d^2*n - b^3*c*d*e*n)*a)*x^(2*n) + (4*a^3*b*c*e^2*n + (4*b*c^2*d^2*n - 4*b^2*c*d*e*n - b^3*e^2*n)*a^2 - (b^3*c*d^2*n - b^4*d*e*n)*a)*x^n) - integrate((b^2*c^2*d^3*(n - 1) - 2*b^3*c*d^2*e*(n - 1) + b^4*d*e^2*(n - 1) + (b*c*e^3*(8*n - 3) - 2*c^2*d*e^2*(4*n - 1))*a^2 + (b*c^2*d^2*e*(8*n - 5) - 2*c^3*d^3*(2*n - 1) - b^3*e^3*(2*n - 1) - 2*b^2*c*d*e^2*(n - 1))*a + (2*a^2*c^2*e^3*(3*n - 1) + b*c^3*d^3*(n - 1) - 2*b^2*c^2*d^2*e*(n - 1) + b^3*c*d*e^2*(n - 1) - (b^2*c*e^3*(2*n - 1) - 2*c^3*d^2*e*(n - 1) + b*c^2*d*e^2*(n - 1))*a)*x^n)/(4*a^5*c*e^4*n + (8*c^2*d^2*e^2*n - 8*b*c*d*e^3*n - b^2*e^4*n)*a^4 + 2*(2*c^3*d^4*n - 4*b*c^2*d^3*e*n + b^2*c*d^2*e^2*n + b^3*d*e^3*n)*a^3 - (b^2*c^2*d^4*n - 2*b^3*c*d^3*e*n + b^4*d^2*e^2*n)*a^2 + (4*a^4*c^2*e^4*n + (8*c^3*d^2*e^2*n - 8*b*c^2*d*e^3*n - b^2*c*e^4*n)*a^3 + 2*(2*c^4*d^4*n - 4*b*c^3*d^3*e*n + b^2*c^2*d^2*e^2*n + b^3*c*d*e^3*n)*a^2 - (b^2*c^3*d^4*n - 2*b^3*c^2*d^3*e*n + b^4*c*d^2*e^2*n)*a)*x^(2*n) + (4*a^4*b*c*e^4*n + (8*b*c^2*d^2*e^2*n - 8*b^2*c*d*e^3*n - b^3*e^4*n)*a^3 + 2*(2*b*c^3*d^4*n - 4*b^2*c^2*d^3*e*n + b^3*c*d^2*e^2*n + b^4*d*e^3*n)*a^2 - (b^3*c^2*d^4*n - 2*b^4*c*d^3*e*n + b^5*d^2*e^2*n)*a)*x^n), x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

integral  $\left( \frac{1}{b^2ex^{3n} + a^2d + (c^2ex^n + c^2d)x^{4n} + 2(bcex^{2n} + acd + (bcd + ace)x^n)x^{2n} + (b^2d + 2abe)x^{2n} + (2abd + a^2e)x^n} \right)$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x^n)/(a+b*x^n+c*x^(2*n))^2,x, algorithm="fricas")
```

```
[Out] integral(1/(b^2*e*x^(3*n) + a^2*d + (c^2*e*x^n + c^2*d)*x^(4*n) + 2*(b*c*e*
x^(2*n) + a*c*d + (b*c*d + a*c*e)*x^n)*x^(2*n) + (b^2*d + 2*a*b*e)*x^(2*n)
+ (2*a*b*d + a^2*e)*x^n), x)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x**n)/(a+b*x**n+c*x**(2*n))**2,x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(cx^{2n} + bx^n + a)^2 (ex^n + d)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x^n)/(a+b*x^n+c*x^(2*n))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^(2*n) + b*x^n + a)^2*(e*x^n + d)), x)
```

$$3.79 \quad \int \frac{1}{(d+ex^n)^2(a+bx^n+cx^{2n})^2} dx$$

**Optimal.** Leaf size=1129

result too large to display

```
[Out] -((x*(2*b^3*c*d*e - 6*a*b*c^2*d*e - b^4*e^2 - b^2*c*(c*d^2 - 4*a*e^2) + 2*a*c^2*(c*d^2 - a*e^2) + c*(2*b^2*c*d*e - 4*a*c^2*d*e - b^3*e^2 - b*c*(c*d^2 - 3*a*e^2))*x^n)/(a*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)^2*n*(a + b*x^n + c*x^(2*n)))) - (2*c*e^2*(3*c^2*d^2 + b*(b + Sqrt[b^2 - 4*a*c])*e^2 - c*e*(3*b*d + 2*Sqrt[b^2 - 4*a*c]*d + a*e))*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])])/(b^2 - 4*a*c - b*Sqrt[b^2 - 4*a*c])*(c*d^2 - b*d*e + a*e^2)^3 + (c*(4*a*c^2*(e*(a*e*(1 - 2*n) + Sqrt[b^2 - 4*a*c]*d*(1 - n)) - c*d^2*(1 - 2*n)) - b^2*c*(e*(a*e*(5 - 7*n) + 2*Sqrt[b^2 - 4*a*c]*d*(1 - n)) - c*d^2*(1 - n)) + b*c*(c*d*(4*a*e*(2 - 3*n) + Sqrt[b^2 - 4*a*c]*d*(1 - n)) - 3*a*Sqrt[b^2 - 4*a*c]*e^2*(1 - n)) + b^4*e^2*(1 - n) - b^3*e*(2*c*d - Sqrt[b^2 - 4*a*c]*e)*(1 - n))*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])])/(a*(b^2 - 4*a*c)*(b^2 - 4*a*c - b*Sqrt[b^2 - 4*a*c])*(c*d^2 - b*d*e + a*e^2)^2*n) - (2*c*e^2*(3*c^2*d^2 + b*(b - Sqrt[b^2 - 4*a*c])*e^2 - c*e*(3*b*d - 2*Sqrt[b^2 - 4*a*c]*d + a*e))*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(b^2 - 4*a*c + b*Sqrt[b^2 - 4*a*c])*(c*d^2 - b*d*e + a*e^2)^3 + (c*(4*a*c^2*(e*(a*e*(1 - 2*n) - Sqrt[b^2 - 4*a*c]*d*(1 - n)) - c*d^2*(1 - 2*n)) - b^2*c*(e*(a*e*(5 - 7*n) - 2*Sqrt[b^2 - 4*a*c]*d*(1 - n)) - c*d^2*(1 - n)) + b*c*(c*d*(4*a*e*(2 - 3*n) - Sqrt[b^2 - 4*a*c]*d*(1 - n)) + 3*a*Sqrt[b^2 - 4*a*c]*e^2*(1 - n)) + b^4*e^2*(1 - n) - b^3*e*(2*c*d + Sqrt[b^2 - 4*a*c]*e)*(1 - n))*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(a*(b^2 - 4*a*c)*(b^2 - 4*a*c + b*Sqrt[b^2 - 4*a*c])*(c*d^2 - b*d*e + a*e^2)^2*n) + (2*e^4*(2*c*d - b*e)*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((e*x^n)/d)])/(d*(c*d^2 - b*d*e + a*e^2)^3) + (e^4*x*Hypergeometric2F1[2, n^(-1), 1 + n^(-1), -((e*x^n)/d)])/(d^2*(c*d^2 - b*d*e + a*e^2)^2)
```

**Rubi [A]** time = 3.34142, antiderivative size = 1129, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 4, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$ , Rules used = {1436, 245, 1430, 1422}

$$\frac{2(2cd - be)x {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{ex^n}{d}\right) e^4}{d(cd^2 - bed + ae^2)^3} + \frac{x {}_2F_1\left(2, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{ex^n}{d}\right) e^4}{d^2(cd^2 - bed + ae^2)^2} - \frac{2c\left(3c^2d^2 + b\left(b + \sqrt{b^2 - 4ac}\right)e^2 - ce\left(3bd + 2\sqrt{b^2 - 4ac}\right)\right)}{\left(b^2 - \sqrt{b^2 - 4ac}b - 4ac\right)}$$

Antiderivative was successfully verified.

[In] Int[1/((d + e\*x^n)^2\*(a + b\*x^n + c\*x^(2\*n))^2),x]

[Out] 
$$-\frac{\left( (x(2b^3cde - 6abc^2de - b^4e^2 - b^2c(c^2d^2 - 4ae^2)) + 2ac^2(c^2d^2 - ae^2) + c(2b^2cde - 4ac^2de - b^3e^2 - b^2c(c^2d^2 - 3ae^2))x^n) \right) / (a(b^2 - 4ac)(c^2d^2 - bde + ae^2)^{2n}(a + b^n + c^{2n})) - (2ce^2(3c^2d^2 + b(b + \sqrt{b^2 - 4ac}))e^2 - ce(3bd + 2\sqrt{b^2 - 4ac}d + ae))x \text{Hypergeometric2F1}[1, n^{-1}, 1 + n^{-1}, (-2cx^n)/(b - \sqrt{b^2 - 4ac})]} \right) / ((b^2 - 4ac - b\sqrt{b^2 - 4ac})(c^2d^2 - bde + ae^2)^3 + (c(4ac^2(e(ae(1 - 2n) + \sqrt{b^2 - 4ac})d(1 - n)) - c^2d^2(1 - 2n)) - b^2c(e(ae(5 - 7n) + 2\sqrt{b^2 - 4ac})d(1 - n)) - cd^2(1 - n)) + bc(c^2d(4ae(2 - 3n) + \sqrt{b^2 - 4ac})d(1 - n) - 3a\sqrt{b^2 - 4ac})e^2(1 - n) + b^4e^2(1 - n) - b^3e(2cd - \sqrt{b^2 - 4ac})e)(1 - n))x \text{Hypergeometric2F1}[1, n^{-1}, 1 + n^{-1}, (-2cx^n)/(b - \sqrt{b^2 - 4ac})]} \right) / (a(b^2 - 4ac)(b^2 - 4ac - b\sqrt{b^2 - 4ac})(c^2d^2 - bde + ae^2)^{2n} - (2ce^2(3c^2d^2 + b(b - \sqrt{b^2 - 4ac}))e^2 - ce(3bd - 2\sqrt{b^2 - 4ac})d + ae))x \text{Hypergeometric2F1}[1, n^{-1}, 1 + n^{-1}, (-2cx^n)/(b + \sqrt{b^2 - 4ac})]} \right) / ((b^2 - 4ac + b\sqrt{b^2 - 4ac})(c^2d^2 - bde + ae^2)^3 + (c(4ac^2(e(ae(1 - 2n) - \sqrt{b^2 - 4ac})d(1 - n)) - c^2d^2(1 - 2n)) - b^2c(e(ae(5 - 7n) - 2\sqrt{b^2 - 4ac})d(1 - n)) - cd^2(1 - n)) + bc(c^2d(4ae(2 - 3n) - \sqrt{b^2 - 4ac})d(1 - n) + 3a\sqrt{b^2 - 4ac})e^2(1 - n) + b^4e^2(1 - n) - b^3e(2cd + \sqrt{b^2 - 4ac})e)(1 - n))x \text{Hypergeometric2F1}[1, n^{-1}, 1 + n^{-1}, (-2cx^n)/(b + \sqrt{b^2 - 4ac})]} \right) / (a(b^2 - 4ac)(b^2 - 4ac + b\sqrt{b^2 - 4ac})(c^2d^2 - bde + ae^2)^{2n} + (2e^4(2cd - b)e)x \text{Hypergeometric2F1}[1, n^{-1}, 1 + n^{-1}, -((e*x^n)/d)] \right) / (d(c^2d^2 - bde + ae^2)^3 + (e^4x \text{Hypergeometric2F1}[2, n^{-1}, 1 + n^{-1}, -((e*x^n)/d)] \right) / (d^2(c^2d^2 - bde + ae^2)^2)$$

### Rule 1436

Int[(((d\_) + (e\_)\*(x\_)^(n\_))^(q\_)\*((a\_) + (b\_)\*(x\_)^(n\_) + (c\_)\*(x\_)^(n2\_))^(p\_), x\_Symbol] :> Int[ExpandIntegrand[(d + e\*x^n)^q\*(a + b\*x^n + c\*x^(2\*n))^p, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && EqQ[n2, 2\*n] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && ((IntegersQ[p, q] && !IntegerQ[n]) || IGtQ[p, 0] || (IGtQ[q, 0] && !IntegerQ[n]))

### Rule 245

Int[(((a\_) + (b\_)\*(x\_)^(n\_))^(p\_), x\_Symbol] :> Simp[a^p\*x\*Hypergeometric2F1[-p, 1/n, 1/n + 1, -((b\*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] || GtQ[a, 0])

Rule 1430

```
Int[((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p
_), x_Symbol] := -Simp[(x*(d*b^2 - a*b*e - 2*a*c*d + (b*d - 2*a*e)*c*x^n)*(
a + b*x^n + c*x^(2*n))^(p + 1))/(a*n*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(a
*n*(p + 1)*(b^2 - 4*a*c)), Int[Simp[(n*p + n + 1)*d*b^2 - a*b*e - 2*a*c*d*(
2*n*p + 2*n + 1) + (2*n*p + 3*n + 1)*(d*b - 2*a*e)*c*x^n, x]*(a + b*x^n + c
*x^(2*n))^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] &
& NeQ[b^2 - 4*a*c, 0] && ILtQ[p, -1]
```

Rule 1422

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] || !IGtQ[n/2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(d + ex^n)^2 (a + bx^n + cx^{2n})^2} dx &= \int \left( \frac{e^4}{(cd^2 - bde + ae^2)^2 (d + ex^n)^2} - \frac{2e^4(-2cd + be)}{(cd^2 - bde + ae^2)^3 (d + ex^n)} + \frac{c^2 d^2 - 2bcde + b^2 e^2}{(cd^2 - bde + ae^2)^3} \right) dx \\
&= \frac{e^2 \int \frac{3c^2 d^2 - 5bcde + 2b^2 e^2 - ace^2 + (-4c^2 de + 2bce^2)x^n}{a + bx^n + cx^{2n}} dx}{(cd^2 - bde + ae^2)^3} + \frac{(2e^4(2cd - be)) \int \frac{1}{d + ex^n} dx}{(cd^2 - bde + ae^2)^3} + \frac{\int \frac{c^2 d^2 - 2bcde + b^2 e^2}{(cd^2 - bde + ae^2)^3} dx}{(cd^2 - bde + ae^2)^3} \\
&= -\frac{x(2b^3cde - 6abc^2de - b^4e^2 - b^2c(cd^2 - 4ae^2) + 2ac^2(cd^2 - ae^2) + c(2b^2cde - 4ac^2d^2 - 2bcde + b^2e^2))}{a(b^2 - 4ac)(cd^2 - bde + ae^2)^2 n(a + bx^n + cx^{2n})} \\
&= -\frac{x(2b^3cde - 6abc^2de - b^4e^2 - b^2c(cd^2 - 4ae^2) + 2ac^2(cd^2 - ae^2) + c(2b^2cde - 4ac^2d^2 - 2bcde + b^2e^2))}{a(b^2 - 4ac)(cd^2 - bde + ae^2)^2 n(a + bx^n + cx^{2n})} \\
&= -\frac{x(2b^3cde - 6abc^2de - b^4e^2 - b^2c(cd^2 - 4ae^2) + 2ac^2(cd^2 - ae^2) + c(2b^2cde - 4ac^2d^2 - 2bcde + b^2e^2))}{a(b^2 - 4ac)(cd^2 - bde + ae^2)^2 n(a + bx^n + cx^{2n})}
\end{aligned}$$

**Mathematica [B]** time = 8.10529, size = 11657, normalized size = 10.33

Result too large to show

Antiderivative was successfully verified.

[In] Integrate[1/((d + e\*x^n)^2\*(a + b\*x^n + c\*x^(2\*n))^2), x]

[Out] Result too large to show

**Maple [F]** time = 0.221, size = 0, normalized size = 0.

$$\int \frac{1}{(d + ex^n)^2 (a + bx^n + cx^{2n})^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(d+e\*x^n)^2/(a+b\*x^n+c\*x^(2\*n))^2,x)

[Out] int(1/(d+e\*x^n)^2/(a+b\*x^n+c\*x^(2\*n))^2,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d+e\*x^n)^2/(a+b\*x^n+c\*x^(2\*n))^2,x, algorithm="maxima")

[Out] (c\*d^2\*e^4\*(5\*n - 1) - b\*d\*e^5\*(3\*n - 1) + a\*e^6\*(n - 1))\*integrate(1/(c^3\*d^8\*n - 3\*b\*c^2\*d^7\*e^n + 3\*b^2\*c\*d^6\*e^2\*n - b^3\*d^5\*e^3\*n + a^3\*d^2\*e^6\*n + 3\*(c\*d^4\*e^4\*n - b\*d^3\*e^5\*n)\*a^2 + 3\*(c^2\*d^6\*e^2\*n - 2\*b\*c\*d^5\*e^3\*n + b^2\*d^4\*e^4\*n)\*a + (c^3\*d^7\*e^n - 3\*b\*c^2\*d^6\*e^2\*n + 3\*b^2\*c\*d^5\*e^3\*n - b^3\*d^4\*e^4\*n + a^3\*d\*e^7\*n + 3\*(c\*d^3\*e^5\*n - b\*d^2\*e^6\*n)\*a^2 + 3\*(c^2\*d^5\*e^3\*n - 2\*b\*c\*d^4\*e^4\*n + b^2\*d^3\*e^5\*n)\*a)\*x^n), x) - ((b\*c^3\*d^3\*e - 2\*b^2\*c^2\*d^2\*e^2 + b^3\*c\*d\*e^3 - 4\*a^2\*c^2\*e^4 + (4\*c^3\*d^2\*e^2 - 3\*b\*c^2\*d\*e^3 + b^2\*c\*e^4)\*a)\*x\*x^(2\*n) + (b\*c^3\*d^4 - b^2\*c^2\*d^3\*e - b^3\*c\*d^2\*e^2 + b^4\*d\*e^3 + 2\*(c^2\*d\*e^3 - 2\*b\*c\*e^4)\*a^2 + (2\*c^3\*d^3\*e + 3\*b\*c^2\*d^2\*e^2 - 4\*b^2\*c\*d\*e^3 + b^3\*e^4)\*a)\*x\*x^n + (b^2\*c^2\*d^4 - 2\*b^3\*c\*d^3\*e + b^4\*

$$\begin{aligned}
& d^2e^2 - 4a^3c^2e^4 + (2c^2d^2e^2 + b^2e^4)a^2 - 2(c^3d^4 - 3b^2c^2d^3e + 2b^2c^2d^2e^2)a)x / (4a^5c^2d^2e^4n + (8c^2d^4e^2n - 8b^2c^2d^3e^3n - b^2d^2e^4n)a^4 + 2(2c^3d^6n - 4b^2c^2d^5e^3n + b^2c^2d^4e^2n + b^3d^3e^3n)a^3 - (b^2c^2d^6n - 2b^3c^2d^5e^3n + b^4d^4e^2n)a^2 + (4a^4c^2d^5e^3n + (8c^3d^3e^3n - 8b^2c^2d^2e^4n - b^2c^2d^5e^3n)a^3 + 2(2c^4d^5e^3n - 4b^2c^3d^4e^2n + b^2c^2d^3e^3n + b^3c^2d^2e^4n)a^2 - (b^2c^3d^5e^3n - 2b^3c^2d^4e^2n + b^4c^2d^3e^3n)a)x^3 + (4(c^2d^2e^4n + b^2c^2d^5e^3n)a^4 + (8c^3d^4e^2n - 9b^2c^2d^2e^4n - b^3d^5e^3n)a^3 + 2(2c^4d^6n - 2b^2c^3d^5e^3n - 3b^2c^2d^4e^2n + 2b^3c^2d^3e^3n + b^4d^2e^4n)a^2 - (b^2c^3d^6n - b^3c^2d^5e^3n - b^4c^2d^4e^2n + b^5d^3e^3n)a)x^2 + (4a^5c^2d^5e^3n + (8c^2d^3e^3n - 4b^2c^2d^2e^4n - b^2d^5e^3n)a^4 + (4c^3d^5e^3n - 6b^2c^2d^3e^3n + b^3d^2e^4n)a^3 + (4b^2c^3d^6n - 9b^2c^2d^5e^3n + 4b^3c^2d^4e^2n + b^4d^3e^3n)a^2 - (b^3c^2d^6n - 2b^4c^2d^5e^3n + b^5d^4e^2n)a)x^n) + \text{integrate}(-(2a^3c^2e^4(4n - 1) + b^2c^3d^4(n - 1) - 3b^3c^2d^3e^3(n - 1) + 3b^4c^2d^2e^2(n - 1) - b^5d^3e^3(n - 1) - 2(b^2c^2e^4(7n - 2) - 2b^2c^2d^2e^3(6n - 1) + 6c^3d^2e^2n)a^2 + (b^4e^4(3n - 1) + 4b^2c^3d^3e^3(3n - 2) - 2c^4d^4(2n - 1) - 2b^3c^2d^2e^2(n - 1) - 9b^2c^2d^2e^2(n - 1))a + (b^2c^4d^4(n - 1) - 3b^2c^3d^3e^3(n - 1) + 3b^3c^2d^2e^2(n - 1) - b^4c^2d^3e^3(n - 1) - (b^2c^2e^4(11n - 3) - 4c^3d^3e^3(5n - 1))a^2 - (b^2c^2d^3e^3(3n + 1) - b^3c^2e^4(3n - 1) - 4c^4d^3e^3(n - 1) + 6b^2c^3d^2e^2(n - 1))a)x^n) / (4a^6c^2e^6n + (12c^2d^2e^4n - 12b^2c^2d^2e^5n - b^2e^6n)a^5 + 3(4c^3d^4e^2n - 8b^2c^2d^3e^3n + 3b^2c^2d^2e^4n + b^3d^5e^3n)a^4 + (4c^4d^6n - 12b^2c^3d^5e^3n + 9b^2c^2d^4e^2n + 2b^3c^2d^3e^3n - 3b^4d^2e^4n)a^3 - (b^2c^3d^6n - 3b^3c^2d^5e^3n + 3b^4c^2d^4e^2n - b^5d^3e^3n)a^2 + (4a^5c^2e^6n + (12c^3d^2e^4n - 12b^2c^2d^2e^5n - b^2c^2e^6n)a^4 + 3(4c^4d^4e^2n - 8b^2c^3d^3e^3n + 3b^2c^2d^2e^4n + b^3c^2d^5e^3n)a^3 + (4c^5d^6n - 12b^2c^4d^5e^3n + 9b^2c^3d^4e^2n + 2b^3c^2d^3e^3n - 3b^4c^2d^2e^4n)a^2 - (b^2c^4d^6n - 3b^3c^3d^5e^3n + 3b^4c^2d^4e^2n - b^5c^2d^3e^3n)a)x^2 + (4a^5b^2c^2e^6n + (12b^2c^2d^2e^4n - 12b^2c^2d^2e^5n - b^3e^6n)a^4 + 3(4b^2c^3d^4e^2n - 8b^2c^2d^3e^3n + 3b^3c^2d^2e^4n + b^4d^5e^3n)a^3 + (4b^2c^4d^6n - 12b^2c^3d^5e^3n + 9b^3c^2d^4e^2n + 2b^4c^2d^3e^3n - 3b^5d^2e^4n)a^2 - (b^3c^3d^6n - 3b^4c^2d^5e^3n + 3b^5c^2d^4e^2n - b^6d^3e^3n)a)x^n), x)
\end{aligned}$$

**Fricas [F]** time = 0., size = 0, normalized size = 0.

integral  $\left( \frac{1}{b^2e^2x^{4n} + a^2d^2 + (c^2e^2x^{2n} + 2c^2dex^n + c^2d^2)x^{4n} + 2(b^2de + abe^2)x^{3n} + 2(bce^2x^{3n} + acd^2 + (2bcde + ace^2)x^2} \right)$



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d+e\*x^n)^2/(a+b\*x^n+c\*x^(2\*n))^2,x, algorithm="fricas")

[Out] integral(1/(b^2\*e^2\*x^(4\*n) + a^2\*d^2 + (c^2\*e^2\*x^(2\*n) + 2\*c^2\*d\*e\*x^n + c^2\*d^2)\*x^(4\*n) + 2\*(b^2\*d\*e + a\*b\*e^2)\*x^(3\*n) + 2\*(b\*c\*e^2\*x^(3\*n) + a\*c\*d^2 + (2\*b\*c\*d\*e + a\*c\*e^2)\*x^(2\*n) + (b\*c\*d^2 + 2\*a\*c\*d\*e)\*x^n)\*x^(2\*n) + (b^2\*d^2 + 4\*a\*b\*d\*e + a^2\*e^2)\*x^(2\*n) + 2\*(a\*b\*d^2 + a^2\*d\*e)\*x^n), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d+e\*x\*\*n)\*\*2/(a+b\*x\*\*n+c\*x\*\*(2\*n))\*\*2,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(cx^{2n} + bx^n + a)^2 (ex^n + d)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d+e\*x^n)^2/(a+b\*x^n+c\*x^(2\*n))^2,x, algorithm="giac")

[Out] integrate(1/((c\*x^(2\*n) + b\*x^n + a)^2\*(e\*x^n + d)^2), x)

$$3.80 \quad \int \frac{(d+ex^n)^3}{(a+bx^n+cx^{2n})^3} dx$$

Optimal. Leaf size=1707

result too large to display

```
[Out] (x*(b^2*c*d^3 - 2*a*c*d*(c*d^2 - 3*a*e^2) - a*b*e*(3*c*d^2 + a*e^2) - (a*b^2*e^3 + 2*a*c*e*(3*c*d^2 - a*e^2) - b*c*d*(c*d^2 + 3*a*e^2))*x^n))/(2*a*c*(b^2 - 4*a*c)*n*(a + b*x^n + c*x^(2*n))^2) + (e^2*x*(3*b^2*c*d - 6*a*c^2*d - b^3*e + a*b*c*e + c*(3*b*c*d - b^2*e - 2*a*c*e)*x^n))/(a*c^2*(b^2 - 4*a*c)*n*(a + b*x^n + c*x^(2*n))) - (x*(a*b^2*c^2*d*(3*a*e^2*(1 - 9*n) - 5*c*d^2*(1 - 3*n)) + 4*a^2*c^3*d*(c*d^2 - 3*a*e^2)*(1 - 4*n) - 2*a*b^5*e^3*n + 2*a^2*b*c^2*e*(3*c*d^2*(2 - 3*n) - 5*a*e^2*n) - 3*a*b^3*c*e*(c*d^2 - 3*a*e^2*n) + b^4*c*d*(c*d^2*(1 - 2*n) + 6*a*e^2*n) + c*(4*a^2*c^2*e*(3*c*d^2 - a*e^2)*(1 - 3*n) - 2*a*b^4*e^3*n - 2*a*b*c^2*d*(c*d^2*(2 - 7*n) + 3*a*e^2*n) + b^3*c*d*(c*d^2*(1 - 2*n) + 6*a*e^2*n) - a*b^2*c*e*(3*c*d^2 - a*e^2*(1 + 2*n)))*x^n))/(2*a^2*c^2*(b^2 - 4*a*c)^2*n^2*(a + b*x^n + c*x^(2*n))) + (e^2*(b*c*(2*a*e*(2 - 5*n) + 3*sqrt[b^2 - 4*a*c]*d*(1 - n)) - 2*a*c*(6*c*d*(1 - 2*n) + sqrt[b^2 - 4*a*c]*e*(1 - n)) - b^3*e*(1 - n) + b^2*(3*c*d - sqrt[b^2 - 4*a*c]*e)*(1 - n))*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2*c*x^n)/(b - sqrt[b^2 - 4*a*c])])/(a*c*(b^2 - 4*a*c)*(b^2 - 4*a*c - b*sqrt[b^2 - 4*a*c])*n) + (((1 - n)*(4*a^2*c^2*e*(3*c*d^2 - a*e^2)*(1 - 3*n) - 2*a*b^4*e^3*n - 2*a*b*c^2*d*(c*d^2*(2 - 7*n) + 3*a*e^2*n) + b^3*c*d*(c*d^2*(1 - 2*n) + 6*a*e^2*n) - a*b^2*c*e*(3*c*d^2 - a*e^2*(1 + 2*n))) - (2*a*b^5*e^3*(1 - n)*n - b^4*c*d*(1 - n)*(c*d^2*(1 - 2*n) + 6*a*e^2*n) - 8*a^2*c^3*d*(c*d^2 - 3*a*e^2)*(1 - 6*n + 8*n^2) + 6*a*b^2*c^2*d*(c*d^2*(1 - 4*n + 3*n^2) - a*e^2*(1 - 10*n + 15*n^2)) - 4*a^2*b*c^2*e*(3*c*d^2*(1 - n - 3*n^2) + a*e^2*(1 - 11*n + 19*n^2)) + a*b^3*c*e*(3*c*d^2*(1 - n) + a*e^2*(1 - 19*n + 30*n^2)))/sqrt[b^2 - 4*a*c])*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2*c*x^n)/(b + sqrt[b^2 - 4*a*c])])/(2*a^2*c*(b^2 - 4*a*c)^2*(b - sqrt[b^2 - 4*a*c])*n^2) + (e^2*(b*c*(2*a*e*(2 - 5*n) - 3*sqrt[b^2 - 4*a*c]*d*(1 - n)) - 2*a*c*(6*c*d*(1 - 2*n) - sqrt[b^2 - 4*a*c]*e*(1 - n)) - b^3*e*(1 - n) + b^2*(3*c*d + sqrt[b^2 - 4*a*c]*e)*(1 - n))*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2*c*x^n)/(b + sqrt[b^2 - 4*a*c])])/(a*c*(b^2 - 4*a*c)*(b^2 - 4*a*c + b*sqrt[b^2 - 4*a*c])*n) + (((1 - n)*(4*a^2*c^2*e*(3*c*d^2 - a*e^2)*(1 - 3*n) - 2*a*b^4*e^3*n - 2*a*b*c^2*d*(c*d^2*(2 - 7*n) + 3*a*e^2*n) + b^3*c*d*(c*d^2*(1 - 2*n) + 6*a*e^2*n) - a*b^2*c*e*(3*c*d^2 - a*e^2*(1 + 2*n))) + (2*a*b^5*e^3*(1 - n)*n - b^4*c*d*(1 - n)*(c*d^2*(1 - 2*n) + 6*a*e^2*n) - 8*a^2*c^3*d*(c*d^2 - 3*a*e^2)*(1 - 6*n + 8*n^2) + 6*a*b^2*c^2*d*(c*d^2*(1 - 4*n + 3*n^2) - a*e^2*(1 - 10*n + 15*n^2)) - 4*a^2*b*c^2*e*(3*c*d^2*(1 - n - 3*n^2) + a*e^2*(1 - 11*n + 19*n^2)) + a*b^3*c*e*(3*c*d^2*(1 - n) + a*e^2*(1 - 19*n + 30*n^2)))/sqrt[b^2 - 4*a*c])*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2*c*x^n)/(b + sqrt[b^2 - 4*a*c])])/(2*a^2*c*(b^2 - 4*a*c)^2*(b + sqrt[b^2 - 4*
```

$a*c]) * n^2)$

**Rubi [A]** time = 5.25273, antiderivative size = 1707, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 4, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$ , Rules used = {1436, 1430, 1422, 245}

result too large to display

Antiderivative was successfully verified.

[In] Int[(d + e\*x^n)^3/(a + b\*x^n + c\*x^(2\*n))^3,x]

[Out]  $(x*(b^2*c*d^3 - 2*a*c*d*(c*d^2 - 3*a*e^2) - a*b*e*(3*c*d^2 + a*e^2) - (a*b^2*e^3 + 2*a*c*e*(3*c*d^2 - a*e^2) - b*c*d*(c*d^2 + 3*a*e^2))*x^n)/(2*a*c*(b^2 - 4*a*c)*n*(a + b*x^n + c*x^(2*n))^2 + (e^2*x*(3*b^2*c*d - 6*a*c^2*d - b^3*e + a*b*c*e + c*(3*b*c*d - b^2*e - 2*a*c*e)*x^n)/(a*c^2*(b^2 - 4*a*c)*n*(a + b*x^n + c*x^(2*n))) - (x*(a*b^2*c^2*d*(3*a*e^2*(1 - 9*n) - 5*c*d^2*(1 - 3*n)) + 4*a^2*c^3*d*(c*d^2 - 3*a*e^2)*(1 - 4*n) - 2*a*b^5*e^3*n + 2*a^2*b*c^2*e*(3*c*d^2*(2 - 3*n) - 5*a*e^2*n) - 3*a*b^3*c*e*(c*d^2 - 3*a*e^2*n) + b^4*c*d*(c*d^2*(1 - 2*n) + 6*a*e^2*n) + c*(4*a^2*c^2*e*(3*c*d^2 - a*e^2)*(1 - 3*n) - 2*a*b^4*e^3*n - 2*a*b*c^2*d*(c*d^2*(2 - 7*n) + 3*a*e^2*n) + b^3*c*d*(c*d^2*(1 - 2*n) + 6*a*e^2*n) - a*b^2*c*e*(3*c*d^2 - a*e^2*(1 + 2*n))*x^n)/(2*a^2*c^2*(b^2 - 4*a*c)^2*n^2*(a + b*x^n + c*x^(2*n))) + (e^2*(b*c*(2*a*e*(2 - 5*n) + 3*sqrt[b^2 - 4*a*c]*d*(1 - n)) - 2*a*c*(6*c*d*(1 - 2*n) + sqrt[b^2 - 4*a*c]*e*(1 - n)) - b^3*e*(1 - n) + b^2*(3*c*d - sqrt[b^2 - 4*a*c]*e)*(1 - n))*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2*c*x^n)/(b - sqrt[b^2 - 4*a*c])]/(a*c*(b^2 - 4*a*c)*(b^2 - 4*a*c - b*sqrt[b^2 - 4*a*c])*n) + (((1 - n)*(4*a^2*c^2*e*(3*c*d^2 - a*e^2)*(1 - 3*n) - 2*a*b^4*e^3*n - 2*a*b*c^2*d*(c*d^2*(2 - 7*n) + 3*a*e^2*n) + b^3*c*d*(c*d^2*(1 - 2*n) + 6*a*e^2*n) - a*b^2*c*e*(3*c*d^2 - a*e^2*(1 + 2*n))) - (2*a*b^5*e^3*(1 - n)*n - b^4*c*d*(1 - n)*(c*d^2*(1 - 2*n) + 6*a*e^2*n) - 8*a^2*c^3*d*(c*d^2 - 3*a*e^2)*(1 - 6*n + 8*n^2) + 6*a*b^2*c^2*d*(c*d^2*(1 - 4*n + 3*n^2) - a*e^2*(1 - 10*n + 15*n^2)) - 4*a^2*b*c^2*e*(3*c*d^2*(1 - n - 3*n^2) + a*e^2*(1 - 11*n + 19*n^2)) + a*b^3*c*e*(3*c*d^2*(1 - n) + a*e^2*(1 - 19*n + 30*n^2)))/sqrt[b^2 - 4*a*c]*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2*c*x^n)/(b - sqrt[b^2 - 4*a*c])]/(2*a^2*c*(b^2 - 4*a*c)^2*(b - sqrt[b^2 - 4*a*c])*n^2) + (e^2*(b*c*(2*a*e*(2 - 5*n) - 3*sqrt[b^2 - 4*a*c]*d*(1 - n)) - 2*a*c*(6*c*d*(1 - 2*n) - sqrt[b^2 - 4*a*c]*e*(1 - n)) - b^3*e*(1 - n) + b^2*(3*c*d + sqrt[b^2 - 4*a*c]*e)*(1 - n))*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2*c*x^n)/(b + sqrt[b^2 - 4*a*c])]/(a*c*(b^2 - 4*a*c)*(b^2 - 4*a*c + b*sqrt[b^2 - 4*a*c])*n) + (((1 - n)*(4*a^2*c^2*e*(3*c*d^2 - a*e^2)*(1 - 3*n) - 2*a*b^4*e^3*n - 2*a*b*c^2*d*(c*d^2*(2 - 7*n) + 3*a*e^2*n) + b^3*c*d*(c*d^2*(1 - 2*n) + 6*a*e^2*n) - a*b^2*c*e*(3*c*d^2 - a*e^2*(1 + 2*n))) - (2*a*b^5*e^3*(1 - n)*n - b^4*c*d*(1 - n)*(c*d^2*(1 - 2*n) + 6*a*e^2*n) - 8*a^2*c^3*d*(c*d^2 - 3*a*e^2)*(1 - 6*n + 8*n^2) + 6*a*b^2*c^2*d*(c*d^2*(1 - 4*n + 3*n^2) - a*e^2*(1 - 10*n + 15*n^2)) - 4*a^2*b*c^2*e*(3*c*d^2*(1 - n - 3*n^2) + a*e^2*(1 - 11*n + 19*n^2)) + a*b^3*c*e*(3*c*d^2*(1 - n) + a*e^2*(1 - 19*n + 30*n^2)))/sqrt[b^2 - 4*a*c]*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2*c*x^n)/(b + sqrt[b^2 - 4*a*c])]/(2*a^2*c*(b^2 - 4*a*c)^2*(b + sqrt[b^2 - 4*a*c])*n^2)$

```

- 2*n) + 6*a*e^2*n) - a*b^2*c*e*(3*c*d^2 - a*e^2*(1 + 2*n))) + (2*a*b^5*e^3
*(1 - n)*n - b^4*c*d*(1 - n)*(c*d^2*(1 - 2*n) + 6*a*e^2*n) - 8*a^2*c^3*d*(c
*d^2 - 3*a*e^2)*(1 - 6*n + 8*n^2) + 6*a*b^2*c^2*d*(c*d^2*(1 - 4*n + 3*n^2)
- a*e^2*(1 - 10*n + 15*n^2)) - 4*a^2*b*c^2*e*(3*c*d^2*(1 - n - 3*n^2) + a*e
^2*(1 - 11*n + 19*n^2)) + a*b^3*c*e*(3*c*d^2*(1 - n) + a*e^2*(1 - 19*n + 30
*n^2)))/Sqrt[b^2 - 4*a*c]*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2*c
*x^n)/(b + Sqrt[b^2 - 4*a*c])]/(2*a^2*c*(b^2 - 4*a*c)^2*(b + Sqrt[b^2 - 4*
a*c])*n^2)

```

### Rule 1436

```

Int[((d_) + (e_)*(x_)^(n_))^(q_)*((a_) + (b_)*(x_)^(n_) + (c_)*(x_)^(n2_))
^(p_), x_Symbol] := Int[ExpandIntegrand[(d + e*x^n)^q*(a + b*x^n + c*x^(2
*n))^p, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && ((IntegersQ[p, q] &&
!IntegerQ[n]) || IGtQ[p, 0] || (IGtQ[q, 0] && !IntegerQ[n]))

```

### Rule 1430

```

Int[((d_) + (e_)*(x_)^(n_))*((a_) + (b_)*(x_)^(n_) + (c_)*(x_)^(n2_))^(p
_), x_Symbol] := -Simp[(x*(d*b^2 - a*b*e - 2*a*c*d + (b*d - 2*a*e)*c*x^n)*(
a + b*x^n + c*x^(2*n))^(p + 1))/(a*n*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(a
*n*(p + 1)*(b^2 - 4*a*c)), Int[Simp[(n*p + n + 1)*d*b^2 - a*b*e - 2*a*c*d*(
2*n*p + 2*n + 1) + (2*n*p + 3*n + 1)*(d*b - 2*a*e)*c*x^n, x]*(a + b*x^n + c
*x^(2*n))^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] &
& NeQ[b^2 - 4*a*c, 0] && ILtQ[p, -1]

```

### Rule 1422

```

Int[((d_) + (e_)*(x_)^(n_))/((a_) + (b_)*(x_)^(n_) + (c_)*(x_)^(n2_)), x
_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] || !IGtQ[n/2, 0])

```

### Rule 245

```

Int[((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])

```

### Rubi steps

$$\begin{aligned}
\int \frac{(d + ex^n)^3}{(a + bx^n + cx^{2n})^3} dx &= \int \left( \frac{c^2 d^3 - 3acde^2 + abe^3 + (3c^2 d^2 e - 3bcde^2 + b^2 e^3 - ace^3) x^n}{c^2 (a + bx^n + cx^{2n})^3} + \frac{e^2 (3cd - be + cex^n)}{c^2 (a + bx^n + cx^{2n})^2} \right) dx \\
&= \frac{\int \frac{c^2 d^3 - 3acde^2 + abe^3 + (3c^2 d^2 e - 3bcde^2 + b^2 e^3 - ace^3) x^n}{(a + bx^n + cx^{2n})^3} dx}{c^2} + \frac{e^2 \int \frac{3cd - be + cex^n}{(a + bx^n + cx^{2n})^2} dx}{c^2} \\
&= \frac{x (b^2 cd^3 - 2acd (cd^2 - 3ae^2) - abe (3cd^2 + ae^2) - (ab^2 e^3 + 2ace (3cd^2 - ae^2) - bcd (cd^2 + 3ae^2))}{2ac (b^2 - 4ac) n (a + bx^n + cx^{2n})^2} \\
&= \frac{x (b^2 cd^3 - 2acd (cd^2 - 3ae^2) - abe (3cd^2 + ae^2) - (ab^2 e^3 + 2ace (3cd^2 - ae^2) - bcd (cd^2 + 3ae^2))}{2ac (b^2 - 4ac) n (a + bx^n + cx^{2n})^2} \\
&= \frac{x (b^2 cd^3 - 2acd (cd^2 - 3ae^2) - abe (3cd^2 + ae^2) - (ab^2 e^3 + 2ace (3cd^2 - ae^2) - bcd (cd^2 + 3ae^2))}{2ac (b^2 - 4ac) n (a + bx^n + cx^{2n})^2} \\
&= \frac{x (b^2 cd^3 - 2acd (cd^2 - 3ae^2) - abe (3cd^2 + ae^2) - (ab^2 e^3 + 2ace (3cd^2 - ae^2) - bcd (cd^2 + 3ae^2))}{2ac (b^2 - 4ac) n (a + bx^n + cx^{2n})^2}
\end{aligned}$$

**Mathematica [B]** time = 7.75207, size = 9060, normalized size = 5.31

Result too large to show

Antiderivative was successfully verified.

[In] Integrate[(d + e\*x^n)^3/(a + b\*x^n + c\*x^(2\*n))^3,x]

[Out] Result too large to show

**Maple [F]** time = 0.104, size = 0, normalized size = 0.

$$\int \frac{(d + ex^n)^3}{(a + bx^n + cx^{2n})^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d+e\*x^n)^3/(a+b\*x^n+c\*x^(2\*n))^3,x)

[Out] int((d+e\*x^n)^3/(a+b\*x^n+c\*x^(2\*n))^3,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e\*x^n)^3/(a+b\*x^n+c\*x^(2\*n))^3,x, algorithm="maxima")

[Out] 
$$\frac{1}{2} \left( (b^3 c^2 d^3 (2n-1) + 4a^3 c^2 e^3 (n+1) + (12c^3 d^2 e (3n-1) + b^2 c e^3 (2n-1) - 18b^2 c^2 d^2 e) a) x^{3n} + (2b^4 c d^3 (2n-1) + 2(b^2 c e^3 (3n+2) + 6c^2 d e^2) a^3 - (3b^2 c d e^2 (9n+1) - 6b^2 c^2 d^2 e (9n-4) - 4c^3 d^3 (4n-1) - b^3 e^3 (3n-1)) a^2 - (b^2 c^2 d^3 (29n-9) - 6b^3 c d^2 e) a) x^{2n} + (b^5 d^3 (2n-1) - 4a^4 c e^3 (n-1) + (b^2 e^3 (10n-1) + 12c^2 d^2 e (5n-1) - 6b^2 c d e^2 (5n-2)) a^3 + (3b^2 c d^2 e (4n-3) - 3b^3 d e^2 (2n+1) - 2b^2 c^2 d^3 n) a^2 - (4b^3 c d^3 (3n-1) - 3b^4 d^2 e) a) x^n + (a^5 b^4 d^3 (3n-1) - 6(2c d e^2 (2n-1) - b e^3 n) a^4 + (4c^2 d^3 (6n-1) + 6b^2 c d^2 e (5n-2) - 3b^2 d e^2 (n+1)) a^3 - (b^2 c d^3 (21n-5) + 3b^3 d^2 e (n-1)) a^2) x \right) / (a^4 b^4 n^2 - 8a^5 b^2 c n^2 + 16a^6 c^2 n^2 + (a^2 b^4 c^2 n^2 - 8a^3 b^2 c^3 n^2 + 16a^4 c^4 n^2) x^{4n} + 2(a^2 b^5 c n^2 - 8a^3 b^3 c^2 n^2 + 16a^4 b^2 c^3 n^2) x^{3n} + (a^2 b^6 n^2 - 6a^3 b^4 c n^2 + 32a^5 c^3 n^2) x^{2n} + 2(a^3 b^5 n^2 - 8a^4 b^3 c n^2 + 16a^5 b^2 c^2 n^2) x^n) + \int \frac{1}{2} \left( (2n^2 - 3n + 1) b^4 d^3 + 6(2c d e^2 (2n-1) - b e^3 n) a^3 + (4(8n^2 - 6n + 1) c^2 d^3 - 6b^2 c d^2 e (5n-2) + 3b^2 d e^2 (n+1)) a^2 - ((16n^2 - 21n + 5) b^2 c d^3 - 3b^3 d^2 e (n-1)) a + ((2n^2 - 3n + 1) b^3 c d^3 + 4(n^2 - 1) a^3 c e^3 + (12(3n^2 - 4n + 1) c^2 d^2 e - 18(n^2 - n) b^2 c d e^2 + (2n^2 - 3n + 1) b^2 e^3) a^2 - (2(7n^2 - 9n + 2) b^2 c^2 d^3 - 3b^2 c d^2 e (n-1)) a) x^n \right) / (a^3 b^4 n^2 - 8a^4 b^2 c n^2 + 16a^5 c^2 n^2 + (a^2 b^4 c n^2 - 8a^3 b^2 c^2 n^2 + 16a^4 c^3 n^2) x^{2n} + (a^2 b^5 n^2 - 8a^3 b^3 c n^2 + 16a^4 b^2 c^2 n^2) x^n), x$$

**Fricas [F]** time = 0., size = 0, normalized size = 0.

integral  $\left( \frac{e^3 x^{3n} + 3 d e^2 x^{2n} + 3 d^2 e x^n + d^3}{c^3 x^{6n} + b^3 x^{3n} + 3 a b^2 x^{2n} + 3 a^2 b x^n + a^3 + 3 (b c^2 x^n + a c^2) x^{4n} + 3 (b^2 c x^{2n} + 2 a b c x^n + a^2 c) x^{2n}}, x \right)$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x^n)^3/(a+b*x^n+c*x^(2*n))^3,x, algorithm="fricas")
```

```
[Out] integral((e^3*x^(3*n) + 3*d*e^2*x^(2*n) + 3*d^2*e*x^n + d^3)/(c^3*x^(6*n) +
  b^3*x^(3*n) + 3*a*b^2*x^(2*n) + 3*a^2*b*x^n + a^3 + 3*(b*c^2*x^n + a*c^2)*
  x^(4*n) + 3*(b^2*c*x^(2*n) + 2*a*b*c*x^n + a^2*c)*x^(2*n)), x)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x**n)**3/(a+b*x**n+c*x**(2*n))**3,x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^n + d)^3}{(cx^{2n} + bx^n + a)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x^n)^3/(a+b*x^n+c*x^(2*n))^3,x, algorithm="giac")
```

```
[Out] integrate((e*x^n + d)^3/(c*x^(2*n) + b*x^n + a)^3, x)
```

$$3.81 \quad \int \frac{(d+ex^n)^2}{(a+bx^n+cx^{2n})^3} dx$$

**Optimal.** Leaf size=1191

result too large to display

```
[Out] (x*(b^2*d^2 - 2*a*b*d*e - 2*a*(c*d^2 - a*e^2) + (b*c*d^2 - 4*a*c*d*e + a*b*
e^2)*x^n))/(2*a*(b^2 - 4*a*c)*n*(a + b*x^n + c*x^(2*n))^2) + (e^2*x*(b^2 -
2*a*c + b*c*x^n))/(a*c*(b^2 - 4*a*c)*n*(a + b*x^n + c*x^(2*n))) + (x*(2*a*b
^3*c*d*e - a*b^2*c*(a*e^2*(1 - 9*n) - 5*c*d^2*(1 - 3*n)) - 4*a^2*c^2*(c*d^2
- a*e^2)*(1 - 4*n) - 4*a^2*b*c^2*d*e*(2 - 3*n) - b^4*(c*d^2*(1 - 2*n) + 2*
a*e^2*n) + c*(2*a*b^2*c*d*e - 8*a^2*c^2*d*e*(1 - 3*n) + 2*a*b*c*(c*d^2*(2 -
7*n) + a*e^2*n) - b^3*(c*d^2*(1 - 2*n) + 2*a*e^2*n))*x^n)/(2*a^2*c*(b^2 -
4*a*c)^2*n^2*(a + b*x^n + c*x^(2*n))) - (e^2*(4*a*c*(1 - 2*n) - b^2*(1 - n
) - b*Sqrt[b^2 - 4*a*c]*(1 - n))*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1),
(-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])])/(a*(b^2 - 4*a*c)*(b^2 - 4*a*c - b*Sqr
t[b^2 - 4*a*c])*n) - (((1 - n)*(2*a*b^2*c*d*e - 8*a^2*c^2*d*e*(1 - 3*n) + 2
*a*b*c*(c*d^2*(2 - 7*n) + a*e^2*n) - b^3*(c*d^2*(1 - 2*n) + 2*a*e^2*n)) + (
2*a*b^3*c*d*e*(1 - n) - b^4*(1 - n)*(c*d^2*(1 - 2*n) + 2*a*e^2*n) - 8*a^2*b
*c^2*d*e*(1 - n - 3*n^2) - 8*a^2*c^2*(c*d^2 - a*e^2)*(1 - 6*n + 8*n^2) + 2*
a*b^2*c*(3*c*d^2*(1 - 4*n + 3*n^2) - a*e^2*(1 - 10*n + 15*n^2)))/Sqrt[b^2 -
4*a*c])*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2*c*x^n)/(b - Sqrt[b^
2 - 4*a*c])])/(2*a^2*(b^2 - 4*a*c)^2*(b - Sqrt[b^2 - 4*a*c])*n^2) - (e^2*(4
*a*c*(1 - 2*n) - b^2*(1 - n) + b*Sqrt[b^2 - 4*a*c]*(1 - n))*x*Hypergeometri
c2F1[1, n^(-1), 1 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(a*(b^2 -
4*a*c)*(b^2 - 4*a*c + b*Sqrt[b^2 - 4*a*c])*n) - (((1 - n)*(2*a*b^2*c*d*e -
8*a^2*c^2*d*e*(1 - 3*n) + 2*a*b*c*(c*d^2*(2 - 7*n) + a*e^2*n) - b^3*(c*d^2*
(1 - 2*n) + 2*a*e^2*n)) - (2*a*b^3*c*d*e*(1 - n) - b^4*(1 - n)*(c*d^2*(1 -
2*n) + 2*a*e^2*n) - 8*a^2*b*c^2*d*e*(1 - n - 3*n^2) - 8*a^2*c^2*(c*d^2 - a*
e^2)*(1 - 6*n + 8*n^2) + 2*a*b^2*c*(3*c*d^2*(1 - 4*n + 3*n^2) - a*e^2*(1 -
10*n + 15*n^2)))/Sqrt[b^2 - 4*a*c])*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-
1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(2*a^2*(b^2 - 4*a*c)^2*(b + Sqrt[b
^2 - 4*a*c])*n^2)
```

**Rubi [A]** time = 4.00994, antiderivative size = 1191, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 5, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$ , Rules used = {1436, 1430, 1422, 245, 1345}

$$\frac{\left(-(-1-n)b^2 - \sqrt{b^2 - 4ac}(1-n)b + 4ac(1-2n)\right) x {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{2cx^n}{b - \sqrt{b^2 - 4ac}}\right) e^2}{a(b^2 - 4ac)\left(b^2 - \sqrt{b^2 - 4ac}b - 4ac\right)n} - \frac{\left(-(-1-n)b^2 + \sqrt{b^2 - 4ac}(1-n)b + 4ac(1-2n)\right) x {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{2cx^n}{b + \sqrt{b^2 - 4ac}}\right) e^2}{a(b^2 - 4ac)\left(b^2 + \sqrt{b^2 - 4ac}b - 4ac\right)n}$$



Antiderivative was successfully verified.

[In] Int[(d + e\*x^n)^2/(a + b\*x^n + c\*x^(2\*n))^3,x]

[Out] (x\*(b^2\*d^2 - 2\*a\*b\*d\*e - 2\*a\*(c\*d^2 - a\*e^2) + (b\*c\*d^2 - 4\*a\*c\*d\*e + a\*b\*e^2)\*x^n))/(2\*a\*(b^2 - 4\*a\*c)\*n\*(a + b\*x^n + c\*x^(2\*n))^2) + (e^2\*x\*(b^2 - 2\*a\*c + b\*c\*x^n))/(a\*c\*(b^2 - 4\*a\*c)\*n\*(a + b\*x^n + c\*x^(2\*n))) + (x\*(2\*a\*b^3\*c\*d\*e - a\*b^2\*c\*(a\*e^2\*(1 - 9\*n) - 5\*c\*d^2\*(1 - 3\*n)) - 4\*a^2\*c^2\*(c\*d^2 - a\*e^2)\*(1 - 4\*n) - 4\*a^2\*b\*c^2\*d\*e\*(2 - 3\*n) - b^4\*(c\*d^2\*(1 - 2\*n) + 2\*a\*e^2\*n) + c\*(2\*a\*b^2\*c\*d\*e - 8\*a^2\*c^2\*d\*e\*(1 - 3\*n) + 2\*a\*b\*c\*(c\*d^2\*(2 - 7\*n) + a\*e^2\*n) - b^3\*(c\*d^2\*(1 - 2\*n) + 2\*a\*e^2\*n))\*x^n)/(2\*a^2\*c\*(b^2 - 4\*a\*c)^2\*n^2\*(a + b\*x^n + c\*x^(2\*n))) - (e^2\*(4\*a\*c\*(1 - 2\*n) - b^2\*(1 - n) - b\*sqrt[b^2 - 4\*a\*c]\*(1 - n))\*x\*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2\*c\*x^n)/(b - sqrt[b^2 - 4\*a\*c])])/(a\*(b^2 - 4\*a\*c)\*(b^2 - 4\*a\*c - b\*sqrt[b^2 - 4\*a\*c])\*n) - (((1 - n)\*(2\*a\*b^2\*c\*d\*e - 8\*a^2\*c^2\*d\*e\*(1 - 3\*n) + 2\*a\*b\*c\*(c\*d^2\*(2 - 7\*n) + a\*e^2\*n) - b^3\*(c\*d^2\*(1 - 2\*n) + 2\*a\*e^2\*n)) + (2\*a\*b^3\*c\*d\*e\*(1 - n) - b^4\*(1 - n)\*(c\*d^2\*(1 - 2\*n) + 2\*a\*e^2\*n) - 8\*a^2\*b\*c^2\*d\*e\*(1 - n - 3\*n^2) - 8\*a^2\*c^2\*(c\*d^2 - a\*e^2)\*(1 - 6\*n + 8\*n^2) + 2\*a\*b^2\*c\*(3\*c\*d^2\*(1 - 4\*n + 3\*n^2) - a\*e^2\*(1 - 10\*n + 15\*n^2)))/sqrt[b^2 - 4\*a\*c])\*x\*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2\*c\*x^n)/(b - sqrt[b^2 - 4\*a\*c])])/(2\*a^2\*(b^2 - 4\*a\*c)^2\*(b - sqrt[b^2 - 4\*a\*c])\*n^2) - (e^2\*(4\*a\*c\*(1 - 2\*n) - b^2\*(1 - n) + b\*sqrt[b^2 - 4\*a\*c]\*(1 - n))\*x\*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2\*c\*x^n)/(b + sqrt[b^2 - 4\*a\*c])])/(a\*(b^2 - 4\*a\*c)\*(b^2 - 4\*a\*c + b\*sqrt[b^2 - 4\*a\*c])\*n) - (((1 - n)\*(2\*a\*b^2\*c\*d\*e - 8\*a^2\*c^2\*d\*e\*(1 - 3\*n) + 2\*a\*b\*c\*(c\*d^2\*(2 - 7\*n) + a\*e^2\*n) - b^3\*(c\*d^2\*(1 - 2\*n) + 2\*a\*e^2\*n)) - (2\*a\*b^3\*c\*d\*e\*(1 - n) - b^4\*(1 - n)\*(c\*d^2\*(1 - 2\*n) + 2\*a\*e^2\*n) - 8\*a^2\*b\*c^2\*d\*e\*(1 - n - 3\*n^2) - 8\*a^2\*c^2\*(c\*d^2 - a\*e^2)\*(1 - 6\*n + 8\*n^2) + 2\*a\*b^2\*c\*(3\*c\*d^2\*(1 - 4\*n + 3\*n^2) - a\*e^2\*(1 - 10\*n + 15\*n^2)))/sqrt[b^2 - 4\*a\*c])\*x\*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2\*c\*x^n)/(b + sqrt[b^2 - 4\*a\*c])])/(2\*a^2\*(b^2 - 4\*a\*c)^2\*(b + sqrt[b^2 - 4\*a\*c])\*n^2)

### Rule 1436

Int[((d\_) + (e\_)\*(x\_)^(n\_))^(q\_)\*((a\_) + (b\_)\*(x\_)^(n\_) + (c\_)\*(x\_)^(n2\_))^(p\_), x\_Symbol] :> Int[ExpandIntegrand[(d + e\*x^n)^q\*(a + b\*x^n + c\*x^(2\*n))^p, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && EqQ[n2, 2\*n] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && ((IntegersQ[p, q] && !IntegerQ[n]) || IGtQ[p, 0] || (IGtQ[q, 0] && !IntegerQ[n]))

### Rule 1430

Int[((d\_) + (e\_)\*(x\_)^(n\_))\*((a\_) + (b\_)\*(x\_)^(n\_) + (c\_)\*(x\_)^(n2\_))^(p\_), x\_Symbol] :> -Simp[(x\*(d\*b^2 - a\*b\*e - 2\*a\*c\*d + (b\*d - 2\*a\*e)\*c\*x^n)\*(a + b\*x^n + c\*x^(2\*n))^(p + 1))/(a\*n\*(p + 1)\*(b^2 - 4\*a\*c)), x] + Dist[1/(a

```
*n*(p + 1)*(b^2 - 4*a*c)), Int[Simp[(n*p + n + 1)*d*b^2 - a*b*e - 2*a*c*d*(
2*n*p + 2*n + 1) + (2*n*p + 3*n + 1)*(d*b - 2*a*e)*c*x^n, x]*(a + b*x^n + c
*x^(2*n))^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] &
& NeQ[b^2 - 4*a*c, 0] && ILtQ[p, -1]
```

### Rule 1422

```
Int[((d_) + (e_)*(x_)^(n_))/((a_) + (b_)*(x_)^(n_) + (c_)*(x_)^(n2_)), x
_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] || !IGtQ[n/2, 0])
```

### Rule 245

```
Int[((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -(b*x^n)/a], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

### Rule 1345

```
Int[((a_) + (c_)*(x_)^(n2_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := -Simp[(
x*(b^2 - 2*a*c + b*c*x^n)*(a + b*x^n + c*x^(2*n))^(p + 1))/(a*n*(p + 1)*(b^
2 - 4*a*c)), x] + Dist[1/(a*n*(p + 1)*(b^2 - 4*a*c)), Int[(b^2 - 2*a*c + n*
(p + 1)*(b^2 - 4*a*c) + b*c*(n*(2*p + 3) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^(
p + 1), x], x] /; FreeQ[{a, b, c, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*
c, 0] && ILtQ[p, -1]
```

### Rubi steps

$$\begin{aligned}
\int \frac{(d + ex^n)^2}{(a + bx^n + cx^{2n})^3} dx &= \int \left( \frac{cd^2 - ae^2 + (2cde - be^2)x^n}{c(a + bx^n + cx^{2n})^3} + \frac{e^2}{c(a + bx^n + cx^{2n})^2} \right) dx \\
&= \frac{\int \frac{cd^2 - ae^2 + (2cde - be^2)x^n}{(a + bx^n + cx^{2n})^3} dx}{c} + \frac{e^2 \int \frac{1}{(a + bx^n + cx^{2n})^2} dx}{c} \\
&= \frac{x(b^2d^2 - 2abde - 2a(cd^2 - ae^2) + (bcd^2 - 4acde + abe^2)x^n)}{2a(b^2 - 4ac)n(a + bx^n + cx^{2n})^2} + \frac{e^2x(b^2 - 2ac + bcx^n)}{ac(b^2 - 4ac)n(a + bx^n + cx^{2n})} \\
&= \frac{x(b^2d^2 - 2abde - 2a(cd^2 - ae^2) + (bcd^2 - 4acde + abe^2)x^n)}{2a(b^2 - 4ac)n(a + bx^n + cx^{2n})^2} + \frac{e^2x(b^2 - 2ac + bcx^n)}{ac(b^2 - 4ac)n(a + bx^n + cx^{2n})} \\
&= \frac{x(b^2d^2 - 2abde - 2a(cd^2 - ae^2) + (bcd^2 - 4acde + abe^2)x^n)}{2a(b^2 - 4ac)n(a + bx^n + cx^{2n})^2} + \frac{e^2x(b^2 - 2ac + bcx^n)}{ac(b^2 - 4ac)n(a + bx^n + cx^{2n})} \\
&= \frac{x(b^2d^2 - 2abde - 2a(cd^2 - ae^2) + (bcd^2 - 4acde + abe^2)x^n)}{2a(b^2 - 4ac)n(a + bx^n + cx^{2n})^2} + \frac{e^2x(b^2 - 2ac + bcx^n)}{ac(b^2 - 4ac)n(a + bx^n + cx^{2n})}
\end{aligned}$$

**Mathematica [B]** time = 6.83908, size = 7402, normalized size = 6.21

Result too large to show

Antiderivative was successfully verified.

[In] Integrate[(d + e\*x^n)^2/(a + b\*x^n + c\*x^(2\*n))^3,x]

[Out] Result too large to show

**Maple [F]** time = 0.1, size = 0, normalized size = 0.

$$\int \frac{(d + ex^n)^2}{(a + bx^n + cx^{2n})^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d+e*x^n)^2/(a+b*x^n+c*x^(2*n))^3,x)`

[Out] `int((d+e*x^n)^2/(a+b*x^n+c*x^(2*n))^3,x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d+e*x^n)^2/(a+b*x^n+c*x^(2*n))^3,x, algorithm="maxima")`

[Out] 
$$\frac{1}{2} \left( (b^3 c^2 d^2 (2n-1) + 2(4c^3 d e (3n-1) - 3b c^2 e^2 n) a^2 - 2(b c^3 d^2 (7n-2) - b^2 c^2 d e) a) x^{3n} + (2b^4 c d^2 (2n-1) + 4a^3 c^2 e^2 - (b^2 c e^2 (9n+1) - 4b c^2 d e (9n-4) - 4c^3 d^2 (4n-1)) a^2 - (b^2 c^2 d^2 (29n-9) - 4b^3 c d e) a) x^{2n} + (b^5 d^2 (2n-1) + 2(4c^2 d e (5n-1) - b c e^2 (5n-2)) a^3 + (2b^2 c d e (4n-3) - b^3 e^2 (2n+1) - 2b c^2 d^2 n) a^2 - 2(2b^3 c d^2 (3n-1) - b^4 d e) a) x^n + (a b^4 d^2 (3n-1) - 4a^4 c e^2 (2n-1) + (4c^2 d^2 (6n-1) + 4b c d e (5n-2) - b^2 e^2 (n+1)) a^3 - (b^2 c d^2 (21n-5) + 2b^3 d e (n-1)) a^2) x \right) / (a^4 b^4 n^2 - 8a^5 b^2 c n^2 + 16a^6 c^2 n^2 + (a^2 b^4 c^2 n^2 - 8a^3 b^2 c^3 n^2 + 16a^4 c^4 n^2) x^{4n} + 2(a^2 b^5 c n^2 - 8a^3 b^3 c^2 n^2 + 16a^4 b c^3 n^2) x^{3n} + (a^2 b^6 n^2 - 6a^3 b^4 c n^2 + 32a^5 c^3 n^2) x^{2n} + 2(a^3 b^5 n^2 - 8a^4 b^3 c n^2 + 16a^5 b c^2 n^2) x^n) - \text{integrate}(-1/2((2n^2 - 3n + 1) b^4 d^2 + 4a^3 c e^2 (2n-1) + (4(8n^2 - 6n + 1) c^2 d^2 - 4b c d e (5n-2) + b^2 e^2 (n+1)) a^2 - ((16n^2 - 21n + 5) b^2 c d^2 - 2b^3 d e (n-1)) a + ((2n^2 - 3n + 1) b^3 c d^2 + 2(4(3n^2 - 4n + 1) c^2 d e - 3(n^2 - n) b c e^2) a^2 - 2((7n^2 - 9n + 2) b c^2 d^2 - b^2 c d e (n-1)) a) x^n) / (a^3 b^4 n^2 - 8a^4 b^2 c n^2 + 16a^5 c^2 n^2 + (a^2 b^4 c n^2 - 8a^3 b^2 c^2 n^2 + 16a^4 c^3 n^2) x^{2n} + (a^2 b^5 n^2 - 8a^3 b^3 c n^2 + 16a^4 b c^2 n^2) x^n), x)$$

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{e^2 x^{2n} + 2 d e x^n + d^2}{c^3 x^{6n} + b^3 x^{3n} + 3 a b^2 x^{2n} + 3 a^2 b x^n + a^3 + 3 (b c^2 x^n + a c^2) x^{4n} + 3 (b^2 c x^{2n} + 2 a b c x^n + a^2 c) x^{2n}}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x^n)^2/(a+b*x^n+c*x^(2*n))^3,x, algorithm="fricas")
```

```
[Out] integral((e^2*x^(2*n) + 2*d*e*x^n + d^2)/(c^3*x^(6*n) + b^3*x^(3*n) + 3*a*b^2*x^(2*n) + 3*a^2*b*x^n + a^3 + 3*(b*c^2*x^n + a*c^2)*x^(4*n) + 3*(b^2*c*x^(2*n) + 2*a*b*c*x^n + a^2*c)*x^(2*n)), x)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x**n)**2/(a+b*x**n+c*x**(2*n))**3,x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^n + d)^2}{(cx^{2n} + bx^n + a)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x^n)^2/(a+b*x^n+c*x^(2*n))^3,x, algorithm="giac")
```

```
[Out] integrate((e*x^n + d)^2/(c*x^(2*n) + b*x^n + a)^3, x)
```

$$3.82 \quad \int \frac{d+ex^n}{(a+bx^n+cx^{2n})^3} dx$$

**Optimal.** Leaf size=713

$$\frac{x \left( cx^n \left( -4a^2ce(1-3n) + ab^2e + 2abcd(2-7n) + b^3(-d)(1-2n) \right) - 2a^2bce(2-3n) - 4a^2c^2d(1-4n) + 5ab^2cd(1-3n) + a \right)}{2a^2n^2 (b^2 - 4ac)^2 (a + bx^n + cx^{2n})}$$

[Out] (x\*(b^2\*d - 2\*a\*c\*d - a\*b\*e + c\*(b\*d - 2\*a\*e)\*x^n))/(2\*a\*(b^2 - 4\*a\*c)\*n\*(a + b\*x^n + c\*x^(2\*n))^2) + (x\*(a\*b^3\*e - 4\*a^2\*c^2\*d\*(1 - 4\*n) + 5\*a\*b^2\*c\*d\*(1 - 3\*n) - 2\*a^2\*b\*c\*e\*(2 - 3\*n) - b^4\*d\*(1 - 2\*n) + c\*(a\*b^2\*e + 2\*a\*b\*c\*d\*(2 - 7\*n) - 4\*a^2\*c\*e\*(1 - 3\*n) - b^3\*d\*(1 - 2\*n))\*x^n))/(2\*a^2\*(b^2 - 4\*a\*c)^2\*n^2\*(a + b\*x^n + c\*x^(2\*n))) + (c\*(a\*b^2\*(Sqrt[b^2 - 4\*a\*c]\*e + 6\*c\*d\*(1 - 3\*n))\*(1 - n) + b^3\*(a\*e - Sqrt[b^2 - 4\*a\*c]\*d\*(1 - 2\*n))\*(1 - n) - b^4\*d\*(1 - 3\*n + 2\*n^2) - 2\*a\*b\*c\*(2\*a\*e\*(1 - n - 3\*n^2) - Sqrt[b^2 - 4\*a\*c]\*d\*(2 - 9\*n + 7\*n^2)) - 4\*a^2\*c\*(Sqrt[b^2 - 4\*a\*c]\*e\*(1 - 4\*n + 3\*n^2) + 2\*c\*d\*(1 - 6\*n + 8\*n^2)))\*x\*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2\*c\*x^n)/(b - Sqrt[b^2 - 4\*a\*c])])/(2\*a^2\*(b^2 - 4\*a\*c)^2\*(b^2 - 4\*a\*c - b\*Sqrt[b^2 - 4\*a\*c])\*n^2) - (c\*(a\*b^2\*(Sqrt[b^2 - 4\*a\*c]\*e - 6\*c\*d\*(1 - 3\*n))\*(1 - n) - b^3\*(a\*e + Sqrt[b^2 - 4\*a\*c]\*d\*(1 - 2\*n))\*(1 - n) + b^4\*d\*(1 - 3\*n + 2\*n^2) + 2\*a\*b\*c\*(2\*a\*e\*(1 - n - 3\*n^2) + Sqrt[b^2 - 4\*a\*c]\*d\*(2 - 9\*n + 7\*n^2)) - 4\*a^2\*c\*(Sqrt[b^2 - 4\*a\*c]\*e\*(1 - 4\*n + 3\*n^2) - 2\*c\*d\*(1 - 6\*n + 8\*n^2)))\*x\*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2\*c\*x^n)/(b + Sqrt[b^2 - 4\*a\*c])])/(2\*a^2\*(b^2 - 4\*a\*c)^2\*(b^2 - 4\*a\*c + b\*Sqrt[b^2 - 4\*a\*c])\*n^2)

**Rubi [A]** time = 1.6635, antiderivative size = 713, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {1430, 1422, 245}

$$\frac{x \left( cx^n \left( -4a^2ce(1-3n) + ab^2e + 2abcd(2-7n) + b^3(-d)(1-2n) \right) - 2a^2bce(2-3n) - 4a^2c^2d(1-4n) + 5ab^2cd(1-3n) + a \right)}{2a^2n^2 (b^2 - 4ac)^2 (a + bx^n + cx^{2n})}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x^n)/(a + b\*x^n + c\*x^(2\*n))^3,x]

[Out] (x\*(b^2\*d - 2\*a\*c\*d - a\*b\*e + c\*(b\*d - 2\*a\*e)\*x^n))/(2\*a\*(b^2 - 4\*a\*c)\*n\*(a + b\*x^n + c\*x^(2\*n))^2) + (x\*(a\*b^3\*e - 4\*a^2\*c^2\*d\*(1 - 4\*n) + 5\*a\*b^2\*c\*

$$d*(1 - 3*n) - 2*a^2*b*c*e*(2 - 3*n) - b^4*d*(1 - 2*n) + c*(a*b^2*e + 2*a*b*c*d*(2 - 7*n) - 4*a^2*c*e*(1 - 3*n) - b^3*d*(1 - 2*n))*x^n)/(2*a^2*(b^2 - 4*a*c)^2*n^2*(a + b*x^n + c*x^(2*n))) + (c*(a*b^2*(Sqrt[b^2 - 4*a*c]*e + 6*c*d*(1 - 3*n))*(1 - n) + b^3*(a*e - Sqrt[b^2 - 4*a*c]*d*(1 - 2*n))*(1 - n) - b^4*d*(1 - 3*n + 2*n^2) - 2*a*b*c*(2*a*e*(1 - n - 3*n^2) - Sqrt[b^2 - 4*a*c]*d*(2 - 9*n + 7*n^2)) - 4*a^2*c*(Sqrt[b^2 - 4*a*c]*e*(1 - 4*n + 3*n^2) + 2*c*d*(1 - 6*n + 8*n^2)))*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])])/(2*a^2*(b^2 - 4*a*c)^2*(b^2 - 4*a*c - b*Sqrt[b^2 - 4*a*c])*n^2) - (c*(a*b^2*(Sqrt[b^2 - 4*a*c]*e - 6*c*d*(1 - 3*n))*(1 - n) - b^3*(a*e + Sqrt[b^2 - 4*a*c]*d*(1 - 2*n))*(1 - n) + b^4*d*(1 - 3*n + 2*n^2) + 2*a*b*c*(2*a*e*(1 - n - 3*n^2) + Sqrt[b^2 - 4*a*c]*d*(2 - 9*n + 7*n^2)) - 4*a^2*c*(Sqrt[b^2 - 4*a*c]*e*(1 - 4*n + 3*n^2) - 2*c*d*(1 - 6*n + 8*n^2)))*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(2*a^2*(b^2 - 4*a*c)^2*(b^2 - 4*a*c + b*Sqrt[b^2 - 4*a*c])*n^2)$$

### Rule 1430

```
Int[((d_) + (e_)*(x_)^(n_))*((a_) + (b_)*(x_)^(n_) + (c_)*(x_)^(n2_))^(p_), x_Symbol] := -Simp[(x*(d*b^2 - a*b*e - 2*a*c*d + (b*d - 2*a*e)*c*x^n)*(a + b*x^n + c*x^(2*n))^(p + 1))/(a*n*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(a*n*(p + 1)*(b^2 - 4*a*c)), Int[Simp[(n*p + n + 1)*d*b^2 - a*b*e - 2*a*c*d*(2*n*p + 2*n + 1) + (2*n*p + 3*n + 1)*(d*b - 2*a*e)*c*x^n, x]*(a + b*x^n + c*x^(2*n))^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && ILtQ[p, -1]
```

### Rule 1422

```
Int[((d_) + (e_)*(x_)^(n_))/((a_) + (b_)*(x_)^(n_) + (c_)*(x_)^(n2_)), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a*c] || !IGtQ[n/2, 0])
```

### Rule 245

```
Int[((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Simp[a^p*x*Hypergeometric2F1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] || GtQ[a, 0])
```

### Rubi steps

$$\begin{aligned}
\int \frac{d + ex^n}{(a + bx^n + cx^{2n})^3} dx &= \frac{x(b^2d - 2acd - abe + c(bd - 2ae)x^n)}{2a(b^2 - 4ac)n(a + bx^n + cx^{2n})^2} - \frac{\int \frac{-abe - 2acd(1-4n) + b^2(d-2dn) + c(bd-2ae)(1-3n)x^n}{(a+bx^n+cx^{2n})^2} dx}{2a(b^2 - 4ac)n} \\
&= \frac{x(b^2d - 2acd - abe + c(bd - 2ae)x^n)}{2a(b^2 - 4ac)n(a + bx^n + cx^{2n})^2} + \frac{x(ab^3e - 4a^2c^2d(1-4n) + 5ab^2cd(1-3n) - 2a^2bce(2 - 2a^2))}{2a^2(b^2 - 4ac)n(a + bx^n + cx^{2n})^2} \\
&= \frac{x(b^2d - 2acd - abe + c(bd - 2ae)x^n)}{2a(b^2 - 4ac)n(a + bx^n + cx^{2n})^2} + \frac{x(ab^3e - 4a^2c^2d(1-4n) + 5ab^2cd(1-3n) - 2a^2bce(2 - 2a^2))}{2a^2(b^2 - 4ac)n(a + bx^n + cx^{2n})^2} \\
&= \frac{x(b^2d - 2acd - abe + c(bd - 2ae)x^n)}{2a(b^2 - 4ac)n(a + bx^n + cx^{2n})^2} + \frac{x(ab^3e - 4a^2c^2d(1-4n) + 5ab^2cd(1-3n) - 2a^2bce(2 - 2a^2))}{2a^2(b^2 - 4ac)n(a + bx^n + cx^{2n})^2}
\end{aligned}$$

**Mathematica [B]** time = 6.53474, size = 5848, normalized size = 8.2

Result too large to show

Antiderivative was successfully verified.

[In] Integrate[(d + e\*x^n)/(a + b\*x^n + c\*x^(2\*n))^3, x]

[Out] Result too large to show

**Maple [F]** time = 0.094, size = 0, normalized size = 0.

$$\int \frac{d + ex^n}{(a + bx^n + cx^{2n})^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d+e\*x^n)/(a+b\*x^n+c\*x^(2\*n))^3, x)

[Out] int((d+e\*x^n)/(a+b\*x^n+c\*x^(2\*n))^3, x)



**Maxima [F]** time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e\*x^n)/(a+b\*x^n+c\*x^(2\*n))^3,x, algorithm="maxima")

[Out] 
$$\frac{1}{2} \left( (4a^2c^3e(3n-1) + b^3c^2d(2n-1) - (2b^3c^3d(7n-2) - b^2c^2e)a) x^{3n} + (2b^4cd(2n-1) + 2(b^2c^2e(9n-4) + 2c^3d(4n-1))a^2 - (b^2c^2d(29n-9) - 2b^3c^3e)a) x^{2n} + (4a^3c^2e(5n-1) + b^5d(2n-1) + (b^2c^2e(4n-3) - 2b^3c^2d)n)a^2 - (4b^3cd(3n-1) - b^4e)a) x^n + (ab^4d(3n-1) + 2(2c^2d(6n-1) + b^3c^2e(5n-2))a^3 - (b^2cd(21n-5) + b^3e(n-1))a^2)x \right) / (a^4b^4n^2 - 8a^5b^2cn^2 + 16a^6c^2n^2 + (a^2b^4c^2n^2 - 8a^3b^2c^3n^2 + 16a^4c^4n^2)x^{4n} + 2(a^2b^5cn^2 - 8a^3b^3c^2n^2 + 16a^4b^3cn^2)x^{3n} + (a^2b^6n^2 - 6a^3b^4cn^2 + 32a^5c^3n^2)x^{2n} + 2(a^3b^5n^2 - 8a^4b^3cn^2 + 16a^5b^3c^2n^2)x^n) + \int \frac{1}{2} \left( (2n^2 - 3n + 1)b^4d + 2(2(8n^2 - 6n + 1)c^2d - b^3c^2e(5n-2))a^2 - ((16n^2 - 21n + 5)b^2cd - b^3e(n-1))a + ((2n^2 - 3n + 1)b^3cd + 4(3n^2 - 4n + 1)a^2c^2e - (2(7n^2 - 9n + 2)b^2cd - b^2c^2e(n-1))a) x \right) / (a^3b^4n^2 - 8a^4b^2cn^2 + 16a^5c^2n^2 + (a^2b^4cn^2 - 8a^3b^2c^2n^2 + 16a^4c^3n^2)x^{2n} + (a^2b^5n^2 - 8a^3b^3cn^2 + 16a^4b^3c^2n^2)x^n), x$$

**Fricas [F]** time = 0., size = 0, normalized size = 0.

integral  $\left( \frac{ex^n + d}{c^3x^{6n} + b^3x^{3n} + 3ab^2x^{2n} + 3a^2bx^n + a^3 + 3(bc^2x^n + ac^2)x^{4n} + 3(b^2cx^{2n} + 2abcx^n + a^2c)x^{2n}}, x \right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e\*x^n)/(a+b\*x^n+c\*x^(2\*n))^3,x, algorithm="fricas")

[Out] integral((e\*x^n + d)/(c^3\*x^(6\*n) + b^3\*x^(3\*n) + 3\*a\*b^2\*x^(2\*n) + 3\*a^2\*b\*x^n + a^3 + 3\*(b\*c^2\*x^n + a\*c^2)\*x^(4\*n) + 3\*(b^2\*c\*x^(2\*n) + 2\*a\*b\*c\*x^n + a^2\*c)\*x^(2\*n)), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e\*x\*\*n)/(a+b\*x\*\*n+c\*x\*\*(2\*n))\*\*3,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{ex^n + d}{(cx^{2n} + bx^n + a)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e\*x^n)/(a+b\*x^n+c\*x^(2\*n))^3,x, algorithm="giac")

[Out] integrate((e\*x^n + d)/(c\*x^(2\*n) + b\*x^n + a)^3, x)

$$3.83 \quad \int \frac{1}{(d+ex^n)(a+bx^n+cx^{2n})^3} dx$$

Optimal. Leaf size=1708

result too large to display

```
[Out] (x*(b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e + c*(b*c*d - b^2*e + 2*a*c*e)*x^n)/(2*a*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)*n*(a + b*x^n + c*x^(2*n))^2) + (e^2*x*(b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e + c*(b*c*d - b^2*e + 2*a*c*e)*x^n)/(a*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)^2*n*(a + b*x^n + c*x^(2*n))) + (x*(2*a^2*b*c^2*e*(4 - 11*n) - 3*a*b^3*c*e*(2 - 5*n) - 4*a^2*c^3*d*(1 - 4*n) + 5*a*b^2*c^2*d*(1 - 3*n) - b^4*c*d*(1 - 2*n) + b^5*(e - 2*e*n) - c*(a*b^2*c*e*(5 - 14*n) - 2*a*b*c^2*d*(2 - 7*n) - 4*a^2*c^2*e*(1 - 3*n) + b^3*c*d*(1 - 2*n) - b^4*e*(1 - 2*n))*x^n)/(2*a^2*(b^2 - 4*a*c)^2*(c*d^2 - b*d*e + a*e^2)*n^2*(a + b*x^n + c*x^(2*n))) - (c*e^4*(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])])/(b^2 - 4*a*c - b*Sqrt[b^2 - 4*a*c])*(c*d^2 - b*d*e + a*e^2)^3 + (c*e^2*(b*c*(2*a*e*(2 - 3*n) + Sqrt[b^2 - 4*a*c]*d*(1 - n)) - 2*a*c*(2*c*d*(1 - 2*n) - Sqrt[b^2 - 4*a*c]*e*(1 - n)) - b^3*e*(1 - n) + b^2*(c*d - Sqrt[b^2 - 4*a*c])*e*(1 - n))*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])])/(a*(b^2 - 4*a*c)*(b^2 - 4*a*c - b*Sqrt[b^2 - 4*a*c])*(c*d^2 - b*d*e + a*e^2)^2*n) - (c*(a*b^2*c*(Sqrt[b^2 - 4*a*c])*e*(5 - 14*n) - 6*c*d*(1 - 3*n))*(1 - n) + b^3*c*(a*e*(7 - 18*n) + Sqrt[b^2 - 4*a*c]*d*(1 - 2*n))*(1 - n) - b^5*e*(1 - 3*n + 2*n^2) + b^4*(c*d - Sqrt[b^2 - 4*a*c])*e*(1 - 3*n + 2*n^2) - 4*a^2*c^2*(Sqrt[b^2 - 4*a*c])*e*(1 - 4*n + 3*n^2) - 2*c*d*(1 - 6*n + 8*n^2)) - 2*a*b*c^2*(Sqrt[b^2 - 4*a*c]*d*(2 - 9*n + 7*n^2) + 2*a*e*(3 - 13*n + 13*n^2))*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])])/(2*a^2*(b^2 - 4*a*c)^2*(b^2 - 4*a*c - b*Sqrt[b^2 - 4*a*c])*(c*d^2 - b*d*e + a*e^2)*n^2) - (c*e^4*(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e)*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(b^2 - 4*a*c + b*Sqrt[b^2 - 4*a*c])*(c*d^2 - b*d*e + a*e^2)^3 + (c*e^2*(b*c*(2*a*e*(2 - 3*n) - Sqrt[b^2 - 4*a*c]*d*(1 - n)) - 2*a*c*(2*c*d*(1 - 2*n) + Sqrt[b^2 - 4*a*c])*e*(1 - n)) - b^3*e*(1 - n) + b^2*(c*d + Sqrt[b^2 - 4*a*c])*e*(1 - n))*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(a*(b^2 - 4*a*c)*(b^2 - 4*a*c + b*Sqrt[b^2 - 4*a*c])*(c*d^2 - b*d*e + a*e^2)^2*n) + (c*(a*b^2*c*(Sqrt[b^2 - 4*a*c])*e*(5 - 14*n) + 6*c*d*(1 - 3*n))*(1 - n) - b^3*c*(a*e*(7 - 18*n) - Sqrt[b^2 - 4*a*c]*d*(1 - 2*n))*(1 - n) + b^5*e*(1 - 3*n + 2*n^2) - b^4*(c*d + Sqrt[b^2 - 4*a*c])*e*(1 - 3*n + 2*n^2) - 4*a^2*c^2*(Sqrt[b^2 - 4*a*c])*e*(1 - 4*n + 3*n^2) + 2*c*d*(1 - 6*n + 8*n^2)) - 2*a*b*c^2*(Sqrt[b^2 - 4*a*c]*d*(2 - 9*n + 7*n^2) - 2*a*e*(3 - 13*n + 13*n^2))*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(2*a^2*(b^2 - 4*a*c)^2*(b^2 - 4*a*c + b*Sqrt[b^2 - 4*a*c])*(c*d^2 - b*d*e +
```

$a*e^2*n^2) + (e^6*x*Hypergeometric2F1[1, n^{(-1)}, 1 + n^{(-1)}, -((e*x^n)/d)])/(d*(c*d^2 - b*d*e + a*e^2)^3)$

**Rubi [A]** time = 5.07442, antiderivative size = 1708, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 4, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$ , Rules used = {1436, 245, 1430, 1422}

result too large to display

Antiderivative was successfully verified.

[In] Int[1/((d + e\*x^n)\*(a + b\*x^n + c\*x^(2\*n))^3), x]

[Out]  $(x*(b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e + c*(b*c*d - b^2*e + 2*a*c*e)*x^n)/(2*a*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)*n*(a + b*x^n + c*x^(2*n))^2) + (e^2*x*(b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e + c*(b*c*d - b^2*e + 2*a*c*e)*x^n)/(a*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)^2*n*(a + b*x^n + c*x^(2*n))) + (x*(2*a^2*b*c^2*e*(4 - 11*n) - 3*a*b^3*c*e*(2 - 5*n) - 4*a^2*c^3*d*(1 - 4*n) + 5*a*b^2*c^2*d*(1 - 3*n) - b^4*c*d*(1 - 2*n) + b^5*(e - 2*e*n) - c*(a*b^2*c*e*(5 - 14*n) - 2*a*b*c^2*d*(2 - 7*n) - 4*a^2*c^2*e*(1 - 3*n) + b^3*c*d*(1 - 2*n) - b^4*e*(1 - 2*n))*x^n)/(2*a^2*(b^2 - 4*a*c)^2*(c*d^2 - b*d*e + a*e^2)*n^2*(a + b*x^n + c*x^(2*n))) - (c*e^4*(2*c*d - (b + Sqrt[b^2 - 4*a*c]))*e)*x*Hypergeometric2F1[1, n^{(-1)}, 1 + n^{(-1)}, (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]/((b^2 - 4*a*c - b*Sqrt[b^2 - 4*a*c])*(c*d^2 - b*d*e + a*e^2)^3) + (c*e^2*(b*c*(2*a*e*(2 - 3*n) + Sqrt[b^2 - 4*a*c]*d*(1 - n)) - 2*a*c*(2*c*d*(1 - 2*n) - Sqrt[b^2 - 4*a*c]*e*(1 - n)) - b^3*e*(1 - n) + b^2*(c*d - Sqrt[b^2 - 4*a*c]*e)*(1 - n))*x*Hypergeometric2F1[1, n^{(-1)}, 1 + n^{(-1)}, (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]/(a*(b^2 - 4*a*c)*(b^2 - 4*a*c - b*Sqrt[b^2 - 4*a*c])*(c*d^2 - b*d*e + a*e^2)^2*n) - (c*(a*b^2*c*(Sqrt[b^2 - 4*a*c]*e*(5 - 14*n) - 6*c*d*(1 - 3*n))*(1 - n) + b^3*c*(a*e*(7 - 18*n) + Sqrt[b^2 - 4*a*c]*d*(1 - 2*n))*(1 - n) - b^5*e*(1 - 3*n + 2*n^2) + b^4*(c*d - Sqrt[b^2 - 4*a*c]*e)*(1 - 3*n + 2*n^2) - 4*a^2*c^2*(Sqrt[b^2 - 4*a*c]*e*(1 - 4*n + 3*n^2) - 2*c*d*(1 - 6*n + 8*n^2)) - 2*a*b*c^2*(Sqrt[b^2 - 4*a*c]*d*(2 - 9*n + 7*n^2) + 2*a*e*(3 - 13*n + 13*n^2)))*x*Hypergeometric2F1[1, n^{(-1)}, 1 + n^{(-1)}, (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]/(2*a^2*(b^2 - 4*a*c)^2*(b^2 - 4*a*c - b*Sqrt[b^2 - 4*a*c])*(c*d^2 - b*d*e + a*e^2)*n^2) - (c*e^4*(2*c*d - (b - Sqrt[b^2 - 4*a*c]))*e)*x*Hypergeometric2F1[1, n^{(-1)}, 1 + n^{(-1)}, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]/((b^2 - 4*a*c + b*Sqrt[b^2 - 4*a*c])*(c*d^2 - b*d*e + a*e^2)^3) + (c*e^2*(b*c*(2*a*e*(2 - 3*n) - Sqrt[b^2 - 4*a*c]*d*(1 - n)) - 2*a*c*(2*c*d*(1 - 2*n) + Sqrt[b^2 - 4*a*c]*e*(1 - n)) - b^3*e*(1 - n) + b^2*(c*d + Sqrt[b^2 - 4*a*c]*e)*(1 - n))*x*Hypergeometric2F1[1, n^{(-1)}, 1 + n^{(-1)}, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]/(a*(b^2 - 4*$

```

a*c)*(b^2 - 4*a*c + b*Sqrt[b^2 - 4*a*c])*(c*d^2 - b*d*e + a*e^2)^2*n) + (c*
(a*b^2*c*(Sqrt[b^2 - 4*a*c]*e*(5 - 14*n) + 6*c*d*(1 - 3*n))*(1 - n) - b^3*c
*(a*e*(7 - 18*n) - Sqrt[b^2 - 4*a*c]*d*(1 - 2*n))*(1 - n) + b^5*e*(1 - 3*n
+ 2*n^2) - b^4*(c*d + Sqrt[b^2 - 4*a*c]*e)*(1 - 3*n + 2*n^2) - 4*a^2*c^2*(S
qrt[b^2 - 4*a*c]*e*(1 - 4*n + 3*n^2) + 2*c*d*(1 - 6*n + 8*n^2)) - 2*a*b*c^2
*(Sqrt[b^2 - 4*a*c]*d*(2 - 9*n + 7*n^2) - 2*a*e*(3 - 13*n + 13*n^2)))*x*Hyp
ergeometric2F1[1, n^(-1), 1 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]/
(2*a^2*(b^2 - 4*a*c)^2*(b^2 - 4*a*c + b*Sqrt[b^2 - 4*a*c])*(c*d^2 - b*d*e +
a*e^2)*n^2) + (e^6*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((e*x^n)/d)
])/((d*(c*d^2 - b*d*e + a*e^2)^3)

```

### Rule 1436

```

Int[((d_) + (e_)*(x_)^(n_))^(q_)*((a_) + (b_)*(x_)^(n_) + (c_)*(x_)^(n2_
))^p, x_Symbol] := Int[ExpandIntegrand[(d + e*x^n)^q*(a + b*x^n + c*x^(2
*n))^p, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && ((IntegerQ[p, q] &&
!IntegerQ[n]) || IGtQ[p, 0] || (IGtQ[q, 0] && !IntegerQ[n]))

```

### Rule 245

```

Int[((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])

```

### Rule 1430

```

Int[((d_) + (e_)*(x_)^(n_))*((a_) + (b_)*(x_)^(n_) + (c_)*(x_)^(n2_))^(p
_), x_Symbol] := -Simp[(x*(d*b^2 - a*b*e - 2*a*c*d + (b*d - 2*a*e)*c*x^n)*(
a + b*x^n + c*x^(2*n))^(p + 1))/(a*n*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(a
*n*(p + 1)*(b^2 - 4*a*c)), Int[Simp[(n*p + n + 1)*d*b^2 - a*b*e - 2*a*c*d*(
2*n*p + 2*n + 1) + (2*n*p + 3*n + 1)*(d*b - 2*a*e)*c*x^n, x]*(a + b*x^n + c
*x^(2*n))^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] &
& NeQ[b^2 - 4*a*c, 0] && ILtQ[p, -1]

```

### Rule 1422

```

Int[((d_) + (e_)*(x_)^(n_))/((a_) + (b_)*(x_)^(n_) + (c_)*(x_)^(n2_)), x
_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] || !IGtQ[n/2, 0])

```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(d + ex^n)(a + bx^n + cx^{2n})^3} dx &= \int \left( \frac{e^6}{(cd^2 - bde + ae^2)^3 (d + ex^n)} + \frac{cd - be - cex^n}{(cd^2 - bde + ae^2)(a + bx^n + cx^{2n})^3} - \frac{e^2}{(cd^2 - bde + ae^2)} \right) dx \\
&= -\frac{e^4 \int \frac{-cd+be+cex^n}{a+bx^n+cx^{2n}} dx}{(cd^2 - bde + ae^2)^3} + \frac{e^6 \int \frac{1}{d+ex^n} dx}{(cd^2 - bde + ae^2)^3} - \frac{e^2 \int \frac{-cd+be+cex^n}{(a+bx^n+cx^{2n})^2} dx}{(cd^2 - bde + ae^2)^2} + \frac{\int \frac{cd-be-cex^n}{(a+bx^n+cx^{2n})^3} dx}{cd^2 - bde + ae^2} \\
&= \frac{x(b^2cd - 2ac^2d - b^3e + 3abce + c(bcd - b^2e + 2ace)x^n)}{2a(b^2 - 4ac)(cd^2 - bde + ae^2)n(a + bx^n + cx^{2n})^2} + \frac{e^2x(b^2cd - 2ac^2d - b^3e + 3abce + c(bcd - b^2e + 2ace)x^n)}{a(b^2 - 4ac)(cd^2 - bde + ae^2)} \\
&= \frac{x(b^2cd - 2ac^2d - b^3e + 3abce + c(bcd - b^2e + 2ace)x^n)}{2a(b^2 - 4ac)(cd^2 - bde + ae^2)n(a + bx^n + cx^{2n})^2} + \frac{e^2x(b^2cd - 2ac^2d - b^3e + 3abce + c(bcd - b^2e + 2ace)x^n)}{a(b^2 - 4ac)(cd^2 - bde + ae^2)} \\
&= \frac{x(b^2cd - 2ac^2d - b^3e + 3abce + c(bcd - b^2e + 2ace)x^n)}{2a(b^2 - 4ac)(cd^2 - bde + ae^2)n(a + bx^n + cx^{2n})^2} + \frac{e^2x(b^2cd - 2ac^2d - b^3e + 3abce + c(bcd - b^2e + 2ace)x^n)}{a(b^2 - 4ac)(cd^2 - bde + ae^2)} \\
&= \frac{x(b^2cd - 2ac^2d - b^3e + 3abce + c(bcd - b^2e + 2ace)x^n)}{2a(b^2 - 4ac)(cd^2 - bde + ae^2)n(a + bx^n + cx^{2n})^2} + \frac{e^2x(b^2cd - 2ac^2d - b^3e + 3abce + c(bcd - b^2e + 2ace)x^n)}{a(b^2 - 4ac)(cd^2 - bde + ae^2)}
\end{aligned}$$

**Mathematica [B]** time = 8.65581, size = 30258, normalized size = 17.72

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[1/((d + e\*x^n)\*(a + b\*x^n + c\*x^(2\*n)))^3, x]

[Out] Result too large to show

---

**Maple [F]** time = 0.329, size = 0, normalized size = 0.

$$\int \frac{1}{(d + ex^n)(a + bx^n + cx^{2n})^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}(1/(d+e*x^n)/(a+b*x^n+c*x^{(2*n)})^3,x)$

[Out]  $\text{int}(1/(d+e*x^n)/(a+b*x^n+c*x^{(2*n)})^3,x)$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}(1/(d+e*x^n)/(a+b*x^n+c*x^{(2*n)})^3,x, \text{algorithm}="maxima")$

[Out] 
$$e^6 \cdot \text{integrate}\left(\frac{1}{(c^3 d^7 - 3 b^2 c^2 d^6 e + 3 b^2 c d^5 e^2 - b^3 d^4 e^3 + a^3 d e^6 + 3(c d^3 e^4 - b d^2 e^5) a^2 + 3(c^2 d^5 e^2 - 2 b^2 c d^4 e^3 + b^2 d^3 e^4) a + (c^3 d^6 e - 3 b^2 c^2 d^5 e^2 + 3 b^2 c d^4 e^3 - b^3 d^3 e^4 + a^3 e^7 + 3(c d^2 e^5 - b d e^6) a^2 + 3(c^2 d^4 e^3 - 2 b^2 c d^3 e^4 + b^2 d^2 e^5) a) x^n}, x\right) - \frac{1}{2} \cdot \left( (4 a^3 c^4 e^3 (7 n - 1) - b^3 c^4 d^3 (2 n - 1) + 2 b^4 c^3 d^2 e (2 n - 1) - b^5 c^2 d e^2 (2 n - 1) - (b^2 c^3 e^3 (26 n - 5) - 4 c^5 d^2 e (3 n - 1) - 10 b^2 c^4 d e^2 n) a^2 - (b^2 c^4 d^2 e (28 n - 9) - 2 b^2 c^5 d^3 (7 n - 2) - 2 b^3 c^3 d e^2 (5 n - 2) - b^4 c^2 e^3 (4 n - 1)) a) x x^{(3 n)} - (2 b^4 c^3 d^3 (2 n - 1) - 4 b^5 c^2 d^2 e (2 n - 1) + 2 b^6 c^2 d e^2 (2 n - 1) - 2 (b^2 c^3 e^3 (37 n - 6) - 2 c^4 d e^2 (8 n - 1)) a^3 - (2 b^2 c^4 d^2 e (25 n - 8) + 3 b^2 c^3 d e^2 (5 n + 1) - 11 b^3 c^2 e^3 (5 n - 1) - 4 c^5 d^3 (4 n - 1)) a^2 - (b^2 c^4 d^3 (29 n - 9) - 2 b^3 c^3 d^2 e (29 n - 10) + 3 b^4 c^2 d e^2 (7 n - 3) + 2 b^5 c e^3 (4 n - 1)) a) x x^{(2 n)} + (4 a^4 c^3 e^3 (9 n - 1) - b^5 c^2 d^3 (2 n - 1) + 2 b^6 c^2 d^2 e (2 n - 1) - b^7 d e^2 (2 n - 1) + (b^2 c^2 e^3 (14 n - 3) - 2 b^2 c^3 d e^2 (13 n - 2) + 4 c^4 d^2 e (5 n - 1)) a^3 - (b^4 c e^3 (24 n - 5) - b^3 c^2 d e^2 (20 n - 1) - 2 b^2 c^4 d^3 n + 3 b^2 c^3 d^2 e) a^2 - (3 b^4 c^2 d^2 e (8 n - 3) - b^6 e^3 (4 n - 1) - 4 b^3 c^3 d^3 (3 n - 1) - 4 b^5 c d e^2 (2 n - 1)) a) x x^n + (2 (b^2 c^2 e^3 (29 n - 4) - 2 c^3 d e^2 (10 n - 1)) a^4 + (2 b^2 c^3 d^2 e (29 n - 6) - 4 c^4 d^3 (6 n - 1) - 6 b^3 c e^3 (6 n - 1) - b^2 c^2 d e^2 (n - 3)) a^3 - (b^3 c^2 d^2 e (43 n - 11) - b^2 c^3 d^3 (21 n - 5) - b^4 c d e^2 (17 n - 5) - b^5 e^3 (5 n - 1)) a^2 - (b^4 c^2 d^3 (3 n - 1) - 2 b^5 c d^2 e (3 n - 1) + b^6 d e^2 (3 n - 1)) a) x \right) / (16 a^8 c^2 e^4 n^2 + 8 (4 c^3 d^2 e^2 n^2 - 4 b^2 c^2 d e^3 n^2 - b^2 c e^4 n^2) a^7 + (16 c^4 d^4 n^2 - 32 b^2 c^3 d^3 e n^2 + 16 b^3 c d e^3 n^2 + b^4 e^4 n^2) a^6 - 2 (4 b^2 c^3 d^4 n^2 - 8 b^3 c^2 d^3 e n^2 + 3 b^4 c d^2 e^2 n^2 + b^5 d e^3 n^2) a^5 + (b^4 c^2 d^4 n^2 - 2 b^5 c d^3 e n^2 + b^6 d^2 e^2 n^2) a^4 + (16 a^6 c^4 e^4 n^2 + 8 (4 c^5 d^2 e^2 n^2 - 4 b^2 c^4 d e^3$$

$$\begin{aligned}
& n^2 - b^2c^3e^4n^2)a^5 + (16c^6d^4n^2 - 32b^2c^5d^3e^4n^2 + 16b^3c^4d^2e^3n^2 + b^4c^3d^2e^2n^2 + b^4c^2e^4n^2)a^4 - 2(4b^2c^5d^4n^2 - 8b^3c^4d^3e^4n^2 + 3b^4c^3d^2e^2n^2 + b^5c^2d^2e^3n^2)a^3 + (b^4c^4d^4n^2 - 2b^5c^3d^3e^4n^2 + b^6c^2d^2e^2n^2)a^2)x^{(4n)} + 2(16a^6b^2c^3e^4n^2 + 8(4b^2c^4d^2e^2n^2 - 4b^2c^3d^3e^3n^2 - b^3c^2e^4n^2)a^5 + (16b^2c^5d^4n^2 - 32b^2c^4d^3e^4n^2 + 16b^4c^2d^2e^3n^2 + b^5c^2e^4n^2)a^4 - 2(4b^3c^4d^4n^2 - 8b^4c^3d^3e^4n^2 + 3b^5c^2d^2e^2n^2 + b^6c^2d^2e^3n^2)a^3 + (b^5c^3d^4n^2 - 2b^6c^2d^3e^4n^2 + b^7c^2d^2e^2n^2)a^2)x^{(3n)} + (32a^7c^3e^4n^2 + 64(c^4d^2e^2n^2 - b^2c^3d^2e^3n^2)a^6 + 2(16c^5d^4n^2 - 32b^2c^4d^3e^4n^2 + 16b^2c^3d^2e^2n^2 - 3b^4c^2e^4n^2)a^5 - (12b^4c^2d^2e^2n^2 - 12b^5c^2d^2e^3n^2 - b^6e^4n^2)a^4 - 2(3b^4c^3d^4n^2 - 6b^5c^2d^3e^4n^2 + 2b^6c^2d^2e^2n^2 + b^7d^2e^3n^2)a^3 + (b^6c^2d^4n^2 - 2b^7c^2d^3e^4n^2 + b^8d^2e^2n^2)a^2)x^{(2n)} + 2(16a^7b^2c^2e^4n^2 + 8(4b^2c^3d^2e^2n^2 - 4b^2c^2d^2e^3n^2 - b^3c^2e^4n^2)a^6 + (16b^2c^4d^4n^2 - 32b^2c^3d^3e^4n^2 + 16b^4c^2d^2e^3n^2 + b^5e^4n^2)a^5 - 2(4b^3c^3d^4n^2 - 8b^4c^2d^3e^4n^2 + 3b^5c^2d^2e^2n^2 + b^6d^2e^3n^2)a^4 + (b^5c^2d^4n^2 - 2b^6c^2d^3e^4n^2 + b^7d^2e^2n^2)a^3)x^n) - \int (-1/2((2n^2 - 3n + 1)b^4c^3d^5 - 3(2n^2 - 3n + 1)b^5c^2d^4e + 3(2n^2 - 3n + 1)b^6c^2d^3e^2 - (2n^2 - 3n + 1)b^7d^2e^3 + 2(2(24n^2 - 10n + 1)c^3d^2e^4 - (48n^2 - 29n + 4)b^2c^2e^5)a^4 + (8(12n^2 - 8n + 1)c^4d^3e^2 - 12(16n^2 - 13n + 2)b^2c^3d^2e^3 + (48n^2 - 59n + 11)b^2c^2d^2e^4 + 6(8n^2 - 6n + 1)b^3c^2e^5)a^3 + (4(8n^2 - 6n + 1)c^5d^5 - 2(48n^2 - 41n + 8)b^2c^4d^4e + 2(24n^2 - 19n + 5)b^2c^3d^3e^2 + 2(32n^2 - 39n + 7)b^3c^2d^2e^3 - (42n^2 - 53n + 11)b^4c^2d^2e^4 - (6n^2 - 5n + 1)b^5e^5)a^2 - ((16n^2 - 21n + 5)b^2c^4d^5 - 16(3n^2 - 4n + 1)b^3c^3d^4e + 3(14n^2 - 19n + 5)b^4c^2d^3e^2 - 2(2n^2 - 3n + 1)b^5c^2d^2e^3 - 2(3n^2 - 4n + 1)b^6d^2e^4)a + ((2n^2 - 3n + 1)b^3c^4d^5 - 3(2n^2 - 3n + 1)b^4c^3d^4e + 3(2n^2 - 3n + 1)b^5c^2d^3e^2 - (2n^2 - 3n + 1)b^6c^2d^2e^3 - 4(15n^2 - 8n + 1)a^4c^3e^5 - (8(5n^2 - 6n + 1)c^4d^2e^3 - 2(9n^2 - 11n + 2)b^2c^3d^2e^4 - (42n^2 - 31n + 5)b^2c^2e^5)a^3 - (4(3n^2 - 4n + 1)c^5d^4e + 12(n^2 - n)b^2c^4d^3e^2 - 2(32n^2 - 39n + 7)b^2c^3d^2e^3 + 9(4n^2 - 5n + 1)b^3c^2d^2e^4 + (6n^2 - 5n + 1)b^4c^2e^5)a^2 - (2(7n^2 - 9n + 2)b^2c^5d^5 - (42n^2 - 55n + 13)b^2c^4d^4e + 12(3n^2 - 4n + 1)b^3c^3d^3e^2 - (2n^2 - 3n + 1)b^4c^2d^2e^3 - 2(3n^2 - 4n + 1)b^5c^2d^2e^4)a)x^n)/(16a^8c^2e^6n^2 + 8(6c^3d^2e^4n^2 - 6b^2c^2d^2e^5n^2 - b^2c^2e^6n^2)a^7 + (48c^4d^4e^2n^2 - 96b^2c^3d^3e^3n^2 + 24b^2c^2d^2e^4n^2 + 24b^3c^2d^2e^5n^2 + b^4e^6n^2)a^6 + (16c^5d^6n^2 - 48b^2c^4d^5e^4n^2 + 24b^2c^3d^4e^2n^2 + 32b^3c^2d^3e^3n^2 - 21b^4c^2d^2e^4n^2 - 3b^5d^2e^5n^2)a^5 - (8b^2c^4d^6n^2 - 24b^3c^3d^5e^4n^2 + 21b^4c^2d^4e^2n^2 - 2b^5c^2d^3e^3n^2 - 3b^6d^2e^4n^2)a^4 + (b^4c^3d^6n^2 - 3b^5c^2d^5e^4n^2 + 3b^6c^2d^4e^2n^2 - b^7d^3e^3n^2)a^3 + (16a^7c^3e^6n^2 + 8(6c^4d^2e^4n^2 - 6b^2c^3d^2e^5n^2 - b^2c^2e^6n^2)
\end{aligned}$$



```
*n^2)*a^6 + (48*c^5*d^4*e^2*n^2 - 96*b*c^4*d^3*e^3*n^2 + 24*b^2*c^3*d^2*e^4
*n^2 + 24*b^3*c^2*d*e^5*n^2 + b^4*c*e^6*n^2)*a^5 + (16*c^6*d^6*n^2 - 48*b*c
^5*d^5*e*n^2 + 24*b^2*c^4*d^4*e^2*n^2 + 32*b^3*c^3*d^3*e^3*n^2 - 21*b^4*c^2
*d^2*e^4*n^2 - 3*b^5*c*d*e^5*n^2)*a^4 - (8*b^2*c^5*d^6*n^2 - 24*b^3*c^4*d^5
*e*n^2 + 21*b^4*c^3*d^4*e^2*n^2 - 2*b^5*c^2*d^3*e^3*n^2 - 3*b^6*c*d^2*e^4*n
^2)*a^3 + (b^4*c^4*d^6*n^2 - 3*b^5*c^3*d^5*e*n^2 + 3*b^6*c^2*d^4*e^2*n^2 -
b^7*c*d^3*e^3*n^2)*a^2)*x^(2*n) + (16*a^7*b*c^2*e^6*n^2 + 8*(6*b*c^3*d^2*e^
4*n^2 - 6*b^2*c^2*d*e^5*n^2 - b^3*c*e^6*n^2)*a^6 + (48*b*c^4*d^4*e^2*n^2 -
96*b^2*c^3*d^3*e^3*n^2 + 24*b^3*c^2*d^2*e^4*n^2 + 24*b^4*c*d*e^5*n^2 + b^5*
e^6*n^2)*a^5 + (16*b*c^5*d^6*n^2 - 48*b^2*c^4*d^5*e*n^2 + 24*b^3*c^3*d^4*e^
2*n^2 + 32*b^4*c^2*d^3*e^3*n^2 - 21*b^5*c*d^2*e^4*n^2 - 3*b^6*d*e^5*n^2)*a^
4 - (8*b^3*c^4*d^6*n^2 - 24*b^4*c^3*d^5*e*n^2 + 21*b^5*c^2*d^4*e^2*n^2 - 2*
b^6*c*d^3*e^3*n^2 - 3*b^7*d^2*e^4*n^2)*a^3 + (b^5*c^3*d^6*n^2 - 3*b^6*c^2*d
^5*e*n^2 + 3*b^7*c*d^4*e^2*n^2 - b^8*d^3*e^3*n^2)*a^2)*x^n), x)
```

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{b^3ex^{4n} + a^3d + (c^3ex^n + c^3d)x^{6n} + 3(bc^2ex^{2n} + ac^2d + (bc^2d + ac^2e)x^n)x^{4n} + (b^3d + 3ab^2e)x^{3n} + 3(b^2cex^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x^n)/(a+b*x^n+c*x^(2*n))^3,x, algorithm="fricas")
```

```
[Out] integral(1/(b^3*e*x^(4*n) + a^3*d + (c^3*e*x^n + c^3*d)*x^(6*n) + 3*(b*c^2*
e*x^(2*n) + a*c^2*d + (b*c^2*d + a*c^2*e)*x^n)*x^(4*n) + (b^3*d + 3*a*b^2*e
)*x^(3*n) + 3*(b^2*c*e*x^(3*n) + a^2*c*d + (b^2*c*d + 2*a*b*c*e)*x^(2*n) +
(2*a*b*c*d + a^2*c*e)*x^n)*x^(2*n) + 3*(a*b^2*d + a^2*b*e)*x^(2*n) + (3*a^2
*b*d + a^3*e)*x^n), x)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x**n)/(a+b*x**n+c*x**(2*n))**3,x)
```

```
[Out] Timed out
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(cx^{2n} + bx^n + a)^3 (ex^n + d)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x^n)/(a+b*x^n+c*x^(2*n))^3,x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^(2*n) + b*x^n + a)^3*(e*x^n + d)), x)
```

$$3.84 \quad \int \frac{1}{(d+ex^n)^2(a+bx^n+cx^{2n})^3} dx$$

Optimal. Leaf size=2446

result too large to display

```
[Out] -(x*(2*b^3*c*d*e - 6*a*b*c^2*d*e - b^4*e^2 - b^2*c*(c*d^2 - 4*a*e^2) + 2*a*c^2*(c*d^2 - a*e^2) + c*(2*b^2*c*d*e - 4*a*c^2*d*e - b^3*e^2 - b*c*(c*d^2 - 3*a*e^2))*x^n)/(2*a*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)^2*n*(a + b*x^n + c*x^(2*n))^2) - (e^2*x*(5*b^3*c*d*e - 14*a*b*c^2*d*e - 2*b^4*e^2 - b^2*c*(3*c*d^2 - 7*a*e^2) + 2*a*c^2*(3*c*d^2 - a*e^2) + c*(5*b^2*c*d*e - 8*a*c^2*d*e - 2*b^3*e^2 - b*c*(3*c*d^2 - 5*a*e^2))*x^n)/(a*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)^3*n*(a + b*x^n + c*x^(2*n))) - (x*(a*b^2*c^2*(a*e^2*(13 - 37*n) - 5*c*d^2*(1 - 3*n)) - b^4*c*(a*e^2*(7 - 17*n) - c*d^2*(1 - 2*n)) - 4*a^2*b*c^3*d*e*(4 - 11*n) + 6*a*b^3*c^2*d*e*(2 - 5*n) + 4*a^2*c^3*(c*d^2 - a*e^2)*(1 - 4*n) - 2*b^5*c*d*e*(1 - 2*n) + b^6*e^2*(1 - 2*n) + c*(2*a*b*c^2*(a*e^2*(4 - 13*n) - c*d^2*(2 - 7*n)) - b^3*c*(2*a*e^2*(3 - 8*n) - c*d^2*(1 - 2*n)) + 2*a*b^2*c^2*d*e*(5 - 14*n) - 8*a^2*c^3*d*e*(1 - 3*n) - 2*b^4*c*d*e*(1 - 2*n) + b^5*e^2*(1 - 2*n))*x^n)/(2*a^2*(b^2 - 4*a*c)^2*(c*d^2 - b*d*e + a*e^2)^2*n^2*(a + b*x^n + c*x^(2*n))) - (c*e^4*(10*c^2*d^2 + 3*b*(b + Sqrt[b^2 - 4*a*c])*e^2 - 2*c*e*(5*b*d + 3*Sqrt[b^2 - 4*a*c]*d + a*e))*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])])/(b^2 - 4*a*c - b*Sqrt[b^2 - 4*a*c])*(c*d^2 - b*d*e + a*e^2)^4) + (c*e^2*(4*a*c^2*(e*(a*e*(1 - 2*n) + 2*Sqrt[b^2 - 4*a*c]*d*(1 - n)) - 3*c*d^2*(1 - 2*n)) - b^2*c*(e*(a*e*(9 - 13*n) + 5*Sqrt[b^2 - 4*a*c]*d*(1 - n)) - 3*c*d^2*(1 - n)) + b*c*(c*d*(4*a*e*(5 - 8*n) + 3*Sqrt[b^2 - 4*a*c]*d*(1 - n)) - 5*a*Sqrt[b^2 - 4*a*c]*e^2*(1 - n) + 2*b^4*e^2*(1 - n) - b^3*e*(5*c*d - 2*Sqrt[b^2 - 4*a*c]*e)*(1 - n))*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])])/(a*(b^2 - 4*a*c)*(b^2 - 4*a*c - b*Sqrt[b^2 - 4*a*c])*(c*d^2 - b*d*e + a*e^2)^3*n) + (c*((2*a*b*c^2*(a*e^2*(4 - 13*n) - c*d^2*(2 - 7*n)) - b^3*c*(2*a*e^2*(3 - 8*n) - c*d^2*(1 - 2*n)) + 2*a*b^2*c^2*d*e*(5 - 14*n) - 8*a^2*c^3*d*e*(1 - 3*n) - 2*b^4*c*d*e*(1 - 2*n) + b^5*e^2*(1 - 2*n))*(1 - n) - (b^4*c*(4*a*e^2*(2 - 5*n) - c*d^2*(1 - 2*n))*(1 - n) + 2*b^5*c*d*e*(1 - 3*n + 2*n^2) - b^6*e^2*(1 - 3*n + 2*n^2) - 8*a^2*c^3*(c*d^2 - a*e^2)*(1 - 6*n + 8*n^2) + 8*a^2*b*c^3*d*e*(3 - 13*n + 13*n^2) - 2*a*b^3*c^2*d*e*(7 - 25*n + 18*n^2) + 2*a*b^2*c^2*(3*c*d^2*(1 - 4*n + 3*n^2) - a*e^2*(9 - 38*n + 35*n^2)))/Sqrt[b^2 - 4*a*c])*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])])/(2*a^2*(b^2 - 4*a*c)^2*(b - Sqrt[b^2 - 4*a*c])*(c*d^2 - b*d*e + a*e^2)^2*n^2) - (c*e^4*(10*c^2*d^2 + 3*b*(b - Sqrt[b^2 - 4*a*c])*e^2 - 2*c*e*(5*b*d - 3*Sqrt[b^2 - 4*a*c]*d + a*e))*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(b^2 - 4*a*c + b*Sqrt[b^2 - 4*a*c])*(c*d^2 - b*d*e + a*e^2)^4) + (c*e^2*(4*a*c^2*(e*(a*e*(1 - 2*n) - 2*Sqrt[b^2 - 4*a*c]*d*(1 - n)) - 3*c*d
```

$$\begin{aligned} &^2*(1 - 2*n)) - b^2*c*(e*(a*e*(9 - 13*n) - 5*\text{Sqrt}[b^2 - 4*a*c]*d*(1 - n)) - \\ &3*c*d^2*(1 - n)) + b*c*(c*d*(4*a*e*(5 - 8*n) - 3*\text{Sqrt}[b^2 - 4*a*c]*d*(1 - \\ &n)) + 5*a*\text{Sqrt}[b^2 - 4*a*c]*e^2*(1 - n)) + 2*b^4*e^2*(1 - n) - b^3*e*(5*c*d \\ &+ 2*\text{Sqrt}[b^2 - 4*a*c]*e)*(1 - n))*x*\text{Hypergeometric2F1}[1, n^{(-1)}, 1 + n^{(-1)} \\ &), (-2*c*x^n)/(b + \text{Sqrt}[b^2 - 4*a*c])]]/(a*(b^2 - 4*a*c)*(b^2 - 4*a*c + b*\text{S} \\ &\text{qrt}[b^2 - 4*a*c])*(c*d^2 - b*d*e + a*e^2)^3*n) + (c*((2*a*b*c^2*(a*e^2*(4 - \\ &13*n) - c*d^2*(2 - 7*n)) - b^3*c*(2*a*e^2*(3 - 8*n) - c*d^2*(1 - 2*n)) + 2 \\ &*a*b^2*c^2*d*e*(5 - 14*n) - 8*a^2*c^3*d*e*(1 - 3*n) - 2*b^4*c*d*e*(1 - 2*n) \\ &+ b^5*e^2*(1 - 2*n))*(1 - n) + (b^4*c*(4*a*e^2*(2 - 5*n) - c*d^2*(1 - 2*n) \\ &)*(1 - n) + 2*b^5*c*d*e*(1 - 3*n + 2*n^2) - b^6*e^2*(1 - 3*n + 2*n^2) - 8*a \\ &^2*c^3*(c*d^2 - a*e^2)*(1 - 6*n + 8*n^2) + 8*a^2*b*c^3*d*e*(3 - 13*n + 13*n \\ &^2) - 2*a*b^3*c^2*d*e*(7 - 25*n + 18*n^2) + 2*a*b^2*c^2*(3*c*d^2*(1 - 4*n + \\ &3*n^2) - a*e^2*(9 - 38*n + 35*n^2)))/\text{Sqrt}[b^2 - 4*a*c])*x*\text{Hypergeometric2F} \\ &1[1, n^{(-1)}, 1 + n^{(-1)}, (-2*c*x^n)/(b + \text{Sqrt}[b^2 - 4*a*c])]]/(2*a^2*(b^2 - \\ &4*a*c)^2*(b + \text{Sqrt}[b^2 - 4*a*c])*(c*d^2 - b*d*e + a*e^2)^2*n^2) + (3*e^6*( \\ &2*c*d - b*e))*x*\text{Hypergeometric2F1}[1, n^{(-1)}, 1 + n^{(-1)}, -((e*x^n)/d)]/(d*( \\ &c*d^2 - b*d*e + a*e^2)^4) + (e^6*x*\text{Hypergeometric2F1}[2, n^{(-1)}, 1 + n^{(-1)}, \\ &-((e*x^n)/d)]/(d^2*(c*d^2 - b*d*e + a*e^2)^3) \end{aligned}$$

**Rubi [A]** time = 8.93528, antiderivative size = 2446, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 4, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$ , Rules used = {1436, 245, 1430, 1422}

result too large to display

Antiderivative was successfully verified.

[In] Int[1/((d + e\*x^n)^2\*(a + b\*x^n + c\*x^(2\*n))^3),x]

[Out] 
$$\begin{aligned} &-(x*(2*b^3*c*d*e - 6*a*b*c^2*d*e - b^4*e^2 - b^2*c*(c*d^2 - 4*a*e^2) + 2*a* \\ &c^2*(c*d^2 - a*e^2) + c*(2*b^2*c*d*e - 4*a*c^2*d*e - b^3*e^2 - b*c*(c*d^2 - \\ &3*a*e^2))*x^n))/(2*a*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)^2*n*(a + b*x^n \\ &+ c*x^(2*n))^2) - (e^2*x*(5*b^3*c*d*e - 14*a*b*c^2*d*e - 2*b^4*e^2 - b^2*c* \\ &(3*c*d^2 - 7*a*e^2) + 2*a*c^2*(3*c*d^2 - a*e^2) + c*(5*b^2*c*d*e - 8*a*c^2* \\ &d*e - 2*b^3*e^2 - b*c*(3*c*d^2 - 5*a*e^2))*x^n))/(a*(b^2 - 4*a*c)*(c*d^2 - \\ &b*d*e + a*e^2)^3*n*(a + b*x^n + c*x^(2*n))) - (x*(a*b^2*c^2*(a*e^2*(13 - 37 \\ &*n) - 5*c*d^2*(1 - 3*n)) - b^4*c*(a*e^2*(7 - 17*n) - c*d^2*(1 - 2*n)) - 4*a \\ &^2*b*c^3*d*e*(4 - 11*n) + 6*a*b^3*c^2*d*e*(2 - 5*n) + 4*a^2*c^3*(c*d^2 - a* \\ &e^2)*(1 - 4*n) - 2*b^5*c*d*e*(1 - 2*n) + b^6*e^2*(1 - 2*n) + c*(2*a*b*c^2*( \\ &a*e^2*(4 - 13*n) - c*d^2*(2 - 7*n)) - b^3*c*(2*a*e^2*(3 - 8*n) - c*d^2*(1 - \\ &2*n)) + 2*a*b^2*c^2*d*e*(5 - 14*n) - 8*a^2*c^3*d*e*(1 - 3*n) - 2*b^4*c*d*e \\ &*(1 - 2*n) + b^5*e^2*(1 - 2*n))*x^n))/(2*a^2*(b^2 - 4*a*c)^2*(c*d^2 - b*d*e \end{aligned}$$

$$\begin{aligned}
& + a e^2)^2 n^2 (a + b x^n + c x^{2n})) - (c e^4 (10 c^2 d^2 + 3 b (b + \text{Sqrt}[b^2 - 4 a c]) e^2 - 2 c e (5 b d + 3 \text{Sqrt}[b^2 - 4 a c] d + a e)) x \text{Hypergeometric2F1}[1, n^{-1}, 1 + n^{-1}, (-2 c x^n)/(b - \text{Sqrt}[b^2 - 4 a c])]) / ((b^2 - 4 a c - b \text{Sqrt}[b^2 - 4 a c]) (c d^2 - b d e + a e^2)^4) + (c e^2 (4 a c^2 (e (a e (1 - 2 n) + 2 \text{Sqrt}[b^2 - 4 a c] d (1 - n)) - 3 c d^2 (1 - 2 n)) - b^2 c (e (a e (9 - 13 n) + 5 \text{Sqrt}[b^2 - 4 a c] d (1 - n)) - 3 c d^2 (1 - n)) + b c (c d (4 a e (5 - 8 n) + 3 \text{Sqrt}[b^2 - 4 a c] d (1 - n)) - 5 a \text{Sqrt}[b^2 - 4 a c] e^2 (1 - n)) + 2 b^4 e^2 (1 - n) - b^3 e (5 c d - 2 \text{Sqrt}[b^2 - 4 a c] e) (1 - n)) x \text{Hypergeometric2F1}[1, n^{-1}, 1 + n^{-1}, (-2 c x^n)/(b - \text{Sqrt}[b^2 - 4 a c])]) / (a (b^2 - 4 a c) (b^2 - 4 a c - b \text{Sqrt}[b^2 - 4 a c]) (c d^2 - b d e + a e^2)^3 n) + (c ((2 a b c^2 (a e^2 (4 - 13 n) - c d^2 (2 - 7 n)) - b^3 c (2 a e^2 (3 - 8 n) - c d^2 (1 - 2 n)) + 2 a b^2 c^2 d e (5 - 14 n) - 8 a^2 c^3 d e (1 - 3 n) - 2 b^4 c d e (1 - 2 n) + b^5 e^2 (1 - 2 n)) (1 - n) - (b^4 c (4 a e^2 (2 - 5 n) - c d^2 (1 - 2 n)) (1 - n) + 2 b^5 c d e (1 - 3 n + 2 n^2) - b^6 e^2 (1 - 3 n + 2 n^2) - 8 a^2 c^3 (c d^2 - a e^2) (1 - 6 n + 8 n^2) + 8 a^2 b c^3 d e (3 - 13 n + 13 n^2) - 2 a b^3 c^2 d e (7 - 25 n + 18 n^2) + 2 a b^2 c^2 (3 c d^2 (1 - 4 n + 3 n^2) - a e^2 (9 - 38 n + 35 n^2))) / \text{Sqrt}[b^2 - 4 a c]) x \text{Hypergeometric2F1}[1, n^{-1}, 1 + n^{-1}, (-2 c x^n)/(b - \text{Sqrt}[b^2 - 4 a c])]) / (2 a^2 (b^2 - 4 a c)^2 (b - \text{Sqrt}[b^2 - 4 a c]) (c d^2 - b d e + a e^2)^2 n^2) - (c e^4 (10 c^2 d^2 + 3 b (b - \text{Sqrt}[b^2 - 4 a c]) e^2 - 2 c e (5 b d - 3 \text{Sqrt}[b^2 - 4 a c] d + a e)) x \text{Hypergeometric2F1}[1, n^{-1}, 1 + n^{-1}, (-2 c x^n)/(b + \text{Sqrt}[b^2 - 4 a c])]) / ((b^2 - 4 a c + b \text{Sqrt}[b^2 - 4 a c]) (c d^2 - b d e + a e^2)^4) + (c e^2 (4 a c^2 (e (a e (1 - 2 n) - 2 \text{Sqrt}[b^2 - 4 a c] d (1 - n)) - 3 c d^2 (1 - 2 n)) - b^2 c (e (a e (9 - 13 n) - 5 \text{Sqrt}[b^2 - 4 a c] d (1 - n)) - 3 c d^2 (1 - n)) + b c (c d (4 a e (5 - 8 n) - 3 \text{Sqrt}[b^2 - 4 a c] d (1 - n)) + 5 a \text{Sqrt}[b^2 - 4 a c] e^2 (1 - n)) + 2 b^4 e^2 (1 - n) - b^3 e (5 c d + 2 \text{Sqrt}[b^2 - 4 a c] e) (1 - n)) x \text{Hypergeometric2F1}[1, n^{-1}, 1 + n^{-1}, (-2 c x^n)/(b + \text{Sqrt}[b^2 - 4 a c])]) / (a (b^2 - 4 a c) (b^2 - 4 a c + b \text{Sqrt}[b^2 - 4 a c]) (c d^2 - b d e + a e^2)^3 n) + (c ((2 a b c^2 (a e^2 (4 - 13 n) - c d^2 (2 - 7 n)) - b^3 c (2 a e^2 (3 - 8 n) - c d^2 (1 - 2 n)) + 2 a b^2 c^2 d e (5 - 14 n) - 8 a^2 c^3 d e (1 - 3 n) - 2 b^4 c d e (1 - 2 n) + b^5 e^2 (1 - 2 n)) (1 - n) + (b^4 c (4 a e^2 (2 - 5 n) - c d^2 (1 - 2 n)) (1 - n) + 2 b^5 c d e (1 - 3 n + 2 n^2) - b^6 e^2 (1 - 3 n + 2 n^2) - 8 a^2 c^3 (c d^2 - a e^2) (1 - 6 n + 8 n^2) + 8 a^2 b c^3 d e (3 - 13 n + 13 n^2) - 2 a b^3 c^2 d e (7 - 25 n + 18 n^2) + 2 a b^2 c^2 (3 c d^2 (1 - 4 n + 3 n^2) - a e^2 (9 - 38 n + 35 n^2))) / \text{Sqrt}[b^2 - 4 a c]) x \text{Hypergeometric2F1}[1, n^{-1}, 1 + n^{-1}, (-2 c x^n)/(b + \text{Sqrt}[b^2 - 4 a c])]) / (2 a^2 (b^2 - 4 a c)^2 (b + \text{Sqrt}[b^2 - 4 a c]) (c d^2 - b d e + a e^2)^2 n^2) + (3 e^6 (2 c d - b e) x \text{Hypergeometric2F1}[1, n^{-1}, 1 + n^{-1}, -((e x^n)/d)]) / (d (c d^2 - b d e + a e^2)^4) + (e^6 x \text{Hypergeometric2F1}[2, n^{-1}, 1 + n^{-1}, -((e x^n)/d)]) / (d^2 (c d^2 - b d e + a e^2)^3)
\end{aligned}$$

Rule 1436

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol] := Int[ExpandIntegrand[(d + e*x^n)^q*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && ((IntegersQ[p, q] && !IntegerQ[n]) || IGtQ[p, 0] || (IGtQ[q, 0] && !IntegerQ[n]))
```

### Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[a^p*x*Hypergeometric2F1[-p, 1/n, 1/n + 1, -(b*x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] || GtQ[a, 0])
```

### Rule 1430

```
Int[((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol] := -Simp[(x*(d*b^2 - a*b*e - 2*a*c*d + (b*d - 2*a*e)*c*x^n)*(a + b*x^n + c*x^(2*n))^(p + 1))/(a*n*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(a*n*(p + 1)*(b^2 - 4*a*c)), Int[Simp[(n*p + n + 1)*d*b^2 - a*b*e - 2*a*c*d*(2*n*p + 2*n + 1) + (2*n*p + 3*n + 1)*(d*b - 2*a*e)*c*x^n, x]*(a + b*x^n + c*x^(2*n))^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && ILtQ[p, -1]
```

### Rule 1422

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a*c] || !IGtQ[n/2, 0])
```

### Rubi steps

$$\begin{aligned}
\int \frac{1}{(d + ex^n)^2 (a + bx^n + cx^{2n})^3} dx &= \int \left( \frac{e^6}{(cd^2 - bde + ae^2)^3 (d + ex^n)^2} - \frac{3e^6(-2cd + be)}{(cd^2 - bde + ae^2)^4 (d + ex^n)} + \frac{c^2 d^2 - 2bcde + b^2 e^2}{(cd^2 - bde + ae^2)^5} \right) dx \\
&= \frac{e^4 \int \frac{5c^2 d^2 - 8bcde + 3b^2 e^2 - ace^2 + (-6c^2 de + 3bce^2)x^n}{a + bx^n + cx^{2n}} dx}{(cd^2 - bde + ae^2)^4} + \frac{(3e^6(2cd - be)) \int \frac{1}{d + ex^n} dx}{(cd^2 - bde + ae^2)^4} + \frac{e^2 \int \frac{3c^2}{(d + ex^n)^2} dx}{(cd^2 - bde + ae^2)^5} \\
&= -\frac{x(2b^3cde - 6abc^2de - b^4e^2 - b^2c(cd^2 - 4ae^2)) + 2ac^2(cd^2 - ae^2) + c(2b^2cde - 4abc^2e)}{2a(b^2 - 4ac)(cd^2 - bde + ae^2)^2 n(a + bx^n + cx^{2n})} \\
&= -\frac{x(2b^3cde - 6abc^2de - b^4e^2 - b^2c(cd^2 - 4ae^2)) + 2ac^2(cd^2 - ae^2) + c(2b^2cde - 4abc^2e)}{2a(b^2 - 4ac)(cd^2 - bde + ae^2)^2 n(a + bx^n + cx^{2n})} \\
&= -\frac{x(2b^3cde - 6abc^2de - b^4e^2 - b^2c(cd^2 - 4ae^2)) + 2ac^2(cd^2 - ae^2) + c(2b^2cde - 4abc^2e)}{2a(b^2 - 4ac)(cd^2 - bde + ae^2)^2 n(a + bx^n + cx^{2n})} \\
&= -\frac{x(2b^3cde - 6abc^2de - b^4e^2 - b^2c(cd^2 - 4ae^2)) + 2ac^2(cd^2 - ae^2) + c(2b^2cde - 4abc^2e)}{2a(b^2 - 4ac)(cd^2 - bde + ae^2)^2 n(a + bx^n + cx^{2n})}
\end{aligned}$$

**Mathematica [B]** time = 10.1094, size = 39502, normalized size = 16.15

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[1/((d + e\*x^n)^2\*(a + b\*x^n + c\*x^(2\*n))^3),x]

[Out] Result too large to show

---

**Maple [F]** time = 0.448, size = 0, normalized size = 0.

$$\int \frac{1}{(d + ex^n)^2 (a + bx^n + cx^{2n})^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}(1/(d+e*x^n)^2/(a+b*x^n+c*x^{(2*n)})^3,x)$

[Out]  $\text{int}(1/(d+e*x^n)^2/(a+b*x^n+c*x^{(2*n)})^3,x)$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}(1/(d+e*x^n)^2/(a+b*x^n+c*x^{(2*n)})^3,x, \text{algorithm}="maxima")$

[Out]  $(c*d^2*e^6*(7*n - 1) - b*d*e^7*(4*n - 1) + a*e^8*(n - 1))*\text{integrate}(1/(c^4*d^{10*n} - 4*b*c^3*d^9*e^n + 6*b^2*c^2*d^8*e^{2*n} - 4*b^3*c*d^7*e^{3*n} + b^4*d^6*e^{4*n} + a^4*d^2*e^{8*n} + 4*(c*d^4*e^6*n - b*d^3*e^7*n)*a^3 + 6*(c^2*d^6*e^{4*n} - 2*b*c*d^5*e^5*n + b^2*d^4*e^6*n)*a^2 + 4*(c^3*d^8*e^{2*n} - 3*b*c^2*d^7*e^{3*n} + 3*b^2*c*d^6*e^4*n - b^3*d^5*e^5*n)*a + (c^4*d^9*e^n - 4*b*c^3*d^8*e^{2*n} + 6*b^2*c^2*d^7*e^3*n - 4*b^3*c*d^6*e^4*n + b^4*d^5*e^5*n + a^4*d*e^9*n + 4*(c*d^3*e^7*n - b*d^2*e^8*n)*a^3 + 6*(c^2*d^5*e^5*n - 2*b*c*d^4*e^6*n + b^2*d^3*e^7*n)*a^2 + 4*(c^3*d^7*e^3*n - 3*b*c^2*d^6*e^4*n + 3*b^2*c*d^5*e^5*n - b^3*d^4*e^6*n)*a)*x^n, x) + 1/2*((b^3*c^5*d^5*e*(2*n - 1) - 3*b^4*c^4*d^4*e^2*(2*n - 1) + 3*b^5*c^3*d^3*e^3*(2*n - 1) - b^6*c^2*d^2*e^4*(2*n - 1) + 32*a^4*c^4*e^6*n + 2*(b*c^4*d*e^5*(33*n - 4) - 4*c^5*d^2*e^4*(11*n - 1) - 8*b^2*c^3*e^6*n)*a^3 + 2*(b^2*c^4*d^2*e^4*(29*n - 1) - 3*b^3*c^3*d*e^5*(7*n - 1) - 4*c^6*d^4*e^2*(3*n - 1) + 6*b*c^5*d^3*e^3*(n - 1) + b^4*c^2*e^6*n)*a^2 - (3*b^3*c^4*d^3*e^3*(12*n - 5) + 2*b*c^6*d^5*e*(7*n - 2) - b^5*c^2*d*e^5*(6*n - 1) - 14*b^2*c^5*d^4*e^2*(3*n - 1) - 2*b^4*c^3*d^2*e^4*(n - 2))*a)*x*x^(4*n) + (b^3*c^5*d^6*(2*n - 1) - b^4*c^4*d^5*e*(2*n - 1) - 3*b^5*c^3*d^4*e^2*(2*n - 1) + 5*b^6*c^2*d^3*e^3*(2*n - 1) - 2*b^7*c*d^2*e^4*(2*n - 1) - 4*(c^4*d*e^5*(8*n - 1) - 16*b*c^3*e^6*n)*a^4 + (b^2*c^3*d*e^5*(163*n - 21) - 6*b*c^4*d^2*e^4*(27*n - 2) - 8*c^5*d^3*e^3*(5*n - 1) - 32*b^3*c^2*e^6*n)*a^3 - (b^4*c^2*d*e^5*(89*n - 13) - b^3*c^3*d^2*e^4*(77*n + 5) - 2*b^2*c^4*d^3*e^3*(50*n - 19) + 8*b*c^5*d^4*e^2*(9*n - 2) + 4*c^6*d^5*e*(2*n - 1) - 4*b^5*c*e^6*n)*a^2 - (b^4*c^3*d^3*e^3*(73*n - 29) - b^3*c^4*d^4*e^2*(51*n - 16) - b^2*c^5*d^5*e*(13*n - 5) - b^5*c^2*d^2*e^4*(11*n - 10) + 2*b*c^6*d^6*(7*n - 2) - 2*b^6*c*d*e^5*(6*n - 1))*a)*x*x^(3*n) + (2*b^4*c^4*d^6*(2*n - 1) - 5*b^5*c^3*d^5*e*(2*n - 1) + 3*b^6*c^2*d^4*e^2*(2*n - 1) + b^7*c*d^3*e^3*(2*n - 1) - b^8*d^2*e^4*(2*n - 1) + 64*a^5*c^3*e^6*n - 2*(2*c^4*d^2*e^4*(34*n - 3) - b*c^3*d*e^5*(23*n - 2))*a^4 + (b^2*c^3*d^2*e^4*(81*n - 11) + b^3*c^2*d*e^5*(48*n - 7) - 8*b*c^4*d^3*e^3*(18*n - 1) + 8*c^5*d^4*e^2*($



$$\begin{aligned}
& n + 1) - 12*b^4*c*e^6*n)*a^3 - (2*b*c^5*d^5*e*(43*n - 14) + b^4*c^2*d^2*e^4 \\
& *(21*n - 10) + 2*b^5*c*d*e^5*(20*n - 3) - 5*b^3*c^3*d^3*e^3*(19*n - 2) - 4* \\
& c^6*d^6*(4*n - 1) - 10*b^2*c^4*d^4*e^2*(4*n - 3) - 2*b^6*e^6*n)*a^2 - (b^4* \\
& c^3*d^4*e^2*(39*n - 19) + b^2*c^5*d^6*(29*n - 9) + b^5*c^2*d^3*e^3*(25*n - \\
& 6) - 3*b^3*c^4*d^5*e*(25*n - 9) - b^7*d*e^5*(6*n - 1) - 6*b^6*c*d^2*e^4*(2* \\
& n - 1))*a)*x*x^(2*n) + (b^5*c^3*d^6*(2*n - 1) - 3*b^6*c^2*d^5*e*(2*n - 1) + \\
& 3*b^7*c*d^4*e^2*(2*n - 1) - b^8*d^3*e^3*(2*n - 1) - 4*(c^3*d*e^5*(10*n - 1 \\
& ) - 16*b*c^2*e^6*n)*a^5 + (b^2*c^2*d*e^5*(115*n - 13) - 2*b*c^3*d^2*e^4*(55 \\
& *n - 4) - 8*c^4*d^3*e^3*(7*n - 1) - 32*b^3*c*e^6*n)*a^4 - (b^4*c*d*e^5*(55* \\
& n - 7) - 3*b^3*c^2*d^2*e^4*(35*n - 2) + 2*b^2*c^3*d^3*e^3*(8*n + 7) + 4*c^5 \\
& *d^5*e*(4*n - 1) + 8*b*c^4*d^4*e^2*(n - 1) - 4*b^5*e^6*n)*a^3 + (b^3*c^3*d^ \\
& 4*e^2*(41*n - 26) - b^5*c*d^2*e^4*(31*n - 1) - b^2*c^4*d^5*e*(23*n - 11) + \\
& b^4*c^2*d^3*e^3*(8*n + 15) + b^6*d*e^5*(7*n - 1) - 2*b*c^5*d^6*n)*a^2 + (3* \\
& b^4*c^3*d^5*e*(13*n - 5) - 3*b^5*c^2*d^4*e^2*(13*n - 6) + b^6*c*d^3*e^3*(9* \\
& n - 7) - 4*b^3*c^4*d^6*(3*n - 1) + 3*b^7*d^2*e^4*n)*a)*x*x^n + (32*a^6*c^2* \\
& e^6*n - 4*(c^3*d^2*e^4*(10*n - 1) + 4*b^2*c*e^6*n)*a^5 + (b^2*c^2*d^2*e^4*( \\
& 115*n - 13) - 12*b*c^3*d^3*e^3*(13*n - 1) + 48*c^4*d^4*e^2*n + 2*b^4*e^6*n) \\
& *a^4 + (b^3*c^2*d^3*e^3*(57*n + 1) - b^4*c*d^2*e^4*(55*n - 7) - 4*b*c^4*d^5 \\
& *e*(23*n - 5) + 6*b^2*c^3*d^4*e^2*(11*n - 4) + 4*c^5*d^6*(6*n - 1))*a^3 + ( \\
& b^3*c^3*d^5*e*(65*n - 17) - b^2*c^4*d^6*(21*n - 5) - 6*b^4*c^2*d^4*e^2*(10* \\
& n - 3) + b^5*c*d^3*e^3*(9*n - 5) + b^6*d^2*e^4*(7*n - 1))*a^2 + (b^4*c^3*d^ \\
& 6*(3*n - 1) - 3*b^5*c^2*d^5*e*(3*n - 1) + 3*b^6*c*d^4*e^2*(3*n - 1) - b^7*d \\
& ^3*e^3*(3*n - 1))*a)*x)/(16*a^9*c^2*d^2*e^6*n^2 + 8*(6*c^3*d^4*e^4*n^2 - 6* \\
& b*c^2*d^3*e^5*n^2 - b^2*c*d^2*e^6*n^2)*a^8 + (48*c^4*d^6*e^2*n^2 - 96*b*c^3 \\
& *d^5*e^3*n^2 + 24*b^2*c^2*d^4*e^4*n^2 + 24*b^3*c*d^3*e^5*n^2 + b^4*d^2*e^6* \\
& n^2)*a^7 + (16*c^5*d^8*n^2 - 48*b*c^4*d^7*e*n^2 + 24*b^2*c^3*d^6*e^2*n^2 + \\
& 32*b^3*c^2*d^5*e^3*n^2 - 21*b^4*c*d^4*e^4*n^2 - 3*b^5*d^3*e^5*n^2)*a^6 - (8 \\
& *b^2*c^4*d^8*n^2 - 24*b^3*c^3*d^7*e*n^2 + 21*b^4*c^2*d^6*e^2*n^2 - 2*b^5*c* \\
& d^5*e^3*n^2 - 3*b^6*d^4*e^4*n^2)*a^5 + (b^4*c^3*d^8*n^2 - 3*b^5*c^2*d^7*e*n \\
& ^2 + 3*b^6*c*d^6*e^2*n^2 - b^7*d^5*e^3*n^2)*a^4 + (16*a^7*c^4*d*e^7*n^2 + 8 \\
& *(6*c^5*d^3*e^5*n^2 - 6*b*c^4*d^2*e^6*n^2 - b^2*c^3*d*e^7*n^2)*a^6 + (48*c^ \\
& 6*d^5*e^3*n^2 - 96*b*c^5*d^4*e^4*n^2 + 24*b^2*c^4*d^3*e^5*n^2 + 24*b^3*c^3* \\
& d^2*e^6*n^2 + b^4*c^2*d*e^7*n^2)*a^5 + (16*c^7*d^7*e*n^2 - 48*b*c^6*d^6*e^2 \\
& *n^2 + 24*b^2*c^5*d^5*e^3*n^2 + 32*b^3*c^4*d^4*e^4*n^2 - 21*b^4*c^3*d^3*e^5 \\
& *n^2 - 3*b^5*c^2*d^2*e^6*n^2)*a^4 - (8*b^2*c^6*d^7*e*n^2 - 24*b^3*c^5*d^6*e \\
& ^2*n^2 + 21*b^4*c^4*d^5*e^3*n^2 - 2*b^5*c^3*d^4*e^4*n^2 - 3*b^6*c^2*d^3*e^5 \\
& *n^2)*a^3 + (b^4*c^5*d^7*e*n^2 - 3*b^5*c^4*d^6*e^2*n^2 + 3*b^6*c^3*d^5*e^3* \\
& n^2 - b^7*c^2*d^4*e^4*n^2)*a^2)*x^(5*n) + (16*(c^4*d^2*e^6*n^2 + 2*b*c^3*d* \\
& e^7*n^2)*a^7 + 8*(6*c^5*d^4*e^4*n^2 + 6*b*c^4*d^3*e^5*n^2 - 13*b^2*c^3*d^2* \\
& e^6*n^2 - 2*b^3*c^2*d*e^7*n^2)*a^6 + (48*c^6*d^6*e^2*n^2 - 168*b^2*c^4*d^4* \\
& e^4*n^2 + 72*b^3*c^3*d^3*e^5*n^2 + 49*b^4*c^2*d^2*e^6*n^2 + 2*b^5*c*d*e^7*n \\
& ^2)*a^5 + (16*c^7*d^8*n^2 - 16*b*c^6*d^7*e*n^2 - 72*b^2*c^5*d^6*e^2*n^2 + 8 \\
& 0*b^3*c^4*d^5*e^3*n^2 + 43*b^4*c^3*d^4*e^4*n^2 - 45*b^5*c^2*d^3*e^5*n^2 - 6 \\
& *b^6*c*d^2*e^6*n^2)*a^4 - (8*b^2*c^6*d^8*n^2 - 8*b^3*c^5*d^7*e*n^2 - 27*b^4 \\
& *c^4*d^6*e^2*n^2 + 40*b^5*c^3*d^5*e^3*n^2 - 7*b^6*c^2*d^4*e^4*n^2 - 6*b^7*c
\end{aligned}$$

$$\begin{aligned}
& *d^3e^5n^2)a^3 + (b^4c^5d^8n^2 - b^5c^4d^7e^n^2 - 3b^6c^3d^6e^2n^2 + 5b^7c^2d^5e^3n^2 - 2b^8c*d^4e^4n^2)a^2) *x^{(4n)} + (32a^8c^3d^7e^n^2 + 32*(3c^4d^3e^5n^2 - 2b*c^3d^2e^6n^2)a^7 + 2*(48c^5d^5e^3n^2 - 48b*c^4d^4e^4n^2 - 8b^3c^2d^2e^6n^2 - 3b^4c*d^7e^n^2)a^6 + (32c^6d^7e^n^2 - 96b^2c^4d^5e^3n^2 + 16b^3c^3d^4e^4n^2 + 30b^4c^2d^3e^5n^2 + 20b^5c*d^2e^6n^2 + b^6d^7e^n^2)a^5 + (32b*c^6d^8n^2 - 96b^2c^5d^7e^n^2 + 48b^3c^4d^6e^2n^2 + 46b^4c^3d^5e^3n^2 - 6b^5c^2d^4e^4n^2 - 21b^6c*d^3e^5n^2 - 3b^7d^2e^6n^2)a^4 - (16b^3c^5d^8n^2 - 42b^4c^4d^7e^n^2 + 24b^5c^3d^6e^2n^2 + 11b^6c^2d^5e^3n^2 - 6b^7c*d^4e^4n^2 - 3b^8d^3e^5n^2)a^3 + (2b^5c^4d^8n^2 - 5b^6c^3d^7e^n^2 + 3b^7c^2d^6e^2n^2 + b^8c*d^5e^3n^2 - b^9d^4e^4n^2)a^2) *x^{(3n)} + (32*(c^3d^2e^6n^2 + b*c^2d^7e^n^2)a^8 + 16*(6c^4d^4e^4n^2 - 6b^2c^2d^2e^6n^2 - b^3c*d^7e^n^2)a^7 + 2*(48c^5d^6e^2n^2 - 48b*c^4d^5e^3n^2 - 48b^2c^3d^4e^4n^2 + 24b^3c^2d^3e^5n^2 + 21b^4c*d^2e^6n^2 + b^5d^7e^n^2)a^6 + (32c^6d^8n^2 - 64b*c^5d^7e^n^2 + 16b^3c^3d^5e^3n^2 + 46b^4c^2d^4e^4n^2 - 24b^5c*d^3e^5n^2 - 5b^6d^2e^6n^2)a^5 - (16b^3c^4d^7e^n^2 - 30b^4c^3d^6e^2n^2 + 6b^5c^2d^5e^3n^2 + 11b^6c*d^4e^4n^2 - 3b^7d^3e^5n^2)a^4 - (6b^4c^4d^8n^2 - 20b^5c^3d^7e^n^2 + 21b^6c^2d^6e^2n^2 - 6b^7c*d^5e^3n^2 - b^8d^4e^4n^2)a^3 + (b^6c^3d^8n^2 - 3b^7c^2d^7e^n^2 + 3b^8c*d^6e^2n^2 - b^9d^5e^3n^2)a^2) *x^{(2n)} + (16a^9c^2d^7e^n^2 + 8*(6c^3d^3e^5n^2 - 2b*c^2d^2e^6n^2 - b^2c*d^7e^n^2)a^8 + (48c^4d^5e^3n^2 - 72b^2c^2d^3e^5n^2 + 8b^3c*d^2e^6n^2 + b^4d^7e^n^2)a^7 + (16c^5d^7e^n^2 + 48b*c^4d^6e^2n^2 - 168b^2c^3d^5e^3n^2 + 80b^3c^2d^4e^4n^2 + 27b^4c*d^3e^5n^2 - b^5d^2e^6n^2)a^6 + (32b*c^5d^8n^2 - 104b^2c^4d^7e^n^2 + 72b^3c^3d^6e^2n^2 + 43b^4c^2d^5e^3n^2 - 40b^5c*d^4e^4n^2 - 3b^6d^3e^5n^2)a^5 - (16b^3c^4d^8n^2 - 49b^4c^3d^7e^n^2 + 45b^5c^2d^6e^2n^2 - 7b^6c*d^5e^3n^2 - 5b^7d^4e^4n^2)a^4 + 2*(b^5c^3d^8n^2 - 3b^6c^2d^7e^n^2 + 3b^7c*d^6e^2n^2 - b^8d^5e^3n^2)a^3) *x^n) + \text{integrate}(1/2*((2n^2 - 3n + 1)*b^4c^4d^6 - 4*(2n^2 - 3n + 1)*b^5c^3d^5e + 6*(2n^2 - 3n + 1)*b^6c^2d^4e^2 - 4*(2n^2 - 3n + 1)*b^7c*d^3e^3 + (2n^2 - 3n + 1)*b^8d^2e^4 - 4*(24n^2 - 10n + 1)*a^5c^3e^6 + (4*(48n^2 - 2n - 1)*c^4d^2e^4 - 4*(96n^2 - 29n + 2)*b*c^3d^5e + (240n^2 - 115n + 13)*b^2c^2e^6)a^4 + (4*(32n^2 - 18n + 1)*c^5d^4e^2 - 8*(48n^2 - 37n + 4)*b*c^4d^3e^3 + (288n^2 - 337n + 49)*b^2c^3d^2e^4 + 2*(32n^2 + 29n - 7)*b^3c^2d^5e - (102n^2 - 55n + 7)*b^4c^2e^6)a^3 + (4*(8n^2 - 6n + 1)*c^6d^6 - 4*(32n^2 - 29n + 6)*b*c^5d^5e + (128n^2 - 137n + 39)*b^2c^4d^4e^2 + 8*(8n^2 - 7n - 1)*b^3c^3d^3e^3 - 4*(37n^2 - 43n + 6)*b^4c^2d^2e^4 + 4*(10n^2 - 16n + 3)*b^5c*d^5e + (12n^2 - 7n + 1)*b^6e^6)a^2 - ((16n^2 - 21n + 5)*b^2c^5d^6 - 2*(32n^2 - 43n + 11)*b^3c^4d^5e + 2*(44n^2 - 61n + 17)*b^4c^3d^4e^2 - 20*(2n^2 - 3n + 1)*b^5c^2d^3e^3 - (8n^2 - 7n - 1)*b^6c*d^2e^4 + 2*(4n^2 - 5n + 1)*b^7d^5e)a + ((2n^2 - 3n + 1)*b^3c^5d^6 - 4*(2n^2 - 3n + 1)*b^4c^4d^5e + 6*(2n^2 - 3n +
\end{aligned}$$

$$\begin{aligned}
& 1) * b^5 * c^3 * d^4 * e^2 - 4 * (2 * n^2 - 3 * n + 1) * b^6 * c^2 * d^3 * e^3 + (2 * n^2 - 3 * n + 1) \\
& ) * b^7 * c * d^2 * e^4 - 2 * (4 * (35 * n^2 - 12 * n + 1) * c^4 * d * e^5 - (81 * n^2 - 37 * n + 4) * \\
& b * c^3 * e^6) * a^4 - 2 * (8 * (7 * n^2 - 8 * n + 1) * c^5 * d^3 * e^3 - (83 * n^2 - 97 * n + 14) * \\
& b * c^4 * d^2 * e^4 - (44 * n^2 + 7 * n - 3) * b^2 * c^3 * d * e^5 + 3 * (15 * n^2 - 8 * n + 1) * b^3 \\
& * c^2 * e^6) * a^3 - (8 * (3 * n^2 - 4 * n + 1) * c^6 * d^5 * e - 2 * (11 * n^2 - 19 * n + 8) * b * c^5 \\
& * d^4 * e^2 - 4 * (22 * n^2 - 23 * n + 1) * b^2 * c^4 * d^3 * e^3 + (136 * n^2 - 159 * n + 23) * \\
& b^3 * c^3 * d^2 * e^4 - 2 * (16 * n^2 - 27 * n + 5) * b^4 * c^2 * d * e^5 - (12 * n^2 - 7 * n + 1) * \\
& b^5 * c * e^6) * a^2 - 2 * ((7 * n^2 - 9 * n + 2) * b * c^6 * d^6 - (28 * n^2 - 37 * n + 9) * b^2 * c^5 \\
& * d^5 * e + 2 * (19 * n^2 - 26 * n + 7) * b^3 * c^4 * d^4 * e^2 - 8 * (2 * n^2 - 3 * n + 1) * b^4 * c^3 \\
& * d^3 * e^3 - 5 * (n^2 - n) * b^5 * c^2 * d^2 * e^4 + (4 * n^2 - 5 * n + 1) * b^6 * c * d * e^5) * \\
& a) * x^n) / (16 * a^9 * c^2 * e^8 * n^2 + 8 * (8 * c^3 * d^2 * e^6 * n^2 - 8 * b * c^2 * d * e^7 * n^2 - b^2 * c * e^8 * n^2) * a^8 + (96 * c^4 * d^4 * e^4 * n^2 - 192 * b * c^3 * d^3 * e^5 * n^2 + 64 * b^2 * c^2 * d^2 * e^6 * n^2 + 32 * b^3 * c * d * e^7 * n^2 + b^4 * e^8 * n^2) * a^7 + 4 * (16 * c^5 * d^6 * e^2 * n^2 - 48 * b * c^4 * d^5 * e^3 * n^2 + 36 * b^2 * c^3 * d^4 * e^4 * n^2 + 8 * b^3 * c^2 * d^3 * e^5 * n^2 - 11 * b^4 * c * d^2 * e^6 * n^2 - b^5 * d * e^7 * n^2) * a^6 + 2 * (8 * c^6 * d^8 * n^2 - 32 * b * c^5 * d^7 * e * n^2 + 32 * b^2 * c^4 * d^6 * e^2 * n^2 + 16 * b^3 * c^3 * d^5 * e^3 * n^2 - 37 * b^4 * c^2 * d^4 * e^4 * n^2 + 10 * b^5 * c * d^3 * e^5 * n^2 + 3 * b^6 * d^2 * e^6 * n^2) * a^5 - 4 * (2 * b^2 * c^5 * d^8 * n^2 - 8 * b^3 * c^4 * d^7 * e * n^2 + 11 * b^4 * c^3 * d^6 * e^2 * n^2 - 5 * b^5 * c^2 * d^5 * e^3 * n^2 - b^6 * c * d^4 * e^4 * n^2 + b^7 * d^3 * e^5 * n^2) * a^4 + (b^4 * c^4 * d^8 * n^2 - 4 * b^5 * c^3 * d^7 * e * n^2 + 6 * b^6 * c^2 * d^6 * e^2 * n^2 - 4 * b^7 * c * d^5 * e^3 * n^2 + b^8 * d^4 * e^4 * n^2) * a^3 + (16 * a^8 * c^3 * e^8 * n^2 + 8 * (8 * c^4 * d^2 * e^6 * n^2 - 8 * b * c^3 * d * e^7 * n^2 - b^2 * c^2 * e^8 * n^2) * a^7 + (96 * c^5 * d^4 * e^4 * n^2 - 192 * b * c^4 * d^3 * e^5 * n^2 + 64 * b^2 * c^3 * d^2 * e^6 * n^2 + 32 * b^3 * c^2 * d * e^7 * n^2 + b^4 * c * e^8 * n^2) * a^6 + 4 * (16 * c^6 * d^6 * e^2 * n^2 - 48 * b * c^5 * d^5 * e^3 * n^2 + 36 * b^2 * c^4 * d^4 * e^4 * n^2 + 8 * b^3 * c^3 * d^3 * e^5 * n^2 - 11 * b^4 * c^2 * d^2 * e^6 * n^2 - b^5 * c * d * e^7 * n^2) * a^5 + 2 * (8 * c^7 * d^8 * n^2 - 32 * b * c^6 * d^7 * e * n^2 + 32 * b^2 * c^5 * d^6 * e^2 * n^2 + 16 * b^3 * c^4 * d^5 * e^3 * n^2 - 37 * b^4 * c^3 * d^4 * e^4 * n^2 + 10 * b^5 * c^2 * d^3 * e^5 * n^2 + 3 * b^6 * c * d^2 * e^6 * n^2) * a^4 - 4 * (2 * b^2 * c^6 * d^8 * n^2 - 8 * b^3 * c^5 * d^7 * e * n^2 + 11 * b^4 * c^4 * d^6 * e^2 * n^2 - 5 * b^5 * c^3 * d^5 * e^3 * n^2 - b^6 * c^2 * d^4 * e^4 * n^2 + b^7 * c * d^3 * e^5 * n^2) * a^3 + (b^4 * c^5 * d^8 * n^2 - 4 * b^5 * c^4 * d^7 * e * n^2 + 6 * b^6 * c^3 * d^6 * e^2 * n^2 - 4 * b^7 * c^2 * d^5 * e^3 * n^2 + b^8 * c * d^4 * e^4 * n^2) * a^2) * x^(2 * n) + (16 * a^8 * b * c^2 * e^8 * n^2 + 8 * (8 * b * c^3 * d^2 * e^6 * n^2 - 8 * b^2 * c^2 * d * e^7 * n^2 - b^3 * c * e^8 * n^2) * a^7 + (96 * b * c^4 * d^4 * e^4 * n^2 - 192 * b^2 * c^3 * d^3 * e^5 * n^2 + 64 * b^3 * c^2 * d^2 * e^6 * n^2 + 32 * b^4 * c * d * e^7 * n^2 + b^5 * e^8 * n^2) * a^6 + 4 * (16 * b * c^5 * d^6 * e^2 * n^2 - 48 * b^2 * c^4 * d^5 * e^3 * n^2 + 36 * b^3 * c^3 * d^4 * e^4 * n^2 + 8 * b^4 * c^2 * d^3 * e^5 * n^2 - 11 * b^5 * c * d^2 * e^6 * n^2 - b^6 * d * e^7 * n^2) * a^5 + 2 * (8 * b * c^6 * d^8 * n^2 - 32 * b^2 * c^5 * d^7 * e * n^2 + 32 * b^3 * c^4 * d^6 * e^2 * n^2 + 16 * b^4 * c^3 * d^5 * e^3 * n^2 - 37 * b^5 * c^2 * d^4 * e^4 * n^2 + 10 * b^6 * c * d^3 * e^5 * n^2 + 3 * b^7 * d^2 * e^6 * n^2) * a^4 - 4 * (2 * b^3 * c^5 * d^8 * n^2 - 8 * b^4 * c^4 * d^7 * e * n^2 + 11 * b^5 * c^3 * d^6 * e^2 * n^2 - 5 * b^6 * c^2 * d^5 * e^3 * n^2 - b^7 * c * d^4 * e^4 * n^2 + b^8 * d^3 * e^5 * n^2) * a^3 + (b^5 * c^4 * d^8 * n^2 - 4 * b^6 * c^3 * d^7 * e * n^2 + 6 * b^7 * c^2 * d^6 * e^2 * n^2 - 4 * b^8 * c * d^5 * e^3 * n^2 + b^9 * d^4 * e^4 * n^2) * a^2) * x^n), x)
\end{aligned}$$

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{1}{b^3 e^2 x^{5n} + a^3 d^2 + (c^3 e^2 x^{2n} + 2 c^3 d e x^n + c^3 d^2) x^{6n} + 3 (b c^2 e^2 x^{3n} + a c^2 d^2 + (2 b c^2 d e + a c^2 e^2) x^{2n} + (b c^2 d^2 + 2 a c^2 d e + a^3 d^2) x^n)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d+e\*x^n)^2/(a+b\*x^n+c\*x^(2\*n))^3,x, algorithm="fricas")

[Out] integral(1/(b^3\*e^2\*x^(5\*n) + a^3\*d^2 + (c^3\*e^2\*x^(2\*n) + 2\*c^3\*d\*e\*x^n + c^3\*d^2)\*x^(6\*n) + 3\*(b\*c^2\*e^2\*x^(3\*n) + a\*c^2\*d^2 + (2\*b\*c^2\*d\*e + a\*c^2\*e^2)\*x^(2\*n) + (b\*c^2\*d^2 + 2\*a\*c^2\*d\*e)\*x^n)\*x^(4\*n) + (2\*b^3\*d\*e + 3\*a\*b^2\*e^2)\*x^(4\*n) + (b^3\*d^2 + 6\*a\*b^2\*d\*e + 3\*a^2\*b\*e^2)\*x^(3\*n) + 3\*(b^2\*c\*e^2\*x^(4\*n) + a^2\*c\*d^2 + 2\*(b^2\*c\*d\*e + a\*b\*c\*e^2)\*x^(3\*n) + (b^2\*c\*d^2 + 4\*a\*b\*c\*d\*e + a^2\*c\*e^2)\*x^(2\*n) + 2\*(a\*b\*c\*d^2 + a^2\*c\*d\*e)\*x^n)\*x^(2\*n) + (3\*a\*b^2\*d^2 + 6\*a^2\*b\*d\*e + a^3\*e^2)\*x^(2\*n) + (3\*a^2\*b\*d^2 + 2\*a^3\*d\*e)\*x^n), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d+e\*x\*\*n)\*\*2/(a+b\*x\*\*n+c\*x\*\*(2\*n))\*\*3,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(c x^{2n} + b x^n + a)^3 (e x^n + d)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d+e\*x^n)^2/(a+b\*x^n+c\*x^(2\*n))^3,x, algorithm="giac")

[Out] integrate(1/((c\*x^(2\*n) + b\*x^n + a)^3\*(e\*x^n + d)^2), x)

### 3.85 $\int (d + ex^n) \sqrt{a + bx^n + cx^{2n}} dx$

**Optimal.** Leaf size=292

$$\frac{dx\sqrt{a + bx^n + cx^{2n}} F_1\left(\frac{1}{n}; -\frac{1}{2}, -\frac{1}{2}; 1 + \frac{1}{n}; -\frac{2cx^n}{b - \sqrt{b^2 - 4ac}}, -\frac{2cx^n}{b + \sqrt{b^2 - 4ac}}\right)}{\sqrt{\frac{2cx^n}{b - \sqrt{b^2 - 4ac}} + 1} \sqrt{\frac{2cx^n}{\sqrt{b^2 - 4ac} + b} + 1}} + \frac{ex^{n+1}\sqrt{a + bx^n + cx^{2n}} F_1\left(1 + \frac{1}{n}; -\frac{1}{2}, -\frac{1}{2}; 2 + \frac{1}{n}; -\frac{2cx^n}{b - \sqrt{b^2 - 4ac}}, -\frac{2cx^n}{b + \sqrt{b^2 - 4ac}}\right)}{(n+1)\sqrt{\frac{2cx^n}{b - \sqrt{b^2 - 4ac}} + 1} \sqrt{\frac{2cx^n}{\sqrt{b^2 - 4ac} + b} + 1}}$$

[Out] (e\*x^(1 + n)\*Sqrt[a + b\*x^n + c\*x^(2\*n)]\*AppellF1[1 + n^(-1), -1/2, -1/2, 2 + n^(-1), (-2\*c\*x^n)/(b - Sqrt[b^2 - 4\*a\*c]), (-2\*c\*x^n)/(b + Sqrt[b^2 - 4\*a\*c])])/((1 + n)\*Sqrt[1 + (2\*c\*x^n)/(b - Sqrt[b^2 - 4\*a\*c])]\*Sqrt[1 + (2\*c\*x^n)/(b + Sqrt[b^2 - 4\*a\*c])]) + (d\*x\*Sqrt[a + b\*x^n + c\*x^(2\*n)]\*AppellF1[n^(-1), -1/2, -1/2, 1 + n^(-1), (-2\*c\*x^n)/(b - Sqrt[b^2 - 4\*a\*c]), (-2\*c\*x^n)/(b + Sqrt[b^2 - 4\*a\*c])])/((Sqrt[1 + (2\*c\*x^n)/(b - Sqrt[b^2 - 4\*a\*c])]\*Sqrt[1 + (2\*c\*x^n)/(b + Sqrt[b^2 - 4\*a\*c])]))

**Rubi [A]** time = 0.355107, antiderivative size = 292, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$ , Rules used = {1432, 1348, 429, 1385, 510}

$$\frac{dx\sqrt{a + bx^n + cx^{2n}} F_1\left(\frac{1}{n}; -\frac{1}{2}, -\frac{1}{2}; 1 + \frac{1}{n}; -\frac{2cx^n}{b - \sqrt{b^2 - 4ac}}, -\frac{2cx^n}{b + \sqrt{b^2 - 4ac}}\right)}{\sqrt{\frac{2cx^n}{b - \sqrt{b^2 - 4ac}} + 1} \sqrt{\frac{2cx^n}{\sqrt{b^2 - 4ac} + b} + 1}} + \frac{ex^{n+1}\sqrt{a + bx^n + cx^{2n}} F_1\left(1 + \frac{1}{n}; -\frac{1}{2}, -\frac{1}{2}; 2 + \frac{1}{n}; -\frac{2cx^n}{b - \sqrt{b^2 - 4ac}}, -\frac{2cx^n}{b + \sqrt{b^2 - 4ac}}\right)}{(n+1)\sqrt{\frac{2cx^n}{b - \sqrt{b^2 - 4ac}} + 1} \sqrt{\frac{2cx^n}{\sqrt{b^2 - 4ac} + b} + 1}}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x^n)\*Sqrt[a + b\*x^n + c\*x^(2\*n)], x]

[Out] (e\*x^(1 + n)\*Sqrt[a + b\*x^n + c\*x^(2\*n)]\*AppellF1[1 + n^(-1), -1/2, -1/2, 2 + n^(-1), (-2\*c\*x^n)/(b - Sqrt[b^2 - 4\*a\*c]), (-2\*c\*x^n)/(b + Sqrt[b^2 - 4\*a\*c])])/((1 + n)\*Sqrt[1 + (2\*c\*x^n)/(b - Sqrt[b^2 - 4\*a\*c])]\*Sqrt[1 + (2\*c\*x^n)/(b + Sqrt[b^2 - 4\*a\*c])]) + (d\*x\*Sqrt[a + b\*x^n + c\*x^(2\*n)]\*AppellF1[n^(-1), -1/2, -1/2, 1 + n^(-1), (-2\*c\*x^n)/(b - Sqrt[b^2 - 4\*a\*c]), (-2\*c\*x^n)/(b + Sqrt[b^2 - 4\*a\*c])])/((Sqrt[1 + (2\*c\*x^n)/(b - Sqrt[b^2 - 4\*a\*c])]\*Sqrt[1 + (2\*c\*x^n)/(b + Sqrt[b^2 - 4\*a\*c])]))

#### Rule 1432

Int[((d\_) + (e\_.)\*(x\_)^(n\_))\*((a\_) + (b\_.)\*(x\_)^(n\_) + (c\_.)\*(x\_)^(n2\_))^(p\_), x\_Symbol] :> Int[ExpandIntegrand[(d + e\*x^n)\*(a + b\*x^n + c\*x^(2\*n))]^p,

$x], x] /;$  FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2\*n] && NeQ[b^2 - 4\*a\*c, 0]

### Rule 1348

Int[((a\_) + (c\_.)\*(x\_)^(n2\_.) + (b\_.)\*(x\_)^(n\_.))^p\_], x\_Symbol] := Dist[(a^IntPart[p]\*(a + b\*x^n + c\*x^(2\*n))^FracPart[p])/((1 + (2\*c\*x^n)/(b + Rt[b^2 - 4\*a\*c, 2]))^FracPart[p]\*(1 + (2\*c\*x^n)/(b - Rt[b^2 - 4\*a\*c, 2]))^FracPart[p]), Int[(1 + (2\*c\*x^n)/(b + Sqrt[b^2 - 4\*a\*c]))^p\*(1 + (2\*c\*x^n)/(b - Sqrt[b^2 - 4\*a\*c]))^p, x], x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2\*n] && NeQ[b^2 - 4\*a\*c, 0] && !IntegerQ[p]

### Rule 429

Int[((a\_) + (b\_.)\*(x\_)^(n\_.))^p\_\*((c\_) + (d\_.)\*(x\_)^(n\_.))^q\_], x\_Symbol] := Simp[a^p\*c^q\*x\*AppellF1[1/n, -p, -q, 1 + 1/n, -((b\*x^n)/a), -((d\*x^n)/c)], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b\*c - a\*d, 0] && NeQ[n, -1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])

### Rule 1385

Int[((d\_.)\*(x\_)^(m\_.)\*((a\_) + (c\_.)\*(x\_)^(n2\_.) + (b\_.)\*(x\_)^(n\_.))^p\_], x\_Symbol] := Dist[(a^IntPart[p]\*(a + b\*x^n + c\*x^(2\*n))^FracPart[p])/((1 + (2\*c\*x^n)/(b + Rt[b^2 - 4\*a\*c, 2]))^FracPart[p]\*(1 + (2\*c\*x^n)/(b - Rt[b^2 - 4\*a\*c, 2]))^FracPart[p]), Int[(d\*x)^m\*(1 + (2\*c\*x^n)/(b + Sqrt[b^2 - 4\*a\*c]))^p\*(1 + (2\*c\*x^n)/(b - Sqrt[b^2 - 4\*a\*c]))^p, x], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2\*n]

### Rule 510

Int[((e\_.)\*(x\_)^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_.))^p\_\*((c\_) + (d\_.)\*(x\_)^(n\_.))^q\_], x\_Symbol] := Simp[(a^p\*c^q\*(e\*x)^(m + 1)\*AppellF1[(m + 1)/n, -p, -q, 1 + (m + 1)/n, -((b\*x^n)/a), -((d\*x^n)/c)]/(e\*(m + 1)), x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b\*c - a\*d, 0] && NeQ[m, -1] && NeQ[m, n - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])

### Rubi steps

$$\begin{aligned}
\int (d + ex^n) \sqrt{a + bx^n + cx^{2n}} dx &= \int \left( d\sqrt{a + bx^n + cx^{2n}} + ex^n \sqrt{a + bx^n + cx^{2n}} \right) dx \\
&= d \int \sqrt{a + bx^n + cx^{2n}} dx + e \int x^n \sqrt{a + bx^n + cx^{2n}} dx \\
&= \frac{\left( d\sqrt{a + bx^n + cx^{2n}} \right) \int \sqrt{1 + \frac{2cx^n}{b - \sqrt{b^2 - 4ac}}} \sqrt{1 + \frac{2cx^n}{b + \sqrt{b^2 - 4ac}}} dx}{\sqrt{1 + \frac{2cx^n}{b - \sqrt{b^2 - 4ac}}} \sqrt{1 + \frac{2cx^n}{b + \sqrt{b^2 - 4ac}}}} + \frac{\left( e\sqrt{a + bx^n + cx^{2n}} \right) \int x^n \sqrt{1 + \frac{2cx^n}{b - \sqrt{b^2 - 4ac}}} \sqrt{1 + \frac{2cx^n}{b + \sqrt{b^2 - 4ac}}} dx}{\sqrt{1 + \frac{2cx^n}{b - \sqrt{b^2 - 4ac}}} \sqrt{1 + \frac{2cx^n}{b + \sqrt{b^2 - 4ac}}}} \\
&= \frac{ex^{1+n} \sqrt{a + bx^n + cx^{2n}} F_1 \left( 1 + \frac{1}{n}; -\frac{1}{2}, -\frac{1}{2}; 2 + \frac{1}{n}; -\frac{2cx^n}{b - \sqrt{b^2 - 4ac}}, -\frac{2cx^n}{b + \sqrt{b^2 - 4ac}} \right)}{(1+n) \sqrt{1 + \frac{2cx^n}{b - \sqrt{b^2 - 4ac}}} \sqrt{1 + \frac{2cx^n}{b + \sqrt{b^2 - 4ac}}}} + \frac{dx \sqrt{a + bx^n + cx^{2n}}}{\sqrt{1 + \frac{2cx^n}{b - \sqrt{b^2 - 4ac}}} \sqrt{1 + \frac{2cx^n}{b + \sqrt{b^2 - 4ac}}}}
\end{aligned}$$

**Mathematica [A]** time = 1.59531, size = 424, normalized size = 1.45

$$x \left( 2(n+1) \left( an \sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^n}{b-\sqrt{b^2-4ac}}} \sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^n}{\sqrt{b^2-4ac}+b}} (2c(2dn+d) - be) F_1 \left( \frac{1}{n}; \frac{1}{2}, \frac{1}{2}; 1 + \frac{1}{n}; -\frac{2cx^n}{b+\sqrt{b^2-4ac}}, \frac{2cx^n}{\sqrt{b^2-4ac}-b} \right) + (a + x^n) \sqrt{a + bx^n + cx^{2n}} \right) \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[(d + e\*x^n)\*Sqrt[a + b\*x^n + c\*x^(2\*n)], x]

[Out] (x\*(-(n\*(-4\*a\*c\*e\*(1+n) + b^2\*e\*(2+n) - 2\*b\*c\*d\*(1+2\*n))\*x^n\*Sqrt[(b - Sqrt[b^2 - 4\*a\*c] + 2\*c\*x^n)/(b - Sqrt[b^2 - 4\*a\*c]])\*Sqrt[(b + Sqrt[b^2 - 4\*a\*c] + 2\*c\*x^n)/(b + Sqrt[b^2 - 4\*a\*c]])\*AppellF1[1 + n^(-1), 1/2, 1/2, 2 + n^(-1), (-2\*c\*x^n)/(b + Sqrt[b^2 - 4\*a\*c]), (2\*c\*x^n)/(-b + Sqrt[b^2 - 4\*a\*c])]) + 2\*(1+n)\*((a + x^n\*(b + c\*x^n))\*(b\*e\*n + 2\*c\*(d + 2\*d\*n + e\*(1+n)\*x^n)) + a\*n\*(-(b\*e) + 2\*c\*(d + 2\*d\*n))\*Sqrt[(b - Sqrt[b^2 - 4\*a\*c] + 2\*c\*x^n)/(b - Sqrt[b^2 - 4\*a\*c]])\*Sqrt[(b + Sqrt[b^2 - 4\*a\*c] + 2\*c\*x^n)/(b + Sqrt[b^2 - 4\*a\*c]])\*AppellF1[n^(-1), 1/2, 1/2, 1 + n^(-1), (-2\*c\*x^n)/(b + Sqrt[b^2 - 4\*a\*c]), (2\*c\*x^n)/(-b + Sqrt[b^2 - 4\*a\*c])])))/(4\*(1+n)^2\*(c + 2\*c\*n)\*Sqrt[a + x^n\*(b + c\*x^n)])

**Maple [F]** time = 0.062, size = 0, normalized size = 0.

$$\int (d + ex^n) \sqrt{a + bx^n + cx^{2n}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d+e*x^n)*(a+b*x^n+c*x^(2*n))^(1/2),x)`

[Out] `int((d+e*x^n)*(a+b*x^n+c*x^(2*n))^(1/2),x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{cx^{2n} + bx^n + a}(ex^n + d) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d+e*x^n)*(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="maxima")`

[Out] `integrate(sqrt(c*x^(2*n) + b*x^n + a)*(e*x^n + d), x)`

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d+e*x^n)*(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="fricas")`

[Out] Exception raised: UnboundLocalError

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int (d + ex^n) \sqrt{a + bx^n + cx^{2n}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d+e*x**n)*(a+b*x**n+c*x**(2*n))**(1/2),x)`

[Out] `Integral((d + e*x**n)*sqrt(a + b*x**n + c*x**(2*n)), x)`



**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{cx^{2n} + bx^n + a}(ex^n + d) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x^n)*(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^(2*n) + b*x^n + a)*(e*x^n + d), x)
```

### 3.86 $\int (d + ex^n) (a + bx^n + cx^{2n})^{3/2} dx$

**Optimal.** Leaf size=294

$$\frac{adx\sqrt{a+bx^n+cx^{2n}}F_1\left(\frac{1}{n};-\frac{3}{2},-\frac{3}{2};1+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}} + \frac{aex^{n+1}\sqrt{a+bx^n+cx^{2n}}F_1\left(1+\frac{1}{n};-\frac{3}{2},-\frac{3}{2};2+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{(n+1)\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}$$

[Out] (a\*e\*x^(1+n)\*Sqrt[a+b\*x^n+c\*x^(2\*n)]\*AppellF1[1+n^(-1), -3/2, -3/2, 2+n^(-1), (-2\*c\*x^n)/(b-Sqrt[b^2-4\*a\*c]), (-2\*c\*x^n)/(b+Sqrt[b^2-4\*a\*c])])/((1+n)\*Sqrt[1+(2\*c\*x^n)/(b-Sqrt[b^2-4\*a\*c])]\*Sqrt[1+(2\*c\*x^n)/(b+Sqrt[b^2-4\*a\*c])]) + (a\*d\*x\*Sqrt[a+b\*x^n+c\*x^(2\*n)]\*AppellF1[n^(-1), -3/2, -3/2, 1+n^(-1), (-2\*c\*x^n)/(b-Sqrt[b^2-4\*a\*c]), (-2\*c\*x^n)/(b+Sqrt[b^2-4\*a\*c])])/(Sqrt[1+(2\*c\*x^n)/(b-Sqrt[b^2-4\*a\*c])]\*Sqrt[1+(2\*c\*x^n)/(b+Sqrt[b^2-4\*a\*c])])

**Rubi [A]** time = 0.347993, antiderivative size = 294, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$ , Rules used = {1432, 1348, 429, 1385, 510}

$$\frac{adx\sqrt{a+bx^n+cx^{2n}}F_1\left(\frac{1}{n};-\frac{3}{2},-\frac{3}{2};1+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}} + \frac{aex^{n+1}\sqrt{a+bx^n+cx^{2n}}F_1\left(1+\frac{1}{n};-\frac{3}{2},-\frac{3}{2};2+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{(n+1)\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x^n)\*(a + b\*x^n + c\*x^(2\*n))^(3/2), x]

[Out] (a\*e\*x^(1+n)\*Sqrt[a+b\*x^n+c\*x^(2\*n)]\*AppellF1[1+n^(-1), -3/2, -3/2, 2+n^(-1), (-2\*c\*x^n)/(b-Sqrt[b^2-4\*a\*c]), (-2\*c\*x^n)/(b+Sqrt[b^2-4\*a\*c])])/((1+n)\*Sqrt[1+(2\*c\*x^n)/(b-Sqrt[b^2-4\*a\*c])]\*Sqrt[1+(2\*c\*x^n)/(b+Sqrt[b^2-4\*a\*c])]) + (a\*d\*x\*Sqrt[a+b\*x^n+c\*x^(2\*n)]\*AppellF1[n^(-1), -3/2, -3/2, 1+n^(-1), (-2\*c\*x^n)/(b-Sqrt[b^2-4\*a\*c]), (-2\*c\*x^n)/(b+Sqrt[b^2-4\*a\*c])])/(Sqrt[1+(2\*c\*x^n)/(b-Sqrt[b^2-4\*a\*c])]\*Sqrt[1+(2\*c\*x^n)/(b+Sqrt[b^2-4\*a\*c])])

#### Rule 1432

Int[((d\_) + (e\_.)\*(x\_)^(n\_))\*((a\_) + (b\_.)\*(x\_)^(n\_) + (c\_.)\*(x\_)^(n2\_))^(p\_), x\_Symbol] := Int[ExpandIntegrand[(d + e\*x^n)\*(a + b\*x^n + c\*x^(2\*n))^(p),

$x], x] /; \text{FreeQ}\{a, b, c, d, e, n\}, x] \&\& \text{EqQ}[n^2, 2*n] \&\& \text{NeQ}[b^2 - 4*a*c, 0]$

### Rule 1348

$\text{Int}[(a_ + (c_)*(x_)^{(n2_)} + (b_)*(x_)^{(n_)})^{(p_)}, x\_Symbol] :> \text{Dist}[(a^{ \text{IntPart}[p]}*(a + b*x^n + c*x^{(2*n)})^{\text{FracPart}[p]})/((1 + (2*c*x^n)/(b + \text{Rt}[b^2 - 4*a*c, 2]))^{\text{FracPart}[p]}*(1 + (2*c*x^n)/(b - \text{Rt}[b^2 - 4*a*c, 2]))^{\text{FracPart}[p]}), \text{Int}[(1 + (2*c*x^n)/(b + \text{Sqrt}[b^2 - 4*a*c]))^p*(1 + (2*c*x^n)/(b - \text{Sqrt}[b^2 - 4*a*c]))^p, x], x] /; \text{FreeQ}\{a, b, c, n, p\}, x] \&\& \text{EqQ}[n^2, 2*n] \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& !\text{IntegerQ}[p]$

### Rule 429

$\text{Int}[(a_ + (b_)*(x_)^{(n_)})^{(p_)}*((c_ + (d_)*(x_)^{(n_)})^{(q_)}), x\_Symbol] :> \text{Simp}[a^p*c^q*x*\text{AppellF1}[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)], x] /; \text{FreeQ}\{a, b, c, d, n, p, q\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[n, -1] \&\& (\text{IntegerQ}[p] || \text{GtQ}[a, 0]) \&\& (\text{IntegerQ}[q] || \text{GtQ}[c, 0])$

### Rule 1385

$\text{Int}[(d_)*(x_)^{(m_)}*((a_ + (c_)*(x_)^{(n2_)} + (b_)*(x_)^{(n_)})^{(p_)}, x\_Symbol] :> \text{Dist}[(a^{\text{IntPart}[p]}*(a + b*x^n + c*x^{(2*n)})^{\text{FracPart}[p]})/((1 + (2*c*x^n)/(b + \text{Rt}[b^2 - 4*a*c, 2]))^{\text{FracPart}[p]}*(1 + (2*c*x^n)/(b - \text{Rt}[b^2 - 4*a*c, 2]))^{\text{FracPart}[p]}), \text{Int}[(d*x)^m*(1 + (2*c*x^n)/(b + \text{Sqrt}[b^2 - 4*a*c]))^p*(1 + (2*c*x^n)/(b - \text{Sqrt}[b^2 - 4*a*c]))^p, x], x] /; \text{FreeQ}\{a, b, c, d, m, n, p\}, x] \&\& \text{EqQ}[n^2, 2*n]$

### Rule 510

$\text{Int}[(e_)*(x_)^{(m_)}*((a_ + (b_)*(x_)^{(n_)})^{(p_)}*((c_ + (d_)*(x_)^{(n_)})^{(q_)}), x\_Symbol] :> \text{Simp}[(a^p*c^q*(e*x)^{(m+1)}*\text{AppellF1}[(m+1)/n, -p, -q, 1 + (m+1)/n, -((b*x^n)/a), -((d*x^n)/c)]/(e*(m+1)), x] /; \text{FreeQ}\{a, b, c, d, e, m, n, p, q\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[m, -1] \&\& \text{NeQ}[m, n - 1] \&\& (\text{IntegerQ}[p] || \text{GtQ}[a, 0]) \&\& (\text{IntegerQ}[q] || \text{GtQ}[c, 0])$

### Rubi steps

$$\begin{aligned}
\int (d + ex^n)(a + bx^n + cx^{2n})^{3/2} dx &= \int \left( d(a + bx^n + cx^{2n})^{3/2} + ex^n(a + bx^n + cx^{2n})^{3/2} \right) dx \\
&= d \int (a + bx^n + cx^{2n})^{3/2} dx + e \int x^n (a + bx^n + cx^{2n})^{3/2} dx \\
&= \frac{\left( ad\sqrt{a + bx^n + cx^{2n}} \right) \int \left( 1 + \frac{2cx^n}{b - \sqrt{b^2 - 4ac}} \right)^{3/2} \left( 1 + \frac{2cx^n}{b + \sqrt{b^2 - 4ac}} \right)^{3/2} dx}{\sqrt{1 + \frac{2cx^n}{b - \sqrt{b^2 - 4ac}}} \sqrt{1 + \frac{2cx^n}{b + \sqrt{b^2 - 4ac}}}} + \frac{\left( ae\sqrt{a + bx^n + cx^{2n}} \right) \int \left( 1 + \frac{2cx^n}{b - \sqrt{b^2 - 4ac}} \right)^{3/2} \left( 1 + \frac{2cx^n}{b + \sqrt{b^2 - 4ac}} \right)^{3/2} dx}{\sqrt{1 + \frac{2cx^n}{b - \sqrt{b^2 - 4ac}}} \sqrt{1 + \frac{2cx^n}{b + \sqrt{b^2 - 4ac}}}} \\
&= \frac{aex^{1+n}\sqrt{a + bx^n + cx^{2n}} F_1 \left( 1 + \frac{1}{n}; -\frac{3}{2}, -\frac{3}{2}; 2 + \frac{1}{n}; -\frac{2cx^n}{b - \sqrt{b^2 - 4ac}}, -\frac{2cx^n}{b + \sqrt{b^2 - 4ac}} \right)}{(1+n)\sqrt{1 + \frac{2cx^n}{b - \sqrt{b^2 - 4ac}}} \sqrt{1 + \frac{2cx^n}{b + \sqrt{b^2 - 4ac}}}} + \frac{adx\sqrt{a + bx^n + cx^{2n}}}{\sqrt{1 + \frac{2cx^n}{b - \sqrt{b^2 - 4ac}}} \sqrt{1 + \frac{2cx^n}{b + \sqrt{b^2 - 4ac}}}}
\end{aligned}$$

**Mathematica [B]** time = 4.54331, size = 690, normalized size = 2.35

$$x \left( 3n^2 x^n \sqrt{\frac{-\sqrt{b^2 - 4ac} + b + 2cx^n}{b - \sqrt{b^2 - 4ac}}} \sqrt{\frac{\sqrt{b^2 - 4ac} + b + 2cx^n}{\sqrt{b^2 - 4ac} + b}} F_1 \left( 1 + \frac{1}{n}; \frac{1}{2}, \frac{1}{2}; 2 + \frac{1}{n}; -\frac{2cx^n}{b + \sqrt{b^2 - 4ac}}, \frac{2cx^n}{\sqrt{b^2 - 4ac} - b} \right) (16a^2 c^2 e (3n^2 + 4n + 1) - 4ab^2 ce) \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[(d + e\*x^n)\*(a + b\*x^n + c\*x^(2\*n))^(3/2), x]

[Out] (x\*(3\*n^2\*(16\*a^2\*c^2\*e\*(1 + 4\*n + 3\*n^2) + b^4\*e\*(4 + 8\*n + 3\*n^2) - 2\*b^3\*c\*d\*(2 + 9\*n + 4\*n^2) - 4\*a\*b^2\*c\*e\*(5 + 14\*n + 6\*n^2) + 8\*a\*b\*c^2\*d\*(2 + 11\*n + 12\*n^2))\*x^n\*Sqrt[(b - Sqrt[b^2 - 4\*a\*c] + 2\*c\*x^n)/(b - Sqrt[b^2 - 4\*a\*c]])\*Sqrt[(b + Sqrt[b^2 - 4\*a\*c] + 2\*c\*x^n)/(b + Sqrt[b^2 - 4\*a\*c]])\*AppellF1[1 + n^(-1), 1/2, 1/2, 2 + n^(-1), (-2\*c\*x^n)/(b + Sqrt[b^2 - 4\*a\*c]), (2\*c\*x^n)/(-b + Sqrt[b^2 - 4\*a\*c])] + 2\*(1 + n)\*((a + x^n\*(b + c\*x^n))\*(-3\*b^3\*e\*n^2\*(2 + 3\*n) + 6\*b^2\*c\*n^2\*(d + 4\*d\*n + e\*(1 + n)\*x^n) + 8\*c^3\*(1 + 3\*n + 2\*n^2)\*x^(2\*n)\*(d + 4\*d\*n + e\*(1 + 3\*n)\*x^n) + 4\*b\*c^2\*(1 + n)\*x^n\*(d\*(2 + 15\*n + 28\*n^2) + e\*(2 + 13\*n + 18\*n^2)\*x^n) + 4\*a\*c\*(3\*b\*e\*n^2\*(2 + 5\*n) + 2\*c\*(d\*(1 + 2\*n)\*(1 + 4\*n)^2 + e\*(1 + 9\*n + 23\*n^2 + 15\*n^3)\*x^n))) + 3\*a\*n^2\*(b^3\*e\*(2 + 3\*n) - 2\*b^2\*c\*d\*(1 + 4\*n) - 4\*a\*b\*c\*e\*(2 + 5\*n) + 8\*a\*c^2\*d\*(1 + 6\*n + 8\*n^2))\*Sqrt[(b - Sqrt[b^2 - 4\*a\*c] + 2\*c\*x^n)/(b - Sqrt[b^2 - 4\*a\*c]])\*Sqrt[(b + Sqrt[b^2 - 4\*a\*c] + 2\*c\*x^n)/(b + Sqrt[b^2 - 4\*a\*c]])\*AppellF1[n^(-1), 1/2, 1/2, 1 + n^(-1), (-2\*c\*x^n)/(b + Sqrt[b^2 - 4\*a\*c]), (2\*c\*x^n)/(-b + Sqrt[b^2 - 4\*a\*c])])))/(16\*c^2\*(1 + n)^2\*(1 + 2\*n)\*(1 + 3\*n)\*(1 + 4\*n)\*Sqrt[a + x^n\*(b + c\*x^n)])

**Maple [F]** time = 0.059, size = 0, normalized size = 0.

$$\int (d + ex^n) (a + bx^n + cx^{2n})^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d+e\*x^n)\*(a+b\*x^n+c\*x^(2\*n))^(3/2),x)

[Out] int((d+e\*x^n)\*(a+b\*x^n+c\*x^(2\*n))^(3/2),x)

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (cx^{2n} + bx^n + a)^{\frac{3}{2}}(ex^n + d) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e\*x^n)\*(a+b\*x^n+c\*x^(2\*n))^(3/2),x, algorithm="maxima")

[Out] integrate((c\*x^(2\*n) + b\*x^n + a)^(3/2)\*(e\*x^n + d), x)

---

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e\*x^n)\*(a+b\*x^n+c\*x^(2\*n))^(3/2),x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x**n)*(a+b*x**n+c*x**(2*n))**(3/2),x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (cx^{2n} + bx^n + a)^{\frac{3}{2}}(ex^n + d) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x^n)*(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^(2*n) + b*x^n + a)^(3/2)*(e*x^n + d), x)
```

$$3.87 \quad \int \frac{d+ex^n}{\sqrt{a+bx^n+cx^{2n}}} dx$$

**Optimal.** Leaf size=292

$$\frac{dx \sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}} + 1 \sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}} + 1F_1\left(\frac{1}{n}; \frac{1}{2}, \frac{1}{2}; 1 + \frac{1}{n}; -\frac{2cx^n}{b-\sqrt{b^2-4ac}}, -\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{a+bx^n+cx^{2n}}} + \frac{ex^{n+1} \sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}} + 1 \sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}} + 1F_1\left(\frac{1}{n+1}; \frac{1}{2}, \frac{1}{2}; 1 + \frac{1}{n+1}; -\frac{2cx^n}{b-\sqrt{b^2-4ac}}, -\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{(n+1)\sqrt{a+bx^n+cx^{2n}}}$$

[Out] (e\*x^(1+n)\*Sqrt[1+(2\*c\*x^n)/(b-Sqrt[b^2-4\*a\*c])]\*Sqrt[1+(2\*c\*x^n)/(b+Sqrt[b^2-4\*a\*c])]\*AppellF1[1+n^(-1), 1/2, 1/2, 2+n^(-1), (-2\*c\*x^n)/(b-Sqrt[b^2-4\*a\*c]), (-2\*c\*x^n)/(b+Sqrt[b^2-4\*a\*c])])/((1+n)\*Sqrt[a+b\*x^n+c\*x^(2\*n)]) + (d\*x\*Sqrt[1+(2\*c\*x^n)/(b-Sqrt[b^2-4\*a\*c])]\*Sqrt[1+(2\*c\*x^n)/(b+Sqrt[b^2-4\*a\*c])]\*AppellF1[n^(-1), 1/2, 1/2, 1+n^(-1), (-2\*c\*x^n)/(b-Sqrt[b^2-4\*a\*c]), (-2\*c\*x^n)/(b+Sqrt[b^2-4\*a\*c])])/Sqrt[a+b\*x^n+c\*x^(2\*n)])

**Rubi [A]** time = 0.338118, antiderivative size = 292, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$ , Rules used = {1432, 1348, 429, 1385, 510}

$$\frac{dx \sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}} + 1 \sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}} + 1F_1\left(\frac{1}{n}; \frac{1}{2}, \frac{1}{2}; 1 + \frac{1}{n}; -\frac{2cx^n}{b-\sqrt{b^2-4ac}}, -\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{a+bx^n+cx^{2n}}} + \frac{ex^{n+1} \sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}} + 1 \sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}} + 1F_1\left(\frac{1}{n+1}; \frac{1}{2}, \frac{1}{2}; 1 + \frac{1}{n+1}; -\frac{2cx^n}{b-\sqrt{b^2-4ac}}, -\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{(n+1)\sqrt{a+bx^n+cx^{2n}}}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x^n)/Sqrt[a + b\*x^n + c\*x^(2\*n)], x]

[Out] (e\*x^(1+n)\*Sqrt[1+(2\*c\*x^n)/(b-Sqrt[b^2-4\*a\*c])]\*Sqrt[1+(2\*c\*x^n)/(b+Sqrt[b^2-4\*a\*c])]\*AppellF1[1+n^(-1), 1/2, 1/2, 2+n^(-1), (-2\*c\*x^n)/(b-Sqrt[b^2-4\*a\*c]), (-2\*c\*x^n)/(b+Sqrt[b^2-4\*a\*c])])/((1+n)\*Sqrt[a+b\*x^n+c\*x^(2\*n)]) + (d\*x\*Sqrt[1+(2\*c\*x^n)/(b-Sqrt[b^2-4\*a\*c])]\*Sqrt[1+(2\*c\*x^n)/(b+Sqrt[b^2-4\*a\*c])]\*AppellF1[n^(-1), 1/2, 1/2, 1+n^(-1), (-2\*c\*x^n)/(b-Sqrt[b^2-4\*a\*c]), (-2\*c\*x^n)/(b+Sqrt[b^2-4\*a\*c])])/Sqrt[a+b\*x^n+c\*x^(2\*n)])

### Rule 1432

Int[((d\_) + (e\_.)\*(x\_)^(n\_))\*((a\_) + (b\_.)\*(x\_)^(n\_) + (c\_.)\*(x\_)^(n2\_))^(p\_), x\_Symbol] :> Int[ExpandIntegrand[(d + e\*x^n)\*(a + b\*x^n + c\*x^(2\*n))]^p, x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2\*n] && NeQ[b^2 - 4\*a\*c,

0]

Rule 1348

```
Int[((a_) + (c_)*(x_)^(n2_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Dist[(a
^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/((1 + (2*c*x^n)/(b + Rt[b^
2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^n)/(b - Rt[b^2 - 4*a*c, 2]))^FracPa
rt[p]), Int[(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^n)/(b - S
qrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n] &
& NeQ[b^2 - 4*a*c, 0] && !IntegerQ[p]
```

Rule 429

```
Int[((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_), x_Symbol]
:= Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
&& (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1385

```
Int[((d_)*(x_)^(m_))*((a_) + (c_)*(x_)^(n2_) + (b_)*(x_)^(n_))^(p_), x
_Symbol] := Dist[(a^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/((1 + (
2*c*x^n)/(b + Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^n)/(b - Rt[b^2 -
4*a*c, 2]))^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]
))^p*(1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rule 510

```
Int[((e_)*(x_)^(m_))*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_
))^(q_), x_Symbol] := Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)]/(e*(m + 1)), x] /; FreeQ[{a,
b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
- 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps



$$\begin{aligned}
\int \frac{d + ex^n}{\sqrt{a + bx^n + cx^{2n}}} dx &= \int \left( \frac{d}{\sqrt{a + bx^n + cx^{2n}}} + \frac{ex^n}{\sqrt{a + bx^n + cx^{2n}}} \right) dx \\
&= d \int \frac{1}{\sqrt{a + bx^n + cx^{2n}}} dx + e \int \frac{x^n}{\sqrt{a + bx^n + cx^{2n}}} dx \\
&= \frac{\left( d \sqrt{1 + \frac{2cx^n}{b - \sqrt{b^2 - 4ac}}} \sqrt{1 + \frac{2cx^n}{b + \sqrt{b^2 - 4ac}}} \right) \int \frac{1}{\sqrt{1 + \frac{2cx^n}{b - \sqrt{b^2 - 4ac}}} \sqrt{1 + \frac{2cx^n}{b + \sqrt{b^2 - 4ac}}} dx}{\sqrt{a + bx^n + cx^{2n}}} + \frac{\left( e \sqrt{1 + \frac{2cx^n}{b - \sqrt{b^2 - 4ac}}} \sqrt{1 + \frac{2cx^n}{b + \sqrt{b^2 - 4ac}}} \right) \int \frac{x^n}{\sqrt{1 + \frac{2cx^n}{b - \sqrt{b^2 - 4ac}}} \sqrt{1 + \frac{2cx^n}{b + \sqrt{b^2 - 4ac}}} dx}{\sqrt{a + bx^n + cx^{2n}}} \\
&= \frac{ex^{1+n} \sqrt{1 + \frac{2cx^n}{b - \sqrt{b^2 - 4ac}}} \sqrt{1 + \frac{2cx^n}{b + \sqrt{b^2 - 4ac}}} F_1 \left( 1 + \frac{1}{n}; \frac{1}{2}, \frac{1}{2}; 2 + \frac{1}{n}; -\frac{2cx^n}{b - \sqrt{b^2 - 4ac}}, -\frac{2cx^n}{b + \sqrt{b^2 - 4ac}} \right) dx \sqrt{1 + \frac{2cx^n}{b - \sqrt{b^2 - 4ac}}} \sqrt{1 + \frac{2cx^n}{b + \sqrt{b^2 - 4ac}}}}{(1+n)\sqrt{a + bx^n + cx^{2n}}} + \frac{dx \sqrt{1 + \frac{2cx^n}{b - \sqrt{b^2 - 4ac}}} \sqrt{1 + \frac{2cx^n}{b + \sqrt{b^2 - 4ac}}}}{(1+n)\sqrt{a + bx^n + cx^{2n}}}
\end{aligned}$$

**Mathematica [A]** time = 0.348705, size = 245, normalized size = 0.84

$$\frac{x \sqrt{\frac{-\sqrt{b^2 - 4ac} + b + 2cx^n}{b - \sqrt{b^2 - 4ac}}} \sqrt{\frac{\sqrt{b^2 - 4ac} + b + 2cx^n}{\sqrt{b^2 - 4ac} + b}} \left( d(n+1) F_1 \left( \frac{1}{n}; \frac{1}{2}, \frac{1}{2}; 1 + \frac{1}{n}; -\frac{2cx^n}{b + \sqrt{b^2 - 4ac}}, \frac{2cx^n}{\sqrt{b^2 - 4ac} - b} \right) + ex^n F_1 \left( 1 + \frac{1}{n}; \frac{1}{2}, \frac{1}{2}; 2 + \frac{1}{n}; -\frac{2cx^n}{b + \sqrt{b^2 - 4ac}}, -\frac{2cx^n}{b + \sqrt{b^2 - 4ac}} \right) \right)}{(n+1)\sqrt{a + x^n(b + cx^n)}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(d + e\*x^n)/Sqrt[a + b\*x^n + c\*x^(2\*n)], x]

[Out] (x\*Sqrt[(b - Sqrt[b^2 - 4\*a\*c] + 2\*c\*x^n)/(b - Sqrt[b^2 - 4\*a\*c]])\*Sqrt[(b + Sqrt[b^2 - 4\*a\*c] + 2\*c\*x^n)/(b + Sqrt[b^2 - 4\*a\*c]])\*(e\*x^n\*AppellF1[1 + n^(-1), 1/2, 1/2, 2 + n^(-1), (-2\*c\*x^n)/(b + Sqrt[b^2 - 4\*a\*c]), (2\*c\*x^n)/(-b + Sqrt[b^2 - 4\*a\*c])] + d\*(1 + n)\*AppellF1[n^(-1), 1/2, 1/2, 1 + n^(-1), (-2\*c\*x^n)/(b + Sqrt[b^2 - 4\*a\*c]), (2\*c\*x^n)/(-b + Sqrt[b^2 - 4\*a\*c])])/(1 + n)\*Sqrt[a + x^n\*(b + c\*x^n)])

**Maple [F]** time = 0.018, size = 0, normalized size = 0.

$$\int (d + ex^n) \frac{1}{\sqrt{a + bx^n + cx^{2n}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d+e\*x^n)/(a+b\*x^n+c\*x^(2\*n))^(1/2), x)

[Out] `int((d+e*x^n)/(a+b*x^n+c*x^(2*n))^(1/2),x)`

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{ex^n + d}{\sqrt{cx^{2n} + bx^n + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d+e*x^n)/(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="maxima")`

[Out] `integrate((e*x^n + d)/sqrt(c*x^(2*n) + b*x^n + a), x)`

---

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d+e*x^n)/(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="fricas")`

[Out] Exception raised: UnboundLocalError

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{d + ex^n}{\sqrt{a + bx^n + cx^{2n}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d+e*x**n)/(a+b*x**n+c*x**(2*n))**(1/2),x)`

[Out] `Integral((d + e*x**n)/sqrt(a + b*x**n + c*x**(2*n)), x)`

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{ex^n + d}{\sqrt{cx^{2n} + bx^n + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x^n)/(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*x^n + d)/sqrt(c*x^(2*n) + b*x^n + a), x)
```

$$3.88 \quad \int \frac{d+ex^n}{(a+bx^n+cx^{2n})^{3/2}} dx$$

**Optimal.** Leaf size=298

$$\frac{dx \sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}} + 1 \sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}} + 1F_1\left(\frac{1}{n}; \frac{3}{2}, \frac{3}{2}; 1 + \frac{1}{n}; -\frac{2cx^n}{b-\sqrt{b^2-4ac}}, -\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{a\sqrt{a+bx^n+cx^{2n}}} + \frac{ex^{n+1} \sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}} + 1 \sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}} + 1F_1\left(1\right)}{a(n+1)\sqrt{a+}}$$

[Out] (e\*x^(1+n)\*Sqrt[1+(2\*c\*x^n)/(b-Sqrt[b^2-4\*a\*c])]\*Sqrt[1+(2\*c\*x^n)/(b+Sqrt[b^2-4\*a\*c])]\*AppellF1[1+n^(-1), 3/2, 3/2, 2+n^(-1), (-2\*c\*x^n)/(b-Sqrt[b^2-4\*a\*c]), (-2\*c\*x^n)/(b+Sqrt[b^2-4\*a\*c])])/(a\*(1+n)\*Sqrt[a+b\*x^n+c\*x^(2\*n)]) + (d\*x\*Sqrt[1+(2\*c\*x^n)/(b-Sqrt[b^2-4\*a\*c])]\*Sqrt[1+(2\*c\*x^n)/(b+Sqrt[b^2-4\*a\*c])]\*AppellF1[n^(-1), 3/2, 3/2, 1+n^(-1), (-2\*c\*x^n)/(b-Sqrt[b^2-4\*a\*c]), (-2\*c\*x^n)/(b+Sqrt[b^2-4\*a\*c])])/(a\*Sqrt[a+b\*x^n+c\*x^(2\*n)])

**Rubi [A]** time = 0.34582, antiderivative size = 298, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$ , Rules used = {1432, 1348, 429, 1385, 510}

$$\frac{dx \sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}} + 1 \sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}} + 1F_1\left(\frac{1}{n}; \frac{3}{2}, \frac{3}{2}; 1 + \frac{1}{n}; -\frac{2cx^n}{b-\sqrt{b^2-4ac}}, -\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{a\sqrt{a+bx^n+cx^{2n}}} + \frac{ex^{n+1} \sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}} + 1 \sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}} + 1F_1\left(1\right)}{a(n+1)\sqrt{a+}}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x^n)/(a + b\*x^n + c\*x^(2\*n))^(3/2), x]

[Out] (e\*x^(1+n)\*Sqrt[1+(2\*c\*x^n)/(b-Sqrt[b^2-4\*a\*c])]\*Sqrt[1+(2\*c\*x^n)/(b+Sqrt[b^2-4\*a\*c])]\*AppellF1[1+n^(-1), 3/2, 3/2, 2+n^(-1), (-2\*c\*x^n)/(b-Sqrt[b^2-4\*a\*c]), (-2\*c\*x^n)/(b+Sqrt[b^2-4\*a\*c])])/(a\*(1+n)\*Sqrt[a+b\*x^n+c\*x^(2\*n)]) + (d\*x\*Sqrt[1+(2\*c\*x^n)/(b-Sqrt[b^2-4\*a\*c])]\*Sqrt[1+(2\*c\*x^n)/(b+Sqrt[b^2-4\*a\*c])]\*AppellF1[n^(-1), 3/2, 3/2, 1+n^(-1), (-2\*c\*x^n)/(b-Sqrt[b^2-4\*a\*c]), (-2\*c\*x^n)/(b+Sqrt[b^2-4\*a\*c])])/(a\*Sqrt[a+b\*x^n+c\*x^(2\*n)])

**Rule 1432**

Int[((d\_) + (e\_.)\*(x\_)^(n\_))\*((a\_) + (b\_.)\*(x\_)^(n\_) + (c\_.)\*(x\_)^(n2\_))^(p\_), x\_Symbol] := Int[ExpandIntegrand[(d + e\*x^n)\*(a + b\*x^n + c\*x^(2\*n))^(p),

$x], x] /; \text{FreeQ}\{a, b, c, d, e, n\}, x] \&\& \text{EqQ}[n^2, 2*n] \&\& \text{NeQ}[b^2 - 4*a*c, 0]$

### Rule 1348

$\text{Int}[(a_ + (c_)*(x_)^{(n2_)} + (b_)*(x_)^{(n_)})^{(p_)}, x\_Symbol] \text{:> Dist}[(a^{IntPart[p]}*(a + b*x^n + c*x^{(2*n)})^{FracPart[p]})/((1 + (2*c*x^n)/(b + \text{Rt}[b^2 - 4*a*c, 2]))^{FracPart[p]}*(1 + (2*c*x^n)/(b - \text{Rt}[b^2 - 4*a*c, 2]))^{FracPart[p]}), \text{Int}[(1 + (2*c*x^n)/(b + \text{Sqrt}[b^2 - 4*a*c]))^p*(1 + (2*c*x^n)/(b - \text{Sqrt}[b^2 - 4*a*c]))^p, x], x] /; \text{FreeQ}\{a, b, c, n, p\}, x] \&\& \text{EqQ}[n^2, 2*n] \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& !\text{IntegerQ}[p]$

### Rule 429

$\text{Int}[(a_ + (b_)*(x_)^{(n_)})^{(p_)}*((c_ + (d_)*(x_)^{(n_)})^{(q_)}), x\_Symbol] \text{:> Simp}[a^p*c^q*x*\text{AppellF1}[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)], x] /; \text{FreeQ}\{a, b, c, d, n, p, q\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[n, -1] \&\& (\text{IntegerQ}[p] || \text{GtQ}[a, 0]) \&\& (\text{IntegerQ}[q] || \text{GtQ}[c, 0])$

### Rule 1385

$\text{Int}[(d_)*(x_)^{(m_)}*((a_ + (c_)*(x_)^{(n2_)} + (b_)*(x_)^{(n_)})^{(p_)}, x\_Symbol] \text{:> Dist}[(a^{IntPart[p]}*(a + b*x^n + c*x^{(2*n)})^{FracPart[p]})/((1 + (2*c*x^n)/(b + \text{Rt}[b^2 - 4*a*c, 2]))^{FracPart[p]}*(1 + (2*c*x^n)/(b - \text{Rt}[b^2 - 4*a*c, 2]))^{FracPart[p]}), \text{Int}[(d*x)^m*(1 + (2*c*x^n)/(b + \text{Sqrt}[b^2 - 4*a*c]))^p*(1 + (2*c*x^n)/(b - \text{Sqrt}[b^2 - 4*a*c]))^p, x], x] /; \text{FreeQ}\{a, b, c, d, m, n, p\}, x] \&\& \text{EqQ}[n^2, 2*n]$

### Rule 510

$\text{Int}[(e_)*(x_)^{(m_)}*((a_ + (b_)*(x_)^{(n_)})^{(p_)}*((c_ + (d_)*(x_)^{(n_)})^{(q_)}), x\_Symbol] \text{:> Simp}[(a^p*c^q*(e*x)^{(m+1)}*\text{AppellF1}[(m+1)/n, -p, -q, 1 + (m+1)/n, -((b*x^n)/a), -((d*x^n)/c)]/(e*(m+1)), x] /; \text{FreeQ}\{a, b, c, d, e, m, n, p, q\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[m, -1] \&\& \text{NeQ}[m, n - 1] \&\& (\text{IntegerQ}[p] || \text{GtQ}[a, 0]) \&\& (\text{IntegerQ}[q] || \text{GtQ}[c, 0])$

### Rubi steps

$$\begin{aligned}
\int \frac{d + ex^n}{(a + bx^n + cx^{2n})^{3/2}} dx &= \int \left( \frac{d}{(a + bx^n + cx^{2n})^{3/2}} + \frac{ex^n}{(a + bx^n + cx^{2n})^{3/2}} \right) dx \\
&= d \int \frac{1}{(a + bx^n + cx^{2n})^{3/2}} dx + e \int \frac{x^n}{(a + bx^n + cx^{2n})^{3/2}} dx \\
&= \frac{\left( d \sqrt{1 + \frac{2cx^n}{b - \sqrt{b^2 - 4ac}}} \sqrt{1 + \frac{2cx^n}{b + \sqrt{b^2 - 4ac}}} \right) \int \frac{1}{\left(1 + \frac{2cx^n}{b - \sqrt{b^2 - 4ac}}\right)^{3/2} \left(1 + \frac{2cx^n}{b + \sqrt{b^2 - 4ac}}\right)^{3/2}} dx}{a \sqrt{a + bx^n + cx^{2n}}} + \frac{\left( e \sqrt{1 + \frac{2cx^n}{b - \sqrt{b^2 - 4ac}}} \sqrt{1 + \frac{2cx^n}{b + \sqrt{b^2 - 4ac}}} \right) \int \frac{x^n}{\left(1 + \frac{2cx^n}{b - \sqrt{b^2 - 4ac}}\right)^{3/2} \left(1 + \frac{2cx^n}{b + \sqrt{b^2 - 4ac}}\right)^{3/2}} dx}{a \sqrt{a + bx^n + cx^{2n}}} \\
&= \frac{ex^{1+n} \sqrt{1 + \frac{2cx^n}{b - \sqrt{b^2 - 4ac}}} \sqrt{1 + \frac{2cx^n}{b + \sqrt{b^2 - 4ac}}} F_1 \left( 1 + \frac{1}{n}; \frac{3}{2}, \frac{3}{2}; 2 + \frac{1}{n}; -\frac{2cx^n}{b - \sqrt{b^2 - 4ac}}, -\frac{2cx^n}{b + \sqrt{b^2 - 4ac}} \right)}{a(1+n) \sqrt{a + bx^n + cx^{2n}}} + \frac{dx \sqrt{1 + \frac{2cx^n}{b - \sqrt{b^2 - 4ac}}} \sqrt{1 + \frac{2cx^n}{b + \sqrt{b^2 - 4ac}}}}{a(1+n) \sqrt{a + bx^n + cx^{2n}}}
\end{aligned}$$

**Mathematica [A]** time = 1.53238, size = 414, normalized size = 1.39

$$\frac{x \left( 2cx^n (bd - 2ae) \sqrt{\frac{-\sqrt{b^2 - 4ac} + b + 2cx^n}{b - \sqrt{b^2 - 4ac}}} \sqrt{\frac{\sqrt{b^2 - 4ac} + b + 2cx^n}{\sqrt{b^2 - 4ac} + b}} F_1 \left( 1 + \frac{1}{n}; \frac{1}{2}, \frac{1}{2}; 2 + \frac{1}{n}; -\frac{2cx^n}{b + \sqrt{b^2 - 4ac}}, \frac{2cx^n}{\sqrt{b^2 - 4ac} - b} \right) - (n + 1) \left( \sqrt{\frac{-\sqrt{b^2 - 4ac} + b + 2cx^n}{b - \sqrt{b^2 - 4ac}}} \sqrt{\frac{\sqrt{b^2 - 4ac} + b + 2cx^n}{\sqrt{b^2 - 4ac} + b}} \right)}{an(n + 1) \left( \sqrt{\frac{-\sqrt{b^2 - 4ac} + b + 2cx^n}{b - \sqrt{b^2 - 4ac}}} \sqrt{\frac{\sqrt{b^2 - 4ac} + b + 2cx^n}{\sqrt{b^2 - 4ac} + b}} \right)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(d + e\*x^n)/(a + b\*x^n + c\*x^(2\*n))^(3/2), x]

[Out] (x\*(2\*c\*(b\*d - 2\*a\*e)\*x^n\*Sqrt[(b - Sqrt[b^2 - 4\*a\*c] + 2\*c\*x^n)/(b - Sqrt[b^2 - 4\*a\*c]])\*Sqrt[(b + Sqrt[b^2 - 4\*a\*c] + 2\*c\*x^n)/(b + Sqrt[b^2 - 4\*a\*c]])\*AppellF1[1 + n^(-1), 1/2, 1/2, 2 + n^(-1), (-2\*c\*x^n)/(b + Sqrt[b^2 - 4\*a\*c]), (2\*c\*x^n)/(-b + Sqrt[b^2 - 4\*a\*c])]) - (1 + n)\*(2\*(b^2\*d + b\*(-a\*e) + c\*d\*x^n) - 2\*a\*c\*(d + e\*x^n)) + (2\*a\*b\*e + b^2\*d\*(-2 + n) - 4\*a\*c\*d\*(-1 + n))\*Sqrt[(b - Sqrt[b^2 - 4\*a\*c] + 2\*c\*x^n)/(b - Sqrt[b^2 - 4\*a\*c]])\*Sqrt[(b + Sqrt[b^2 - 4\*a\*c] + 2\*c\*x^n)/(b + Sqrt[b^2 - 4\*a\*c]])\*AppellF1[n^(-1), 1/2, 1/2, 1 + n^(-1), (-2\*c\*x^n)/(b + Sqrt[b^2 - 4\*a\*c]), (2\*c\*x^n)/(-b + Sqrt[b^2 - 4\*a\*c])]))/(a\*(-b^2 + 4\*a\*c)\*n\*(1 + n)\*Sqrt[a + x^n\*(b + c\*x^n)])

**Maple [F]** time = 0.013, size = 0, normalized size = 0.

$$\int (d + ex^n) (a + bx^n + cx^{2n})^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d+e*x^n)/(a+b*x^n+c*x^(2*n))^(3/2),x)`

[Out] `int((d+e*x^n)/(a+b*x^n+c*x^(2*n))^(3/2),x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{ex^n + d}{(cx^{2n} + bx^n + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d+e*x^n)/(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="maxima")`

[Out] `integrate((e*x^n + d)/(c*x^(2*n) + b*x^n + a)^(3/2), x)`

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d+e*x^n)/(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="fricas")`

[Out] Exception raised: UnboundLocalError

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d+e*x**n)/(a+b*x**n+c*x**(2*n))**(3/2),x)`

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{ex^n + d}{(cx^{2n} + bx^n + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e\*x^n)/(a+b\*x^n+c\*x^(2\*n))^(3/2),x, algorithm="giac")

[Out] integrate((e\*x^n + d)/(c\*x^(2\*n) + b\*x^n + a)^(3/2), x)



$$3.89 \quad \int \frac{d+ex^n}{(a+bx^n+cx^{2n})^{5/2}} dx$$

**Optimal.** Leaf size=298

$$\frac{dx \sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}} + 1 \sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}} + 1F_1\left(\frac{1}{n}; \frac{5}{2}, \frac{5}{2}; 1 + \frac{1}{n}; -\frac{2cx^n}{b-\sqrt{b^2-4ac}}, -\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{a^2 \sqrt{a+bx^n+cx^{2n}}} + \frac{ex^{n+1} \sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}} + 1 \sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}} + 1F_1\left(\frac{1}{n}; \frac{5}{2}, \frac{5}{2}; 1 + \frac{1}{n}; -\frac{2cx^n}{b-\sqrt{b^2-4ac}}, -\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{a^2(n+1)\sqrt{a+bx^n+cx^{2n}}}$$

[Out] (e\*x^(1+n)\*Sqrt[1+(2\*c\*x^n)/(b-Sqrt[b^2-4\*a\*c])]\*Sqrt[1+(2\*c\*x^n)/(b+Sqrt[b^2-4\*a\*c])]\*AppellF1[1+n^(-1), 5/2, 5/2, 2+n^(-1), (-2\*c\*x^n)/(b-Sqrt[b^2-4\*a\*c]), (-2\*c\*x^n)/(b+Sqrt[b^2-4\*a\*c])])/(a^2\*(1+n)\*Sqrt[a+b\*x^n+c\*x^(2\*n)]) + (d\*x\*Sqrt[1+(2\*c\*x^n)/(b-Sqrt[b^2-4\*a\*c])]\*Sqrt[1+(2\*c\*x^n)/(b+Sqrt[b^2-4\*a\*c])]\*AppellF1[n^(-1), 5/2, 5/2, 1+n^(-1), (-2\*c\*x^n)/(b-Sqrt[b^2-4\*a\*c]), (-2\*c\*x^n)/(b+Sqrt[b^2-4\*a\*c])])/(a^2\*Sqrt[a+b\*x^n+c\*x^(2\*n)])

**Rubi [A]** time = 0.347054, antiderivative size = 298, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$ , Rules used = {1432, 1348, 429, 1385, 510}

$$\frac{dx \sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}} + 1 \sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}} + 1F_1\left(\frac{1}{n}; \frac{5}{2}, \frac{5}{2}; 1 + \frac{1}{n}; -\frac{2cx^n}{b-\sqrt{b^2-4ac}}, -\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{a^2 \sqrt{a+bx^n+cx^{2n}}} + \frac{ex^{n+1} \sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}} + 1 \sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}} + 1F_1\left(\frac{1}{n}; \frac{5}{2}, \frac{5}{2}; 1 + \frac{1}{n}; -\frac{2cx^n}{b-\sqrt{b^2-4ac}}, -\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{a^2(n+1)\sqrt{a+bx^n+cx^{2n}}}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x^n)/(a + b\*x^n + c\*x^(2\*n))^(5/2), x]

[Out] (e\*x^(1+n)\*Sqrt[1+(2\*c\*x^n)/(b-Sqrt[b^2-4\*a\*c])]\*Sqrt[1+(2\*c\*x^n)/(b+Sqrt[b^2-4\*a\*c])]\*AppellF1[1+n^(-1), 5/2, 5/2, 2+n^(-1), (-2\*c\*x^n)/(b-Sqrt[b^2-4\*a\*c]), (-2\*c\*x^n)/(b+Sqrt[b^2-4\*a\*c])])/(a^2\*(1+n)\*Sqrt[a+b\*x^n+c\*x^(2\*n)]) + (d\*x\*Sqrt[1+(2\*c\*x^n)/(b-Sqrt[b^2-4\*a\*c])]\*Sqrt[1+(2\*c\*x^n)/(b+Sqrt[b^2-4\*a\*c])]\*AppellF1[n^(-1), 5/2, 5/2, 1+n^(-1), (-2\*c\*x^n)/(b-Sqrt[b^2-4\*a\*c]), (-2\*c\*x^n)/(b+Sqrt[b^2-4\*a\*c])])/(a^2\*Sqrt[a+b\*x^n+c\*x^(2\*n)])

**Rule 1432**

Int[((d\_) + (e\_.)\*(x\_)^(n\_))\*((a\_) + (b\_.)\*(x\_)^(n\_) + (c\_.)\*(x\_)^(n2\_))^(p\_), x\_Symbol] :> Int[ExpandIntegrand[(d + e\*x^n)\*(a + b\*x^n + c\*x^(2\*n))^p,

$x], x] /;$  FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2\*n] && NeQ[b^2 - 4\*a\*c, 0]

### Rule 1348

Int[((a\_) + (c\_.)\*(x\_)^(n2\_.) + (b\_.)\*(x\_)^(n\_.))^p\_], x\_Symbol] := Dist[(a^IntPart[p]\*(a + b\*x^n + c\*x^(2\*n))^FracPart[p])/((1 + (2\*c\*x^n)/(b + Rt[b^2 - 4\*a\*c, 2]))^FracPart[p]\*(1 + (2\*c\*x^n)/(b - Rt[b^2 - 4\*a\*c, 2]))^FracPart[p]), Int[(1 + (2\*c\*x^n)/(b + Sqrt[b^2 - 4\*a\*c]))^p\*(1 + (2\*c\*x^n)/(b - Sqrt[b^2 - 4\*a\*c]))^p, x], x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2\*n] && NeQ[b^2 - 4\*a\*c, 0] && !IntegerQ[p]

### Rule 429

Int[((a\_) + (b\_.)\*(x\_)^(n\_.))^p\_\*((c\_) + (d\_.)\*(x\_)^(n\_.))^q\_], x\_Symbol] := Simp[a^p\*c^q\*x\*AppellF1[1/n, -p, -q, 1 + 1/n, -((b\*x^n)/a), -((d\*x^n)/c)], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b\*c - a\*d, 0] && NeQ[n, -1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])

### Rule 1385

Int[((d\_.)\*(x\_)^(m\_.)\*((a\_) + (c\_.)\*(x\_)^(n2\_.) + (b\_.)\*(x\_)^(n\_.))^p\_], x\_Symbol] := Dist[(a^IntPart[p]\*(a + b\*x^n + c\*x^(2\*n))^FracPart[p])/((1 + (2\*c\*x^n)/(b + Rt[b^2 - 4\*a\*c, 2]))^FracPart[p]\*(1 + (2\*c\*x^n)/(b - Rt[b^2 - 4\*a\*c, 2]))^FracPart[p]), Int[(d\*x)^m\*(1 + (2\*c\*x^n)/(b + Sqrt[b^2 - 4\*a\*c]))^p\*(1 + (2\*c\*x^n)/(b - Sqrt[b^2 - 4\*a\*c]))^p, x], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2\*n]

### Rule 510

Int[((e\_.)\*(x\_)^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_.))^p\_\*((c\_) + (d\_.)\*(x\_)^(n\_.))^q\_], x\_Symbol] := Simp[(a^p\*c^q\*(e\*x)^(m + 1)\*AppellF1[(m + 1)/n, -p, -q, 1 + (m + 1)/n, -((b\*x^n)/a), -((d\*x^n)/c)]/(e\*(m + 1)), x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b\*c - a\*d, 0] && NeQ[m, -1] && NeQ[m, n - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])

### Rubi steps

$$\begin{aligned}
\int \frac{d + ex^n}{(a + bx^n + cx^{2n})^{5/2}} dx &= \int \left( \frac{d}{(a + bx^n + cx^{2n})^{5/2}} + \frac{ex^n}{(a + bx^n + cx^{2n})^{5/2}} \right) dx \\
&= d \int \frac{1}{(a + bx^n + cx^{2n})^{5/2}} dx + e \int \frac{x^n}{(a + bx^n + cx^{2n})^{5/2}} dx \\
&= \frac{\left( d \sqrt{1 + \frac{2cx^n}{b - \sqrt{b^2 - 4ac}}} \sqrt{1 + \frac{2cx^n}{b + \sqrt{b^2 - 4ac}}} \int \frac{1}{\left(1 + \frac{2cx^n}{b - \sqrt{b^2 - 4ac}}\right)^{5/2} \left(1 + \frac{2cx^n}{b + \sqrt{b^2 - 4ac}}\right)^{5/2}} dx \right)}{a^2 \sqrt{a + bx^n + cx^{2n}}} + \frac{\left( e \sqrt{1 + \frac{2cx^n}{b - \sqrt{b^2 - 4ac}}} \sqrt{1 + \frac{2cx^n}{b + \sqrt{b^2 - 4ac}}} \int \frac{x^n}{\left(1 + \frac{2cx^n}{b - \sqrt{b^2 - 4ac}}\right)^{5/2} \left(1 + \frac{2cx^n}{b + \sqrt{b^2 - 4ac}}\right)^{5/2}} dx \right)}{a^2 \sqrt{a + bx^n + cx^{2n}}} \\
&= \frac{ex^{1+n} \sqrt{1 + \frac{2cx^n}{b - \sqrt{b^2 - 4ac}}} \sqrt{1 + \frac{2cx^n}{b + \sqrt{b^2 - 4ac}}} F_1 \left( 1 + \frac{1}{n}; \frac{5}{2}, \frac{5}{2}; 2 + \frac{1}{n}; -\frac{2cx^n}{b - \sqrt{b^2 - 4ac}}, -\frac{2cx^n}{b + \sqrt{b^2 - 4ac}} \right)}{a^2 (1+n) \sqrt{a + bx^n + cx^{2n}}} + \frac{dx \sqrt{1 + \frac{2cx^n}{b - \sqrt{b^2 - 4ac}}} \sqrt{1 + \frac{2cx^n}{b + \sqrt{b^2 - 4ac}}}}{a^2 (1+n) \sqrt{a + bx^n + cx^{2n}}}
\end{aligned}$$

**Mathematica [B]** time = 6.55774, size = 6648, normalized size = 22.31

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[(d + e\*x^n)/(a + b\*x^n + c\*x^(2\*n))^(5/2), x]

[Out] Result too large to show

**Maple [F]** time = 0.013, size = 0, normalized size = 0.

$$\int (d + ex^n) (a + bx^n + cx^{2n})^{-\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d+e\*x^n)/(a+b\*x^n+c\*x^(2\*n))^(5/2), x)

[Out] int((d+e\*x^n)/(a+b\*x^n+c\*x^(2\*n))^(5/2), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{ex^n + d}{(cx^{2n} + bx^n + a)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e\*x^n)/(a+b\*x^n+c\*x^(2\*n))^(5/2),x, algorithm="maxima")

[Out] integrate((e\*x^n + d)/(c\*x^(2\*n) + b\*x^n + a)^(5/2), x)

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e\*x^n)/(a+b\*x^n+c\*x^(2\*n))^(5/2),x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e\*x\*\*n)/(a+b\*x\*\*n+c\*x\*\*(2\*n))\*\*(5/2),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{ex^n + d}{(cx^{2n} + bx^n + a)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x^n)/(a+b*x^n+c*x^(2*n))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((e*x^n + d)/(c*x^(2*n) + b*x^n + a)^(5/2), x)
```

$$3.90 \quad \int (d + ex^n)^q (a + bx^n + cx^{2n})^p dx$$

**Optimal.** Leaf size=28

$$\text{Unintegrable}\left((d + ex^n)^q (a + bx^n + cx^{2n})^p, x\right)$$

[Out] Defer[Int][(d + e\*x^n)^q\*(a + b\*x^n + c\*x^(2\*n))^p, x]

**Rubi [A]** time = 0.0100255, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int (d + ex^n)^q (a + bx^n + cx^{2n})^p dx$$

Verification is Not applicable to the result.

[In] Int[(d + e\*x^n)^q\*(a + b\*x^n + c\*x^(2\*n))^p, x]

[Out] Defer[Int][(d + e\*x^n)^q\*(a + b\*x^n + c\*x^(2\*n))^p, x]

Rubi steps

$$\int (d + ex^n)^q (a + bx^n + cx^{2n})^p dx = \int (d + ex^n)^q (a + bx^n + cx^{2n})^p dx$$

**Mathematica [A]** time = 0.305652, size = 0, normalized size = 0.

$$\int (d + ex^n)^q (a + bx^n + cx^{2n})^p dx$$

Verification is Not applicable to the result.

[In] Integrate[(d + e\*x^n)^q\*(a + b\*x^n + c\*x^(2\*n))^p, x]

[Out] Integrate[(d + e\*x^n)^q\*(a + b\*x^n + c\*x^(2\*n))^p, x]

**Maple [A]** time = 0.125, size = 0, normalized size = 0.

$$\int (d + ex^n)^q (a + bx^n + cx^{2n})^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d+e\*x^n)^q\*(a+b\*x^n+c\*x^(2\*n))^p,x)

[Out] int((d+e\*x^n)^q\*(a+b\*x^n+c\*x^(2\*n))^p,x)

---

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int (cx^{2n} + bx^n + a)^p (ex^n + d)^q dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e\*x^n)^q\*(a+b\*x^n+c\*x^(2\*n))^p,x, algorithm="maxima")

[Out] integrate((c\*x^(2\*n) + b\*x^n + a)^p\*(e\*x^n + d)^q, x)

---

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(cx^{2n} + bx^n + a\right)^p (ex^n + d)^q, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e\*x^n)^q\*(a+b\*x^n+c\*x^(2\*n))^p,x, algorithm="fricas")

[Out] integral((c\*x^(2\*n) + b\*x^n + a)^p\*(e\*x^n + d)^q, x)

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x**n)**q*(a+b*x**n+c*x**(2*n))**p,x)
```

```
[Out] Timed out
```

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int (cx^{2n} + bx^n + a)^p (ex^n + d)^q dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x^n)^q*(a+b*x^n+c*x^(2*n))^p,x, algorithm="giac")
```

```
[Out] integrate((c*x^(2*n) + b*x^n + a)^p*(e*x^n + d)^q, x)
```



### 3.91 $\int (d + ex^n)^3 (a + bx^n + cx^{2n})^p dx$

**Optimal.** Leaf size=606

$$\frac{3d^2ex^{n+1} \left( \frac{2cx^n}{b-\sqrt{b^2-4ac}} + 1 \right)^{-p} \left( \frac{2cx^n}{\sqrt{b^2-4ac}+b} + 1 \right)^{-p} (a + bx^n + cx^{2n})^p F_1 \left( 1 + \frac{1}{n}; -p, -p; 2 + \frac{1}{n}; -\frac{2cx^n}{b-\sqrt{b^2-4ac}}, -\frac{2cx^n}{b+\sqrt{b^2-4ac}} \right)}{n+1} + d^3x$$

[Out] (3\*d^2\*e\*x^(1+n)\*(a+b\*x^n+c\*x^(2\*n))^p\*AppellF1[1+n^(-1),-p,-p,2+n^(-1),(-2\*c\*x^n)/(b-Sqrt[b^2-4\*a\*c]),(-2\*c\*x^n)/(b+Sqrt[b^2-4\*a\*c])])/((1+n)\*(1+(2\*c\*x^n)/(b-Sqrt[b^2-4\*a\*c]))^p\*(1+(2\*c\*x^n)/(b+Sqrt[b^2-4\*a\*c]))^p)+(3\*d\*e^2\*x^(1+2\*n)\*(a+b\*x^n+c\*x^(2\*n))^p\*AppellF1[2+n^(-1),-p,-p,3+n^(-1),(-2\*c\*x^n)/(b-Sqrt[b^2-4\*a\*c]),(-2\*c\*x^n)/(b+Sqrt[b^2-4\*a\*c])])/((1+2\*n)\*(1+(2\*c\*x^n)/(b-Sqrt[b^2-4\*a\*c]))^p\*(1+(2\*c\*x^n)/(b+Sqrt[b^2-4\*a\*c]))^p)+(e^3\*x^(1+3\*n)\*(a+b\*x^n+c\*x^(2\*n))^p\*AppellF1[3+n^(-1),-p,-p,4+n^(-1),(-2\*c\*x^n)/(b-Sqrt[b^2-4\*a\*c]),(-2\*c\*x^n)/(b+Sqrt[b^2-4\*a\*c])])/((1+3\*n)\*(1+(2\*c\*x^n)/(b-Sqrt[b^2-4\*a\*c]))^p\*(1+(2\*c\*x^n)/(b+Sqrt[b^2-4\*a\*c]))^p)+(d^3\*x\*(a+b\*x^n+c\*x^(2\*n))^p\*AppellF1[n^(-1),-p,-p,1+n^(-1),(-2\*c\*x^n)/(b-Sqrt[b^2-4\*a\*c]),(-2\*c\*x^n)/(b+Sqrt[b^2-4\*a\*c])])/((1+(2\*c\*x^n)/(b-Sqrt[b^2-4\*a\*c]))^p\*(1+(2\*c\*x^n)/(b+Sqrt[b^2-4\*a\*c]))^p)

**Rubi [A]** time = 0.621973, antiderivative size = 606, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 5, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$ , Rules used = {1436, 1348, 429, 1385, 510}

$$\frac{3d^2ex^{n+1} \left( \frac{2cx^n}{b-\sqrt{b^2-4ac}} + 1 \right)^{-p} \left( \frac{2cx^n}{\sqrt{b^2-4ac}+b} + 1 \right)^{-p} (a + bx^n + cx^{2n})^p F_1 \left( 1 + \frac{1}{n}; -p, -p; 2 + \frac{1}{n}; -\frac{2cx^n}{b-\sqrt{b^2-4ac}}, -\frac{2cx^n}{b+\sqrt{b^2-4ac}} \right)}{n+1} + d^3x$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x^n)^3\*(a + b\*x^n + c\*x^(2\*n))^p,x]

[Out] (3\*d^2\*e\*x^(1+n)\*(a+b\*x^n+c\*x^(2\*n))^p\*AppellF1[1+n^(-1),-p,-p,2+n^(-1),(-2\*c\*x^n)/(b-Sqrt[b^2-4\*a\*c]),(-2\*c\*x^n)/(b+Sqrt[b^2-4\*a\*c])])/((1+n)\*(1+(2\*c\*x^n)/(b-Sqrt[b^2-4\*a\*c]))^p\*(1+(2\*c\*x^n)/(b+Sqrt[b^2-4\*a\*c]))^p)+(3\*d\*e^2\*x^(1+2\*n)\*(a+b\*x^n+c\*x^(2\*n))^p\*AppellF1[2+n^(-1),-p,-p,3+n^(-1),(-2\*c\*x^n)/(b-Sqrt[b^2-4\*a\*c]),(-2\*c\*x^n)/(b+Sqrt[b^2-4\*a\*c])])/((1+2\*n)\*(1+(2\*c\*x^n)/(b-Sqrt[b^2-4\*a\*c]))^p\*(1+(2\*c\*x^n)/(b+Sqrt[b^2-4\*a\*c]))^p)+(e^3\*x^(1+3\*n)\*(a+b\*x^n+c\*x^(2\*n))^p\*AppellF1[3+n^(-1),-p,-p,4+n^(-1),(-2\*c\*x^n)/(b-Sqrt[b^2-4\*a\*c]),(-2\*c\*x^n)/(b+Sqrt[b^2-4\*a\*c])])/((1+3\*n)\*(1+(2\*c\*x^n)/(b-Sqrt[b^2-4\*a\*c]))^p\*(1+(2\*c\*x^n)/(b+Sqrt[b^2-4\*a\*c]))^p)+(d^3\*x\*(a+b\*x^n+c\*x^(2\*n))^p\*AppellF1[n^(-1),-p,-p,1+n^(-1),(-2\*c\*x^n)/(b-Sqrt[b^2-4\*a\*c]),(-2\*c\*x^n)/(b+Sqrt[b^2-4\*a\*c])])/((1+(2\*c\*x^n)/(b-Sqrt[b^2-4\*a\*c]))^p\*(1+(2\*c\*x^n)/(b+Sqrt[b^2-4\*a\*c]))^p)

$$3^n * (a + b*x^n + c*x^{2n})^p * \text{AppellF1}[3 + n^{-1}, -p, -p, 4 + n^{-1}, (-2*c*x^n)/(b - \text{Sqrt}[b^2 - 4*a*c]), (-2*c*x^n)/(b + \text{Sqrt}[b^2 - 4*a*c])]/((1 + 3^n)*(1 + (2*c*x^n)/(b - \text{Sqrt}[b^2 - 4*a*c]))^p*(1 + (2*c*x^n)/(b + \text{Sqrt}[b^2 - 4*a*c]))^p) + (d^3*x*(a + b*x^n + c*x^{2n})^p * \text{AppellF1}[n^{-1}, -p, -p, 1 + n^{-1}, (-2*c*x^n)/(b - \text{Sqrt}[b^2 - 4*a*c]), (-2*c*x^n)/(b + \text{Sqrt}[b^2 - 4*a*c])]/((1 + (2*c*x^n)/(b - \text{Sqrt}[b^2 - 4*a*c]))^p*(1 + (2*c*x^n)/(b + \text{Sqrt}[b^2 - 4*a*c]))^p)$$
Rule 1436

```
Int[((d_) + (e_)*(x_)^(n_))^(q_)*((a_) + (b_)*(x_)^(n_) + (c_)*(x_)^(n2_))^(p_), x_Symbol] := Int[ExpandIntegrand[(d + e*x^n)^q*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && ((IntegersQ[p, q] && !IntegerQ[n]) || IGtQ[p, 0] || (IGtQ[q, 0] && !IntegerQ[n]))
```

Rule 1348

```
Int[((a_) + (c_)*(x_)^(n2_)) + (b_)*(x_)^(n_)]^(p_), x_Symbol] := Dist[(a^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/((1 + (2*c*x^n)/(b + Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^n)/(b - Rt[b^2 - 4*a*c, 2]))^FracPart[p]), Int[(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && !IntegerQ[p]
```

Rule 429

```
Int[((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_), x_Symbol] := Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1385

```
Int[((d_)*(x_)^(m_))*((a_) + (c_)*(x_)^(n2_)) + (b_)*(x_)^(n_)]^(p_), x_Symbol] := Dist[(a^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/((1 + (2*c*x^n)/(b + Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^n)/(b - Rt[b^2 - 4*a*c, 2]))^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rule 510

```
Int[((e_)*(x_)^(m_))*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_), x_Symbol] := Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
```

q, 1 + (m + 1)/n, -((b\*x^n)/a), -((d\*x^n)/c)]/(e\*(m + 1)), x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b\*c - a\*d, 0] && NeQ[m, -1] && NeQ[m, n - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])

### Rubi steps

$$\begin{aligned} \int (d + ex^n)^3 (a + bx^n + cx^{2n})^p dx &= \int \left( d^3 (a + bx^n + cx^{2n})^p + 3d^2 ex^n (a + bx^n + cx^{2n})^p + 3de^2 x^{2n} (a + bx^n + cx^{2n})^p + \right. \\ &= d^3 \int (a + bx^n + cx^{2n})^p dx + (3d^2 e) \int x^n (a + bx^n + cx^{2n})^p dx + (3de^2) \int x^{2n} (a + bx^n + cx^{2n})^p dx \\ &= \left( d^3 \left( 1 + \frac{2cx^n}{b - \sqrt{b^2 - 4ac}} \right)^{-p} \left( 1 + \frac{2cx^n}{b + \sqrt{b^2 - 4ac}} \right)^{-p} (a + bx^n + cx^{2n})^p \right) \int \left( 1 + \frac{2cx^n}{b - \sqrt{b^2 - 4ac}} \right)^{-p} \\ &= \frac{3d^2 ex^{1+n} \left( 1 + \frac{2cx^n}{b - \sqrt{b^2 - 4ac}} \right)^{-p} \left( 1 + \frac{2cx^n}{b + \sqrt{b^2 - 4ac}} \right)^{-p} (a + bx^n + cx^{2n})^p F_1 \left( 1 + \frac{1}{n}; -p, -p; 2 + \frac{1}{n} \right)}{1 + n} \end{aligned}$$

**Mathematica [A]** time = 1.50296, size = 438, normalized size = 0.72

$$\frac{x \left( \frac{-\sqrt{b^2 - 4ac} + b + 2cx^n}{b - \sqrt{b^2 - 4ac}} \right)^{-p} \left( \frac{\sqrt{b^2 - 4ac} + b + 2cx^n}{\sqrt{b^2 - 4ac} + b} \right)^{-p} (a + x^n (b + cx^n))^p \left( (n + 1) \left( (2n + 1) \left( d^3 (3n + 1) F_1 \left( \frac{1}{n}; -p, -p; 1 + \frac{1}{n}; -\frac{2cx^n}{b + \sqrt{b^2 - 4ac}} \right) \right) \right)}{1 + n}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(d + e\*x^n)^3\*(a + b\*x^n + c\*x^(2\*n))^p,x]

[Out] (x\*(a + x^n\*(b + c\*x^n))^p\*(3\*d^2\*e\*(1 + 5\*n + 6\*n^2)\*x^n\*AppellF1[1 + n^(-1), -p, -p, 2 + n^(-1), (-2\*c\*x^n)/(b + Sqrt[b^2 - 4\*a\*c]), (2\*c\*x^n)/(-b + Sqrt[b^2 - 4\*a\*c])] + (1 + n)\*(3\*d\*e^2\*(1 + 3\*n)\*x^(2\*n)\*AppellF1[2 + n^(-1), -p, -p, 3 + n^(-1), (-2\*c\*x^n)/(b + Sqrt[b^2 - 4\*a\*c]), (2\*c\*x^n)/(-b + Sqrt[b^2 - 4\*a\*c])] + (1 + 2\*n)\*(e^3\*x^(3\*n)\*AppellF1[3 + n^(-1), -p, -p, 4 + n^(-1), (-2\*c\*x^n)/(b + Sqrt[b^2 - 4\*a\*c]), (2\*c\*x^n)/(-b + Sqrt[b^2 - 4\*a\*c])]) + d^3\*(1 + 3\*n)\*AppellF1[n^(-1), -p, -p, 1 + n^(-1), (-2\*c\*x^n)/(b + Sqrt[b^2 - 4\*a\*c]), (2\*c\*x^n)/(-b + Sqrt[b^2 - 4\*a\*c])])))/((1 + n)\*(1 + 2\*n)\*(1 + 3\*n)\*((b - Sqrt[b^2 - 4\*a\*c] + 2\*c\*x^n)/(b - Sqrt[b^2 - 4\*a\*c]))^p\*((b + Sqrt[b^2 - 4\*a\*c] + 2\*c\*x^n)/(b + Sqrt[b^2 - 4\*a\*c]))^p)

**Maple [F]** time = 0.099, size = 0, normalized size = 0.

$$\int (d + ex^n)^3 (a + bx^n + cx^{2n})^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d+e*x^n)^3*(a+b*x^n+c*x^(2*n))^p,x)`

[Out] `int((d+e*x^n)^3*(a+b*x^n+c*x^(2*n))^p,x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (ex^n + d)^3 (cx^{2n} + bx^n + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d+e*x^n)^3*(a+b*x^n+c*x^(2*n))^p,x, algorithm="maxima")`

[Out] `integrate((e*x^n + d)^3*(c*x^(2*n) + b*x^n + a)^p, x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(e^3x^{3n} + 3de^2x^{2n} + 3d^2ex^n + d^3\right)(cx^{2n} + bx^n + a)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d+e*x^n)^3*(a+b*x^n+c*x^(2*n))^p,x, algorithm="fricas")`

[Out] `integral((e^3*x^(3*n) + 3*d*e^2*x^(2*n) + 3*d^2*e*x^n + d^3)*(c*x^(2*n) + b*x^n + a)^p, x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d+e*x**n)**3*(a+b*x**n+c*x**(2*n))**p,x)`

[Out] Timed out

---

**Giac [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d+e*x^n)^3*(a+b*x^n+c*x^(2*n))^p,x, algorithm="giac")`

[Out] Exception raised: TypeError

### 3.92 $\int (d + ex^n)^2 (a + bx^n + cx^{2n})^p dx$

**Optimal.** Leaf size=447

$$d^2x \left( \frac{2cx^n}{b - \sqrt{b^2 - 4ac}} + 1 \right)^{-p} \left( \frac{2cx^n}{\sqrt{b^2 - 4ac} + b} + 1 \right)^{-p} (a + bx^n + cx^{2n})^p F_1 \left( \frac{1}{n}; -p, -p; 1 + \frac{1}{n}; -\frac{2cx^n}{b - \sqrt{b^2 - 4ac}}, -\frac{2cx^n}{b + \sqrt{b^2 - 4ac}} \right)$$

[Out] (2\*d\*e\*x^(1 + n)\*(a + b\*x^n + c\*x^(2\*n))^p\*AppellF1[1 + n^(-1), -p, -p, 2 + n^(-1), (-2\*c\*x^n)/(b - Sqrt[b^2 - 4\*a\*c]), (-2\*c\*x^n)/(b + Sqrt[b^2 - 4\*a\*c])])/((1 + n)\*(1 + (2\*c\*x^n)/(b - Sqrt[b^2 - 4\*a\*c]))^p\*(1 + (2\*c\*x^n)/(b + Sqrt[b^2 - 4\*a\*c]))^p) + (e^2\*x^(1 + 2\*n)\*(a + b\*x^n + c\*x^(2\*n))^p\*AppellF1[2 + n^(-1), -p, -p, 3 + n^(-1), (-2\*c\*x^n)/(b - Sqrt[b^2 - 4\*a\*c]), (-2\*c\*x^n)/(b + Sqrt[b^2 - 4\*a\*c])])/((1 + 2\*n)\*(1 + (2\*c\*x^n)/(b - Sqrt[b^2 - 4\*a\*c]))^p\*(1 + (2\*c\*x^n)/(b + Sqrt[b^2 - 4\*a\*c]))^p) + (d^2\*x\*(a + b\*x^n + c\*x^(2\*n))^p\*AppellF1[n^(-1), -p, -p, 1 + n^(-1), (-2\*c\*x^n)/(b - Sqrt[b^2 - 4\*a\*c]), (-2\*c\*x^n)/(b + Sqrt[b^2 - 4\*a\*c])])/((1 + (2\*c\*x^n)/(b - Sqrt[b^2 - 4\*a\*c]))^p\*(1 + (2\*c\*x^n)/(b + Sqrt[b^2 - 4\*a\*c]))^p)

**Rubi [A]** time = 0.461183, antiderivative size = 447, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$ , Rules used = {1436, 1348, 429, 1385, 510}

$$d^2x \left( \frac{2cx^n}{b - \sqrt{b^2 - 4ac}} + 1 \right)^{-p} \left( \frac{2cx^n}{\sqrt{b^2 - 4ac} + b} + 1 \right)^{-p} (a + bx^n + cx^{2n})^p F_1 \left( \frac{1}{n}; -p, -p; 1 + \frac{1}{n}; -\frac{2cx^n}{b - \sqrt{b^2 - 4ac}}, -\frac{2cx^n}{b + \sqrt{b^2 - 4ac}} \right)$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x^n)^2\*(a + b\*x^n + c\*x^(2\*n))^p,x]

[Out] (2\*d\*e\*x^(1 + n)\*(a + b\*x^n + c\*x^(2\*n))^p\*AppellF1[1 + n^(-1), -p, -p, 2 + n^(-1), (-2\*c\*x^n)/(b - Sqrt[b^2 - 4\*a\*c]), (-2\*c\*x^n)/(b + Sqrt[b^2 - 4\*a\*c])])/((1 + n)\*(1 + (2\*c\*x^n)/(b - Sqrt[b^2 - 4\*a\*c]))^p\*(1 + (2\*c\*x^n)/(b + Sqrt[b^2 - 4\*a\*c]))^p) + (e^2\*x^(1 + 2\*n)\*(a + b\*x^n + c\*x^(2\*n))^p\*AppellF1[2 + n^(-1), -p, -p, 3 + n^(-1), (-2\*c\*x^n)/(b - Sqrt[b^2 - 4\*a\*c]), (-2\*c\*x^n)/(b + Sqrt[b^2 - 4\*a\*c])])/((1 + 2\*n)\*(1 + (2\*c\*x^n)/(b - Sqrt[b^2 - 4\*a\*c]))^p\*(1 + (2\*c\*x^n)/(b + Sqrt[b^2 - 4\*a\*c]))^p) + (d^2\*x\*(a + b\*x^n + c\*x^(2\*n))^p\*AppellF1[n^(-1), -p, -p, 1 + n^(-1), (-2\*c\*x^n)/(b - Sqrt[b^2 - 4\*a\*c]), (-2\*c\*x^n)/(b + Sqrt[b^2 - 4\*a\*c])])/((1 + (2\*c\*x^n)/(b - Sqrt[b^2 - 4\*a\*c]))^p\*(1 + (2\*c\*x^n)/(b + Sqrt[b^2 - 4\*a\*c]))^p)

Rule 1436

```
Int[((d_) + (e_)*(x_)^(n_))^(q_)*((a_) + (b_)*(x_)^(n_) + (c_)*(x_)^(n2_))^(p_), x_Symbol] := Int[ExpandIntegrand[(d + e*x^n)^q*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && ((IntegersQ[p, q] && !IntegerQ[n]) || IGtQ[p, 0] || (IGtQ[q, 0] && !IntegerQ[n]))
```

Rule 1348

```
Int[((a_) + (c_)*(x_)^(n2_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Dist[(a^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/((1 + (2*c*x^n)/(b + Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^n)/(b - Rt[b^2 - 4*a*c, 2]))^FracPart[p]), Int[(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && !IntegerQ[p]
```

Rule 429

```
Int[((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_), x_Symbol] := Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -(b*x^n)/a, -(d*x^n)/c], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1385

```
Int[((d_)*(x_)^(m_)*((a_) + (c_)*(x_)^(n2_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Dist[(a^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/((1 + (2*c*x^n)/(b + Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^n)/(b - Rt[b^2 - 4*a*c, 2]))^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rule 510

```
Int[((e_)*(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_), x_Symbol] := Simp[(a^p*c^q*(e*x)^(m+1)*AppellF1[(m+1)/n, -p, -q, 1 + (m+1)/n, -(b*x^n)/a, -(d*x^n)/c]]/(e*(m+1)), x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\int (d + ex^n)^2 (a + bx^n + cx^{2n})^p dx &= \int \left( d^2 (a + bx^n + cx^{2n})^p + 2dex^n (a + bx^n + cx^{2n})^p + e^2 x^{2n} (a + bx^n + cx^{2n})^p \right) dx \\
&= d^2 \int (a + bx^n + cx^{2n})^p dx + (2de) \int x^n (a + bx^n + cx^{2n})^p dx + e^2 \int x^{2n} (a + bx^n + cx^{2n})^p dx \\
&= \left( d^2 \left( 1 + \frac{2cx^n}{b - \sqrt{b^2 - 4ac}} \right)^{-p} \left( 1 + \frac{2cx^n}{b + \sqrt{b^2 - 4ac}} \right)^{-p} (a + bx^n + cx^{2n})^p \right) \int \left( 1 + \frac{2cx^n}{b - \sqrt{b^2 - 4ac}} \right)^{-p} \left( 1 + \frac{2cx^n}{b + \sqrt{b^2 - 4ac}} \right)^{-p} dx \\
&= \frac{2dex^{1+n} \left( 1 + \frac{2cx^n}{b - \sqrt{b^2 - 4ac}} \right)^{-p} \left( 1 + \frac{2cx^n}{b + \sqrt{b^2 - 4ac}} \right)^{-p} (a + bx^n + cx^{2n})^p F_1 \left( 1 + \frac{1}{n}; -p, -p; 2 + \frac{1}{n} \right)}{1 + n}
\end{aligned}$$

**Mathematica [A]** time = 1.03717, size = 338, normalized size = 0.76

$$x \left( \frac{-\sqrt{b^2 - 4ac} + b + 2cx^n}{b - \sqrt{b^2 - 4ac}} \right)^{-p} \left( \frac{\sqrt{b^2 - 4ac} + b + 2cx^n}{\sqrt{b^2 - 4ac} + b} \right)^{-p} (a + x^n (b + cx^n))^p \left( (n+1) \left( d^2 (2n+1) F_1 \left( \frac{1}{n}; -p, -p; 1 + \frac{1}{n}; -\frac{2cx^n}{b + \sqrt{b^2 - 4ac}}, \frac{2cx^n}{\sqrt{b^2 - 4ac} + b} \right) \right) \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[(d + e\*x^n)^2\*(a + b\*x^n + c\*x^(2\*n))^p,x]

[Out] (x\*(a + x^n\*(b + c\*x^n))^p\*(2\*d\*e\*(1 + 2\*n)\*x^n\*AppellF1[1 + n^(-1), -p, -p, 2 + n^(-1), (-2\*c\*x^n)/(b + Sqrt[b^2 - 4\*a\*c]), (2\*c\*x^n)/(-b + Sqrt[b^2 - 4\*a\*c])]) + (1 + n)\*(e^2\*x^(2\*n)\*AppellF1[2 + n^(-1), -p, -p, 3 + n^(-1), (-2\*c\*x^n)/(b + Sqrt[b^2 - 4\*a\*c]), (2\*c\*x^n)/(-b + Sqrt[b^2 - 4\*a\*c])]) + d^2\*(1 + 2\*n)\*AppellF1[n^(-1), -p, -p, 1 + n^(-1), (-2\*c\*x^n)/(b + Sqrt[b^2 - 4\*a\*c]), (2\*c\*x^n)/(-b + Sqrt[b^2 - 4\*a\*c])]))/((1 + n)\*(1 + 2\*n)\*((b - Sqrt[b^2 - 4\*a\*c] + 2\*c\*x^n)/(b - Sqrt[b^2 - 4\*a\*c]))^p\*((b + Sqrt[b^2 - 4\*a\*c] + 2\*c\*x^n)/(b + Sqrt[b^2 - 4\*a\*c]))^p)

**Maple [F]** time = 0.083, size = 0, normalized size = 0.

$$\int (d + ex^n)^2 (a + bx^n + cx^{2n})^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d+e\*x^n)^2\*(a+b\*x^n+c\*x^(2\*n))^p,x)

[Out] int((d+e\*x^n)^2\*(a+b\*x^n+c\*x^(2\*n))^p,x)



---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (ex^n + d)^2 (cx^{2n} + bx^n + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e\*x^n)^2\*(a+b\*x^n+c\*x^(2\*n))^p,x, algorithm="maxima")

[Out] integrate((e\*x^n + d)^2\*(c\*x^(2\*n) + b\*x^n + a)^p, x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(e^2x^{2n} + 2dex^n + d^2\right)\left(cx^{2n} + bx^n + a\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e\*x^n)^2\*(a+b\*x^n+c\*x^(2\*n))^p,x, algorithm="fricas")

[Out] integral((e^2\*x^(2\*n) + 2\*d\*e\*x^n + d^2)\*(c\*x^(2\*n) + b\*x^n + a)^p, x)

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e\*x\*\*n)\*\*2\*(a+b\*x\*\*n+c\*x\*\*(2\*n))\*\*p,x)

[Out] Timed out

---

**Giac [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x^n)^2*(a+b*x^n+c*x^(2*n))^p,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

### 3.93 $\int (d + ex^n) (a + bx^n + cx^{2n})^p dx$

**Optimal.** Leaf size=288

$$dx \left( \frac{2cx^n}{b - \sqrt{b^2 - 4ac}} + 1 \right)^{-p} \left( \frac{2cx^n}{\sqrt{b^2 - 4ac} + b} + 1 \right)^{-p} (a + bx^n + cx^{2n})^p F_1 \left( \frac{1}{n}; -p, -p; 1 + \frac{1}{n}; -\frac{2cx^n}{b - \sqrt{b^2 - 4ac}}, -\frac{2cx^n}{b + \sqrt{b^2 - 4ac}} \right)$$

```
[Out] (e*x^(1 + n)*(a + b*x^n + c*x^(2*n)))^p*AppellF1[1 + n^(-1), -p, -p, 2 + n^(-1), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])] / ((1 + n)*(1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]))^p) + (d*x*(a + b*x^n + c*x^(2*n)))^p*AppellF1[n^(-1), -p, -p, 1 + n^(-1), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])] / ((1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

**Rubi [A]** time = 0.286561, antiderivative size = 288, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$ , Rules used = {1432, 1348, 429, 1385, 510}

$$dx \left( \frac{2cx^n}{b - \sqrt{b^2 - 4ac}} + 1 \right)^{-p} \left( \frac{2cx^n}{\sqrt{b^2 - 4ac} + b} + 1 \right)^{-p} (a + bx^n + cx^{2n})^p F_1 \left( \frac{1}{n}; -p, -p; 1 + \frac{1}{n}; -\frac{2cx^n}{b - \sqrt{b^2 - 4ac}}, -\frac{2cx^n}{b + \sqrt{b^2 - 4ac}} \right)$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^n)*(a + b*x^n + c*x^(2*n))^p,x]
```

```
[Out] (e*x^(1 + n)*(a + b*x^n + c*x^(2*n)))^p*AppellF1[1 + n^(-1), -p, -p, 2 + n^(-1), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])] / ((1 + n)*(1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]))^p) + (d*x*(a + b*x^n + c*x^(2*n)))^p*AppellF1[n^(-1), -p, -p, 1 + n^(-1), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])] / ((1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

#### Rule 1432

```
Int[((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
```

0]

Rule 1348

```
Int[((a_) + (c_)*(x_)^(n2_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Dist[(a
^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/((1 + (2*c*x^n)/(b + Rt[b^
2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^n)/(b - Rt[b^2 - 4*a*c, 2]))^FracPa
rt[p]), Int[(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^n)/(b - S
qrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n] &
& NeQ[b^2 - 4*a*c, 0] && !IntegerQ[p]
```

Rule 429

```
Int[((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_), x_Symbol]
:= Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
&& (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1385

```
Int[((d_)*(x_)^(m_))*((a_) + (c_)*(x_)^(n2_) + (b_)*(x_)^(n_))^(p_), x
_Symbol] := Dist[(a^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/((1 + (
2*c*x^n)/(b + Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^n)/(b - Rt[b^2 -
4*a*c, 2]))^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]
))^p*(1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rule 510

```
Int[((e_)*(x_)^(m_))*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_
))^^(q_), x_Symbol] := Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)]/(e*(m + 1)), x] /; FreeQ[{a,
b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
- 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\int (d + ex^n)(a + bx^n + cx^{2n})^p dx &= \int \left( d(a + bx^n + cx^{2n})^p + ex^n(a + bx^n + cx^{2n})^p \right) dx \\
&= d \int (a + bx^n + cx^{2n})^p dx + e \int x^n (a + bx^n + cx^{2n})^p dx \\
&= \left( d \left( 1 + \frac{2cx^n}{b - \sqrt{b^2 - 4ac}} \right)^{-p} \left( 1 + \frac{2cx^n}{b + \sqrt{b^2 - 4ac}} \right)^{-p} (a + bx^n + cx^{2n})^p \right) \int \left( 1 + \frac{2cx^n}{b - \sqrt{b^2 - 4ac}} \right)^{-p} \\
&= \frac{ex^{1+n} \left( 1 + \frac{2cx^n}{b - \sqrt{b^2 - 4ac}} \right)^{-p} \left( 1 + \frac{2cx^n}{b + \sqrt{b^2 - 4ac}} \right)^{-p} (a + bx^n + cx^{2n})^p F_1 \left( 1 + \frac{1}{n}; -p, -p; 2 + \frac{1}{n} \right)}{1 + n}
\end{aligned}$$

**Mathematica [A]** time = 0.688071, size = 243, normalized size = 0.84

$$\frac{x \left( \frac{-\sqrt{b^2 - 4ac} + b + 2cx^n}{b - \sqrt{b^2 - 4ac}} \right)^{-p} \left( \frac{\sqrt{b^2 - 4ac} + b + 2cx^n}{\sqrt{b^2 - 4ac} + b} \right)^{-p} (a + x^n (b + cx^n))^p \left( d(n+1) F_1 \left( \frac{1}{n}; -p, -p; 1 + \frac{1}{n}; -\frac{2cx^n}{b + \sqrt{b^2 - 4ac}}, \frac{2cx^n}{\sqrt{b^2 - 4ac} - b} \right) + ex^n F_1 \left( 1 + \frac{1}{n}; -p, -p; 2 + \frac{1}{n}; -\frac{2cx^n}{b - \sqrt{b^2 - 4ac}}, \frac{2cx^n}{\sqrt{b^2 - 4ac} - b} \right) \right)}{n + 1}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(d + e\*x^n)\*(a + b\*x^n + c\*x^(2\*n))^p,x]

[Out] (x\*(a + x^n\*(b + c\*x^n))^p\*(e\*x^n\*AppellF1[1 + n^(-1), -p, -p, 2 + n^(-1), (-2\*c\*x^n)/(b + Sqrt[b^2 - 4\*a\*c]), (2\*c\*x^n)/(-b + Sqrt[b^2 - 4\*a\*c])]) + d\*(1 + n)\*AppellF1[n^(-1), -p, -p, 1 + n^(-1), (-2\*c\*x^n)/(b + Sqrt[b^2 - 4\*a\*c]), (2\*c\*x^n)/(-b + Sqrt[b^2 - 4\*a\*c])]))/((1 + n)\*((b - Sqrt[b^2 - 4\*a\*c] + 2\*c\*x^n)/(b - Sqrt[b^2 - 4\*a\*c]))^p\*((b + Sqrt[b^2 - 4\*a\*c] + 2\*c\*x^n)/(b + Sqrt[b^2 - 4\*a\*c]))^p)

**Maple [F]** time = 0.085, size = 0, normalized size = 0.

$$\int (d + ex^n)(a + bx^n + cx^{2n})^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d+e\*x^n)\*(a+b\*x^n+c\*x^(2\*n))^p,x)

[Out] int((d+e\*x^n)\*(a+b\*x^n+c\*x^(2\*n))^p,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (ex^n + d)(cx^{2n} + bx^n + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e\*x^n)\*(a+b\*x^n+c\*x^(2\*n))^p,x, algorithm="maxima")

[Out] integrate((e\*x^n + d)\*(c\*x^(2\*n) + b\*x^n + a)^p, x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((ex^n + d)(cx^{2n} + bx^n + a)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e\*x^n)\*(a+b\*x^n+c\*x^(2\*n))^p,x, algorithm="fricas")

[Out] integral((e\*x^n + d)\*(c\*x^(2\*n) + b\*x^n + a)^p, x)

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e\*x\*\*n)\*(a+b\*x\*\*n+c\*x\*\*(2\*n))\*\*p,x)

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (ex^n + d)(cx^{2n} + bx^n + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x^n)*(a+b*x^n+c*x^(2*n))^p,x, algorithm="giac")
```

```
[Out] integrate((e*x^n + d)*(c*x^(2*n) + b*x^n + a)^p, x)
```

$$3.94 \quad \int \frac{(a+bx^n+cx^{2n})^p}{d+ex^n} dx$$

**Optimal.** Leaf size=28

$$\text{Unintegrable} \left( \frac{(a+bx^n+cx^{2n})^p}{d+ex^n}, x \right)$$

[Out] Defer[Int][(a + b\*x^n + c\*x^(2\*n))^p/(d + e\*x^n), x]

**Rubi [A]** time = 0.0112573, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{(a+bx^n+cx^{2n})^p}{d+ex^n} dx$$

Verification is Not applicable to the result.

[In] Int[(a + b\*x^n + c\*x^(2\*n))^p/(d + e\*x^n), x]

[Out] Defer[Int][(a + b\*x^n + c\*x^(2\*n))^p/(d + e\*x^n), x]

Rubi steps

$$\int \frac{(a+bx^n+cx^{2n})^p}{d+ex^n} dx = \int \frac{(a+bx^n+cx^{2n})^p}{d+ex^n} dx$$

**Mathematica [A]** time = 0.159444, size = 0, normalized size = 0.

$$\int \frac{(a+bx^n+cx^{2n})^p}{d+ex^n} dx$$

Verification is Not applicable to the result.

[In] Integrate[(a + b\*x^n + c\*x^(2\*n))^p/(d + e\*x^n), x]



[Out] Integrate[(a + b\*x^n + c\*x^(2\*n))^p/(d + e\*x^n), x]

**Maple [A]** time = 0.085, size = 0, normalized size = 0.

$$\int \frac{(a + bx^n + cx^{2n})^p}{d + ex^n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*x^n+c\*x^(2\*n))^p/(d+e\*x^n), x)

[Out] int((a+b\*x^n+c\*x^(2\*n))^p/(d+e\*x^n), x)

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx^{2n} + bx^n + a)^p}{ex^n + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*x^n+c\*x^(2\*n))^p/(d+e\*x^n), x, algorithm="maxima")

[Out] integrate((c\*x^(2\*n) + b\*x^n + a)^p/(e\*x^n + d), x)

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(cx^{2n} + bx^n + a)^p}{ex^n + d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*x^n+c\*x^(2\*n))^p/(d+e\*x^n), x, algorithm="fricas")

[Out] integral((c\*x^(2\*n) + b\*x^n + a)^p/(e\*x^n + d), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*x\*\*n+c\*x\*\*(2\*n))\*\*p/(d+e\*x\*\*n),x)

[Out] Timed out

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx^{2n} + bx^n + a)^p}{ex^n + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*x^n+c\*x^(2\*n))^p/(d+e\*x^n),x, algorithm="giac")

[Out] integrate((c\*x^(2\*n) + b\*x^n + a)^p/(e\*x^n + d), x)

$$3.95 \quad \int \frac{(a+bx^n+cx^{2n})^p}{(d+ex^n)^2} dx$$

**Optimal.** Leaf size=28

$$\text{Unintegrable} \left( \frac{(a + bx^n + cx^{2n})^p}{(d + ex^n)^2}, x \right)$$

[Out] Defer[Int] [(a + b\*x^n + c\*x^(2\*n))^p/(d + e\*x^n)^2, x]

**Rubi [A]** time = 0.0113417, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{(a + bx^n + cx^{2n})^p}{(d + ex^n)^2} dx$$

Verification is Not applicable to the result.

[In] Int[(a + b\*x^n + c\*x^(2\*n))^p/(d + e\*x^n)^2, x]

[Out] Defer[Int] [(a + b\*x^n + c\*x^(2\*n))^p/(d + e\*x^n)^2, x]

Rubi steps

$$\int \frac{(a + bx^n + cx^{2n})^p}{(d + ex^n)^2} dx = \int \frac{(a + bx^n + cx^{2n})^p}{(d + ex^n)^2} dx$$

**Mathematica [A]** time = 0.196551, size = 0, normalized size = 0.

$$\int \frac{(a + bx^n + cx^{2n})^p}{(d + ex^n)^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[(a + b\*x^n + c\*x^(2\*n))^p/(d + e\*x^n)^2, x]

[Out] Integrate[(a + b\*x^n + c\*x^(2\*n))^p/(d + e\*x^n)^2, x]

**Maple [A]** time = 0.059, size = 0, normalized size = 0.

$$\int \frac{(a + bx^n + cx^{2n})^p}{(d + ex^n)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*x^n+c\*x^(2\*n))^p/(d+e\*x^n)^2,x)

[Out] int((a+b\*x^n+c\*x^(2\*n))^p/(d+e\*x^n)^2,x)

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx^{2n} + bx^n + a)^p}{(ex^n + d)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*x^n+c\*x^(2\*n))^p/(d+e\*x^n)^2,x, algorithm="maxima")

[Out] integrate((c\*x^(2\*n) + b\*x^n + a)^p/(e\*x^n + d)^2, x)

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(cx^{2n} + bx^n + a)^p}{e^2x^{2n} + 2dex^n + d^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*x^n+c\*x^(2\*n))^p/(d+e\*x^n)^2,x, algorithm="fricas")

[Out] integral((c\*x^(2\*n) + b\*x^n + a)^p/(e^2\*x^(2\*n) + 2\*d\*e\*x^n + d^2), x)

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*x\*\*n+c\*x\*\*(2\*n))\*\*p/(d+e\*x\*\*n)\*\*2,x)

[Out] Timed out

---

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx^{2n} + bx^n + a)^p}{(ex^n + d)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*x^n+c\*x^(2\*n))^p/(d+e\*x^n)^2,x, algorithm="giac")

[Out] integrate((c\*x^(2\*n) + b\*x^n + a)^p/(e\*x^n + d)^2, x)

$$3.96 \quad \int \frac{(a+bx^n+cx^{2n})^p}{(d+ex^n)^3} dx$$

**Optimal.** Leaf size=28

$$\text{Unintegrable} \left( \frac{(a+bx^n+cx^{2n})^p}{(d+ex^n)^3}, x \right)$$

[Out] Defer[Int] [(a + b\*x^n + c\*x^(2\*n))^p/(d + e\*x^n)^3, x]

**Rubi [A]** time = 0.0118203, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{(a+bx^n+cx^{2n})^p}{(d+ex^n)^3} dx$$

Verification is Not applicable to the result.

[In] Int [(a + b\*x^n + c\*x^(2\*n))^p/(d + e\*x^n)^3, x]

[Out] Defer[Int] [(a + b\*x^n + c\*x^(2\*n))^p/(d + e\*x^n)^3, x]

Rubi steps

$$\int \frac{(a+bx^n+cx^{2n})^p}{(d+ex^n)^3} dx = \int \frac{(a+bx^n+cx^{2n})^p}{(d+ex^n)^3} dx$$

**Mathematica [A]** time = 0.709793, size = 0, normalized size = 0.

$$\int \frac{(a+bx^n+cx^{2n})^p}{(d+ex^n)^3} dx$$

Verification is Not applicable to the result.

[In] Integrate [(a + b\*x^n + c\*x^(2\*n))^p/(d + e\*x^n)^3, x]

[Out] Integrate[(a + b\*x^n + c\*x^(2\*n))^p/(d + e\*x^n)^3, x]

**Maple [A]** time = 0.074, size = 0, normalized size = 0.

$$\int \frac{(a + bx^n + cx^{2n})^p}{(d + ex^n)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*x^n+c\*x^(2\*n))^p/(d+e\*x^n)^3,x)

[Out] int((a+b\*x^n+c\*x^(2\*n))^p/(d+e\*x^n)^3,x)

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx^{2n} + bx^n + a)^p}{(ex^n + d)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*x^n+c\*x^(2\*n))^p/(d+e\*x^n)^3,x, algorithm="maxima")

[Out] integrate((c\*x^(2\*n) + b\*x^n + a)^p/(e\*x^n + d)^3, x)

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(cx^{2n} + bx^n + a)^p}{e^3x^{3n} + 3de^2x^{2n} + 3d^2ex^n + d^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*x^n+c\*x^(2\*n))^p/(d+e\*x^n)^3,x, algorithm="fricas")

[Out] integral((c\*x^(2\*n) + b\*x^n + a)^p/(e^3\*x^(3\*n) + 3\*d\*e^2\*x^(2\*n) + 3\*d^2\*e\*x^n + d^3), x)

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*x\*\*n+c\*x\*\*(2\*n))\*\*p/(d+e\*x\*\*n)\*\*3,x)

[Out] Timed out

---

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx^{2n} + bx^n + a)^p}{(ex^n + d)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*x^n+c\*x^(2\*n))^p/(d+e\*x^n)^3,x, algorithm="giac")

[Out] integrate((c\*x^(2\*n) + b\*x^n + a)^p/(e\*x^n + d)^3, x)



# Chapter 4

## Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

### 4.0.1 Mathematica and Rubi grading function

```
1 (* Original version thanks to Albert Rich emailed on 03/21/2017 *)
2 (* ::Package:: *)
3
4 (* ::Subsection:: *)
5 (*GradeAntiderivative[result,optimal]*)
6
7
8 (* ::Text:: *)
9 (*If result and optimal are mathematical expressions, *)
10 (*      GradeAntiderivative[result,optimal] returns*)
11 (* "F" if the result fails to integrate an expression that*)
12 (*      is integrable*)
13 (* "C" if result involves higher level functions than necessary*)
14 (* "B" if result is more than twice the size of the optimal*)
15 (*      antiderivative*)
16 (* "A" if result can be considered optimal*)
17
18
19 GradeAntiderivative[result_,optimal_] :=
20   If[ExpnType[result]<=ExpnType[optimal],
21     If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
```

```

22     If[LeafCount[result]<=2*LeafCount[optimal],
23         "A",
24         "B"],
25     "C"],
26 If[FreeQ[result,Integrate] && FreeQ[result,Int],
27     "C",
28     "F"]]
29
30
31 (* ::Text:: *)
32 (*The following summarizes the type number assigned an *)
33 (*expression based on the functions it involves*)
34 (*1 = rational function*)
35 (*2 = algebraic function*)
36 (*3 = elementary function*)
37 (*4 = special function*)
38 (*5 = hyperpergeometric function*)
39 (*6 = appell function*)
40 (*7 = rootsum function*)
41 (*8 = integrate function*)
42 (*9 = unknown function*)
43
44
45 ExpnType[expn_] :=
46     If[AtomQ[expn],
47         1,
48     If[ListQ[expn],
49         Max[Map[ExpnType,expn]],
50     If[Head[expn]===Power,
51         If[IntegerQ[expn[[2]]],
52             ExpnType[expn[[1]]],
53         If[Head[expn[[2]]]===Rational,
54             If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
55                 1,
56                 Max[ExpnType[expn[[1]],2]],
57             Max[ExpnType[expn[[1]],ExpnType[expn[[2]],3]],
58     If[Head[expn]===Plus || Head[expn]===Times,
59         Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
60     If[ElementaryFunctionQ[Head[expn]],
61         Max[3,ExpnType[expn[[1]]]],
62     If[SpecialFunctionQ[Head[expn]],
63         Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
64     If[HypergeometricFunctionQ[Head[expn]],
65         Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
66     If[AppellFunctionQ[Head[expn]],
67         Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
68     If[Head[expn]===RootSum,

```

```

69   Apply[Max, Append[Map[ExpnType, Apply[List, expn]], 7]],
70   If[Head[expn]===Integrate || Head[expn]===Int,
71     Apply[Max, Append[Map[ExpnType, Apply[List, expn]], 8]],
72   9]]]]]]]]]]
73
74
75 ElementaryFunctionQ[func_] :=
76   MemberQ[{
77     Exp, Log,
78     Sin, Cos, Tan, Cot, Sec, Csc,
79     ArcSin, ArcCos, ArcTan, ArcCot, ArcSec, ArcCsc,
80     Sinh, Cosh, Tanh, Coth, Sech, Csch,
81     ArcSinh, ArcCosh, ArcTanh, ArcCoth, ArcSech, ArcCsch
82   }, func]
83
84
85 SpecialFunctionQ[func_] :=
86   MemberQ[{
87     Erf, Erfc, Erfi,
88     FresnelS, FresnelC,
89     ExpIntegralE, ExpIntegralEi, LogIntegral,
90     SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
91     Gamma, LogGamma, PolyGamma,
92     Zeta, PolyLog, ProductLog,
93     EllipticF, EllipticE, EllipticPi
94   }, func]
95
96
97 HypergeometricFunctionQ[func_] :=
98   MemberQ[{Hypergeometric1F1, Hypergeometric2F1, HypergeometricPFQ}, func]
99
100
101 AppellFunctionQ[func_] :=
102   MemberQ[{AppellF1}, func]

```

## 4.0.2 Maple grading function

```

1 # File: GradeAntiderivative.mpl
2 # Original version thanks to Albert Rich emailed on 03/21/2017
3
4 #Nasser 03/22/2017 Use Maple leaf count instead since buildin
5 #Nasser 03/23/2017 missing 'ln' for ElementaryFunctionQ added
6 #Nasser 03/24/2017 corrected the check for complex result
7 #Nasser 10/27/2017 check for leafsize and do not call ExpnType()
8 #
9 #Nasser 12/22/2019 Added debug flag, added 'dilog' to special functions
10 #
11 see problem 156, file Apostol_Problems

```

```

11
12 GradeAntiderivative := proc(result,optimal)
13 local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
    debug:=false;
14
15     leaf_count_result:=leafcount(result);
16     #do NOT call ExpnType() if leaf size is too large. Recursion problem
17     if leaf_count_result > 500000 then
18         return "B";
19     fi;
20
21     leaf_count_optimal:=leafcount(optimal);
22
23     ExpnType_result:=ExpnType(result);
24     ExpnType_optimal:=ExpnType(optimal);
25
26     if debug then
27         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
    ExpnType_optimal);
28     fi;
29
30 # If result and optimal are mathematical expressions,
31 # GradeAntiderivative[result,optimal] returns
32 #   "F" if the result fails to integrate an expression that
33 #     is integrable
34 #   "C" if result involves higher level functions than necessary
35 #   "B" if result is more than twice the size of the optimal
36 #     antiderivative
37 #   "A" if result can be considered optimal
38
39 #This check below actually is not needed, since I only
40 #call this grading only for passed integrals. i.e. I check
41 #for "F" before calling this. But no harm of keeping it here.
42 #just in case.
43
44
45 if not type(result,freeof('int')) then
46     return "F";
47 end if;
48
49
50 if ExpnType_result<=ExpnType_optimal then
51     if debug then
52         print("ExpnType_result<=ExpnType_optimal");
53     fi;
54     if is_contains_complex(result) then
55         if is_contains_complex(optimal) then

```

```

56     if debug then
57         print("both result and optimal complex");
58     fi;
59     #both result and optimal complex
60     if leaf_count_result<=2*leaf_count_optimal then
61         return "A";
62     else
63         return "B";
64     end if
65 else #result contains complex but optimal is not
66     if debug then
67         print("result contains complex but optimal is not");
68     fi;
69     return "C";
70 end if
71 else # result do not contain complex
72     # this assumes optimal do not as well
73     if debug then
74         print("result do not contain complex, this assumes optimal do not
as well");
75     fi;
76     if leaf_count_result<=2*leaf_count_optimal then
77         if debug then
78             print("leaf_count_result<=2*leaf_count_optimal");
79         fi;
80         return "A";
81     else
82         if debug then
83             print("leaf_count_result>2*leaf_count_optimal");
84         fi;
85         return "B";
86     end if
87 end if
88 else #ExpnType(result) > ExpnType(optimal)
89     if debug then
90         print("ExpnType(result) > ExpnType(optimal)");
91     fi;
92     return "C";
93 end if
94
95 end proc:
96
97 #
98 # is_contains_complex(result)
99 # takes expressions and returns true if it contains "I" else false
100 #
101 #Nasser 032417

```

```

102 is_contains_complex:= proc(expression)
103   return (has(expression,I));
104 end proc:
105
106 # The following summarizes the type number assigned an expression
107 # based on the functions it involves
108 # 1 = rational function
109 # 2 = algebraic function
110 # 3 = elementary function
111 # 4 = special function
112 # 5 = hyperpergeometric function
113 # 6 = appell function
114 # 7 = rootsum function
115 # 8 = integrate function
116 # 9 = unknown function
117
118 ExpnType := proc(expn)
119   if type(expn,'atomic') then
120     1
121   elif type(expn,'list') then
122     apply(max,map(ExpnType,expn))
123   elif type(expn,'sqrt') then
124     if type(op(1,expn),'rational') then
125       1
126     else
127       max(2,ExpnType(op(1,expn)))
128     end if
129   elif type(expn,'^^') then
130     if type(op(2,expn),'integer') then
131       ExpnType(op(1,expn))
132     elif type(op(2,expn),'rational') then
133       if type(op(1,expn),'rational') then
134         1
135       else
136         max(2,ExpnType(op(1,expn)))
137       end if
138     else
139       max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
140     end if
141   elif type(expn,'^+') or type(expn,'^*') then
142     max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
143   elif ElementaryFunctionQ(op(0,expn)) then
144     max(3,ExpnType(op(1,expn)))
145   elif SpecialFunctionQ(op(0,expn)) then
146     max(4,apply(max,map(ExpnType,[op(expn)])))
147   elif HypergeometricFunctionQ(op(0,expn)) then
148     max(5,apply(max,map(ExpnType,[op(expn)])))

```

```

149   elif AppellFunctionQ(op(0,expn)) then
150       max(6,apply(max,map(ExpnType,[op(expn)])))
151   elif op(0,expn)='int' then
152       max(8,apply(max,map(ExpnType,[op(expn)]))) else
153       9
154   end if
155 end proc:
156
157
158 ElementaryFunctionQ := proc(func)
159     member(func,[
160         exp,log,ln,
161         sin,cos,tan,cot,sec,csc,
162         arcsin,arccos,arctan,arccot,arcsec,arccsc,
163         sinh,cosh,tanh,coth,sech,csch,
164         arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
165 end proc:
166
167 SpecialFunctionQ := proc(func)
168     member(func,[
169         erf,erfc,erfi,
170         FresnelS,FresnelC,
171         Ei,Ei,Li,Si,Ci,Shi,Chi,
172         GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
173         EllipticF,EllipticE,EllipticPi])
174 end proc:
175
176 HypergeometricFunctionQ := proc(func)
177     member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
178 end proc:
179
180 AppellFunctionQ := proc(func)
181     member(func,[AppellF1])
182 end proc:
183
184 # u is a sum or product. rest(u) returns all but the
185 # first term or factor of u.
186 rest := proc(u) local v;
187     if nops(u)=2 then
188         op(2,u)
189     else
190         apply(op(0,u),op(2..nops(u),u))
191     end if
192 end proc:
193
194 #leafcount(u) returns the number of nodes in u.
195 #Nasser 3/23/17 Replaced by build-in leafCount from package in Maple

```

```

196 leafcount := proc(u)
197     MmaTranslator[Mma][LeafCount](u);
198 end proc:

```

### 4.0.3 Sympy grading function

```

1 #Dec 24, 2019. Nasser M. Abbasi:
2 #           Port of original Maple grading function by
3 #           Albert Rich to use with Sympy/Python
4 #Dec 27, 2019 Nasser. Added `RootSum`. See problem 177, Timofeev file
5 #           added 'exp_polar'
6 from sympy import *
7
8 def leaf_count(expr):
9     #sympy do not have leaf count function. This is approximation
10    return round(1.7*count_ops(expr))
11
12 def is_sqrt(expr):
13     if isinstance(expr,Pow):
14         if expr.args[1] == Rational(1,2):
15             return True
16         else:
17             return False
18     else:
19         return False
20
21 def is_elementary_function(func):
22     return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
23                   asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
24                   asinh,acosh,atanh,acoth,asech,acsch
25                   ]
26
27 def is_special_function(func):
28     return func in [ erf,erfc,erfi,
29                   fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
30                   gamma,loggamma,digamma,zeta,polylog,LambertW,
31                   elliptic_f,elliptic_e,elliptic_pi,exp_polar
32                   ]
33
34 def is_hypergeometric_function(func):
35     return func in [hyper]
36
37 def is_appell_function(func):
38     return func in [appellf1]
39
40 def is_atom(expn):
41     try:

```



```

42     if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
43         return True
44     else:
45         return False
46
47     except AttributeError as error:
48         return False
49
50 def expnType(expn):
51     debug=False
52     if debug:
53         print("expn=",expn,"type(expn)=",type(expn))
54
55     if is_atom(expn):
56         return 1
57     elif isinstance(expn,list):
58         return max(map(expnType, expn)) #apply(max,map(ExpnType,expn))
59     elif is_sqrt(expn):
60         if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
61             return 1
62         else:
63             return max(2,expnType(expn.args[0])) #max(2,ExpnType(op(1,expn)))
64     elif isinstance(expn,Pow): #type(expn,'^^')
65         if isinstance(expn.args[1],Integer): #type(op(2,expn),'integer')
66             return expnType(expn.args[0]) #ExpnType(op(1,expn))
67         elif isinstance(expn.args[1],Rational): #type(op(2,expn),'rational')
68             if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
69                 return 1
70             else:
71                 return max(2,expnType(expn.args[0])) #max(2,ExpnType(op(1,expn)
72 ))
73     else:
74         return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
75 ExpnType(op(1,expn)),ExpnType(op(2,expn)))
76     elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'+' or type
77 (expn,'*')
78         m1 = expnType(expn.args[0])
79         m2 = expnType(list(expn.args[1:]))
80         return max(m1,m2) #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
81     elif is_elementary_function(expn.func): #ElementaryFunctionQ(op(0,expn))
82         return max(3,expnType(expn.args[0])) #max(3,ExpnType(op(1,expn)))
83     elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
84         m1 = max(map(expnType, list(expn.args)))
85         return max(4,m1) #max(4,apply(max,map(ExpnType,[op(expn)])))
86     elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
87 expn))
88         m1 = max(map(expnType, list(expn.args)))

```

```

85     return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
86 elif is_appell_function(expn.func):
87     m1 = max(map(expnType, list(expn.args)))
88     return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
89 elif isinstance(expn,RootSum):
90     m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,
Apply[List,expn]],7]],
91     return max(7,m1)
92 elif str(expn).find("Integral") != -1:
93     m1 = max(map(expnType, list(expn.args)))
94     return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
95 else:
96     return 9
97
98 #main function
99 def grade_antiderivative(result,optimal):
100
101     leaf_count_result  = leaf_count(result)
102     leaf_count_optimal = leaf_count(optimal)
103
104     expnType_result  = expnType(result)
105     expnType_optimal = expnType(optimal)
106
107     if str(result).find("Integral") != -1:
108         return "F"
109
110     if expnType_result <= expnType_optimal:
111         if result.has(I):
112             if optimal.has(I): #both result and optimal complex
113                 if leaf_count_result <= 2*leaf_count_optimal:
114                     return "A"
115                 else:
116                     return "B"
117             else: #result contains complex but optimal is not
118                 return "C"
119         else: # result do not contain complex, this assumes optimal do not as
well
120             if leaf_count_result <= 2*leaf_count_optimal:
121                 return "A"
122             else:
123                 return "B"
124     else:
125         return "C"

```

## 4.0.4 SageMath grading function

1 #Dec 24, 2019. Nasser: Ported original Maple grading function by

```

2 #           Albert Rich to use with Sagemath. This is used to
3 #           grade Fracas, Giac and Maxima results.
4 #Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
5 #           'arctan2','floor','abs','log_integral'
6
7 from sage.all import *
8 from sage.symbolic.operators import add_vararg, mul_vararg
9
10 def tree(expr):
11     debug=False;
12     if debug:
13         print ("Enter tree(expr), expr=",expr)
14         print ("expr.operator()=",expr.operator())
15         print ("expr.operands()=",expr.operands())
16         print ("map(tree, expr.operands()=",map(tree, expr.operands()))
17
18     if expr.operator() is None:
19         return expr
20     else:
21         return [expr.operator()+list(map(tree, expr.operands()))
22
23 def leaf_count(anti):
24     debug=False;
25
26     if debug: print ("Enter leaf_count, anti=", anti, " len(anti)=", len(anti))
27
28     if len(anti) == 0: #special check for optimal being 0 for some test cases.
29         if debug: print ("len(anti) == 0")
30         return 1
31     else:
32         if debug: print ("round(1.35*len(flatten(tree(anti))))=",round(1.35*len(
33         flatten(tree(anti))))
34         return round(1.35*len(flatten(tree(anti)))) #fudge factor
35         #since this estimate of leaf count is bit lower than
36         #what it should be compared to Mathematica's
37
38 def is_sqrt(expr):
39     debug=False;
40     if expr.operator() == operator.pow: #isinstance(expr,Pow):
41         if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
42             if debug: print ("expr is sqrt")
43             return True
44         else:
45             return False
46     else:
47         return False

```

```

48 def is_elementary_function(func):
49     debug = False
50
51     m = func.name() in ['exp','log','ln',
52         'sin','cos','tan','cot','sec','csc',
53         'arcsin','arccos','arctan','arccot','arcsec','arccsc',
54         'sinh','cosh','tanh','coth','sech','csch',
55         'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
56         'arctan2','floor','abs'
57     ]
58     if debug:
59         if m:
60             print ("func ", func , " is elementary_function")
61         else:
62             print ("func ", func , " is NOT elementary_function")
63
64
65     return m
66
67 def is_special_function(func):
68     debug = False
69
70     if debug: print ("type(func)=", type(func))
71
72     m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
73         'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
74     sinh_integral'
75         'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
76         'polylog','lambert_w','elliptic_f','elliptic_e',
77         'elliptic_pi','exp_integral_e','log_integral']
78
79     if debug:
80         print ("m=",m)
81         if m:
82             print ("func ", func ," is special_function")
83         else:
84             print ("func ", func ," is NOT special_function")
85
86     return m
87
88
89 def is_hypergeometric_function(func):
90     return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U
91     ']
92
93 def is_appell_function(func):

```

```

93     return func.name() in ['hypergeometric']    #[appellf1] can't find this in
          sagemath
94
95 def is_atom(expn):
96
97     #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-type-in-maple/
98     try:
99         if expn.parent() is SR:
100             return expn.operator() is None
101         if expn.parent() in (ZZ, QQ, AA, QQbar):
102             return expn in expn.parent() # Should always return True
103         if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
104             return expn in expn.parent().base_ring() or expn in expn.parent().
          gens()
105         return False
106
107     except AttributeError as error:
108         return False
109
110
111 def expnType(expn):
112     debug=False
113
114     if debug:
115         print(">>>>Enter expnType, expn=", expn)
116         print(">>>>is_atom(expn)=", is_atom(expn))
117
118     if is_atom(expn):
119         return 1
120     elif type(expn)==list:    #isinstance(expn,list):
121         return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
122     elif is_sqrt(expn):
123         if type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],
          Rational):
124             return 1
125         else:
126             return max(2,expnType(expn.operands()[0]))    #max(2,expnType(expn.
          args[0]))
127     elif expn.operator() == operator.pow:    #isinstance(expn,Pow)
128         if type(expn.operands()[1])==Integer:    #isinstance(expn.args[1],Integer)
129             return expnType(expn.operands()[0])    #expnType(expn.args[0])
130         elif type(expn.operands()[1])==Rational:    #isinstance(expn.args[1],
          Rational)
131             if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],
          Rational)
132                 return 1

```

```

133         else:
134             return max(2,expnType(expn.operands()[0])) #max(2,expnType(expn.
args[0]))
135         else:
136             return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
[1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
137         elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
isinstance(expn,Add) or isinstance(expn,Mul)
138             m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
139             m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
140             return max(m1,m2) #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
141         elif is_elementary_function(expn.operator()): #is_elementary_function(expn.
func)
142             return max(3,expnType(expn.operands()[0]))
143         elif is_special_function(expn.operator()): #is_special_function(expn.func)
144             m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))
145             return max(4,m1) #max(4,m1)
146         elif is_hypergeometric_function(expn.operator()): #
is_hypergeometric_function(expn.func)
147             m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))
148             return max(5,m1) #max(5,m1)
149         elif is_appell_function(expn.operator()):
150             m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))
151             return max(6,m1) #max(6,m1)
152         elif str(expn).find("Integral") != -1: #this will never happen, since it
153             #is checked before calling the grading function that is passed.
154             #but kept it here.
155             m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))
156             return max(8,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
157         else:
158             return 9
159
160 #main function
161 def grade_antiderivative(result,optimal):
162     debug = False;
163
164     if debug: print ("Enter grade_antiderivative for sagemath")
165
166     leaf_count_result = leaf_count(result)
167     leaf_count_optimal = leaf_count(optimal)
168
169     if debug: print ("leaf_count_result=", leaf_count_result, "
leaf_count_optimal=",leaf_count_optimal)

```

```
170
171
172     expnType_result = expnType(result)
173     expnType_optimal = expnType(optimal)
174
175     if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
176                     expnType_optimal)
177
178     if expnType_result <= expnType_optimal:
179         if result.has(I):
180             if optimal.has(I): #both result and optimal complex
181                 if leaf_count_result <= 2*leaf_count_optimal:
182                     return "A"
183             else:
184                 return "B"
185         else: #result contains complex but optimal is not
186             return "C"
187     else: # result do not contain complex, this assumes optimal do not as
188         well
189         if leaf_count_result <= 2*leaf_count_optimal:
190             return "A"
191         else:
192             return "B"
193     else:
194         return "C"
```